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### Solutions Manual for

### Machine Elements in Mechanical Design, 5th ed.
### By: Robert L. Mott


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**MDESIGN Software** - Its application to

***Machine Elements in Mechanical Design, 5th edition***

**By: Robert L. Mott**

**Published by: Pearson Education - Prentice-Hall**

## General Description of the Software

A powerful computer-aided calculation software package called MDESIGN is included with each purchase of this book. A total of 66 modules divided among 15 categories make up the complete package, outlined in the Introduction to the software. The software is an updated version of one that first appeared in the 4th edition of *Machine Elements in Mechanical Design.*

The software was created by TEDATA, GMBH, a German company that has a long history of producing such software for professional use throughout Europe and many other parts of the world. The version included with this book has 32 modules that have general applicability or that were produced specially for the book, following the analysis and design methods presented in the book, most of which are patterned on methods and standards commonly used in the United States. The other 34 modules were developed primarily for use by professionals and conform to common practice in Europe as represented by DIN standards, VDI publications, and the popular reference book on machine elements often called Roloff/Matek Machine Elements, written by Herbert Wittel, Dieter Muhs, Dieter Jannasch, and Joachim Vobiek and published by Vieweg+Teubner, Wiesbasen, Germany, 2009.

***Identification of the two types of MDESIGN modules listed in the Introduction to the software:***

- The list identifies the 32 modules most closely aligned with this book by the symbol ⊕. The chapter and section of the book most relevant to each module is indicated. The text includes several sections where a special icon appears to indicate that the use of MDESIGN is pertinent to that topic.
- The other 34 modules are denoted by the symbol ⊗ and are more closely aligned to European standards. They may use terminology, notations, and symbols unfamiliar to those experienced primarily in U.S.-based practices.

It is important to note that the inclusion of this extensive and diverse set of modules can be useful to users of this book throughout the world as a means of expanding the breadth of knowledge of design practices in different regions. Furthermore, many users of the book are likely to engage in projects with industrial companies, design services, consultants, and university faculty members from many parts of the world and having these modules available can aid in communicating across traditional geographic boundaries and between different technical cultures.

## Advice on Use of the MDESIGN Software

The following comments are directed primarily to those using this book as a learning tool either in college and university degree programs or in professional self-study.

The author's approach to the inclusion of calculation aids within initial learning of technical subject matter is:

- ***Users of computer software and calculation aids must have solid understanding of the relevant principles of design and stress analysis to ensure that design decisions are based on reliable foundations.***
- ***Software should be used only after mastering a given design methodology by careful study and practicing manual techniques.***
- ***Then, data with known results can be applied to the software as a check on the understanding of the program's input data requirements, symbols and notation used, limits on the range of acceptable data, and analysis methods.***
- ***Only then should users rely on implementation of design decisions based on output results from the software.***

**General HELP for Running MDESIGN**

An extensive 123-page help file can be accessed from the main menu ribbon. Particular attention should be paid to the Graphical User Interface section on pages 33-39 for those few modules that permit graphical data input.

**Recommended Primary Uses for MDESIGN Software with this Book**

Upon launching the MDESIGN software package, reading the Introduction, and opening the software, the left side of the initial screen will include the list of 15 categories of modules. Each category name is preceded by a plus-sign (+) that, when selected, yields the list of modules in that category. Double-clicking will open any selected module. Alternatively, you can right click and select Open.

The 32 modules most closely aligned with the presentations in the book, identified by the symbol ⊕ in the Introduction, are obviously those that should be considered first for incorporation into courses and individual study. Pertinent sections of the book for which these modules may be useful are indicated by the graphic symbol in the left margin.

Particularly for design projects and where multiple trials for design decisions are to be expected and where the large catalogs of data in MDESIGN can be accessed, the following modules enable learners to try many options in a short amount of time after learning the basic fundamentals. The following modules serve well these purposes.

| | | |
|---|---|---|
| Beam Calculations | Column Analysis | ISO Fit System |
| Statically determinate beams | Column Design | Parallel Keys |
| Statically indeterminate beams | Ball and Roller Bearings | V-belts |
| Helical Compression Springs | Plain Surface/Journal Bearings | Synchronous Belts |
| Helical Extension Springs | Clutches and Brakes (5 modules) | Roller Chains |
| Helical Torsion Springs | Combined Stresses/Mohr's Circle | Shafts-U.S. Standards |

Certainly, in an academic learning situation, instructors must enforce expectations on when and where use of the MDESIGN software is accepted, expected, or prohibited.

The process for using any module should be as follows:

1. Open the relevant module and read the General Text Help screen in the lower left part of the opening page. This outlines the basic functions of the module, shows the technical bases for the analyses performed, and identifies relevant references, terms, and symbols. Be aware that for some modules, the text help has been translated from the original German language and the result may not be in adequate standard English.
2. Use the pull-down menu on that screen to peruse what other textual aids are included. These often elaborate on design approaches by,
    a. Explaining unfamiliar terms
    b. Stating typical units for input data or results
    c. Setting acceptable limits on values of certain variables
    d. Providing tables of data from which some input data must be selected by the user
3. Observe the graphic aids in the lower right of the opening page, again scrolling among all available topics. Some of these can be accessed directly from the Input page.
4. Peruse the data required for the input screen. Open any available help icons for text, data, or "choice" options to determine requirements or available options.
5. Under the Tools tab on the main menu ribbon, select the pull down menu on the Measures System icon and select U.S. System, Metric System, or All Systems. These choices set the primary units in which data are to be entered and for which output results will be shown. In any case, you have the option to change units for any item by passing the cursor over the unit and pulling down the local menu.
    a. Pay special attention to the precision of results data shown on the Output pages. At times, only one or two significant figures of accuracy are displayed and that may not be adequate for your use. You may be able to select a smaller unit that will show higher precision. For example if a length of diameter measurement shows 6.0 in, selecting the *mil* unit (0.001 – thousandths) may show 6075 mils, indicating 6.075 in.
    b. Note that the standard European Metric system uses the comma rather than the decimal point for separating digits in floating point calculations. For example, In the U.S. system a number may be 12.456; in Metric it will appear 12,456.
6. When all data are entered, select the Calculation tab on the main menu or, simply, press the F10 key on the keyboard to initiate the module's calculations. Note the following:
    a. In some modules, intermediate data entry screens pop up for which some initial calculations have generated data on which subsequent design decisions are based. You are asked then to make the final decision before the complete results are found.

b. After a short time for completing the calculations (typically only a few seconds), select the Output Page option at the upper left of the data page to see results.

c. Carefully evaluate results for reasonableness and check that proper units have been selected.

d. Some modules include internal checks on output to assess acceptability, with unacceptable results shown in RED. If that occurs, you must return to the Input Page and change design decisions, recalculate results, and re-evaluate their acceptability.

e. Consider the degree of optimization of any particular result and, where possible, make adjustments to hone into a more optimum design. ***It is typical for mechanical design analyses to require proposing and analyzing several alternative solutions to achieve the most efficient and effective design.***

7. When the final result has been found, use the Print command to print out both the Input Page and the Output Page. It is essential for an instructor or a client to see complete records of the data used along with the results.

## Descriptions of Selected Modules

The following sections describe certain topics from this book for which the use of MDESIGN is particularly pertinent. Suggestions for applying the modules are also given, but practice with known data is a good way to gain skill at entering data and seeking optimum results. Use of data taken from Example Problems from the text is highly recommended. However, there may be slight differences between results in book problems and those from MDESIGN because of rounding of numbers and slightly modified ways of making calculations.

### *Combined Stress and Mohr's Circle* – Chapter 4

**Module group: Shafts, Axles, and Beams.** This module solves problems of the type featured in Sections 4-4 and 4-5 of the text in which data for applied normal and shear stresses in one plane are known and the program computes the maximum principal stress, the minimum principal stress, and the maximum shear stress. The complete Mohr's circle and the pertinent stress elements are also developed and included in the output.

### *Columns* – Chapter 6 – Two modules: Column Analysis and Column Design

**Module group: Shafts, Axles, and Beams.** These modules follow closely the methodology used in the text for applying the Euler formula for long columns and the J. B. Johnson formula for short columns to either analysis or design problems. Loading can be either central or eccentric

and both straight and crooked columns can be analyzed. Problems of the types shown in the text in Example Problems 6-1 to 6-6

***Belt Drives and Chain Drives* – Chapter 7 – Three modules: V-Belts, U.S. Standards, Synchronous Belts, and Roller Chains ISO 10823**

**Module Group: Belt-, Chain Drives:** These modules are pertinent to Sections 7-4 to 7-7. Each module contains large databases of commercially available products that can be selected for designs of power transmission drives. It is recommended that the use of this module be combined with student use of actual online catalogs of belts, sheaves, chains, and sprockets to specify part numbers and model numbers that can be specified for purchase.

- **Comments: V-Belts, U.S. Standards:** This module is pertinent to Section 7-4 in the text. Data entry and calculations are modeled after the method demonstrated in the book in Example Problem 7-1. Users are given options for selecting the belt size (3V, 5V, or 8V) and Figure 7-9 is used by the program to suggest a choice, which may be overridden by selecting another size. Selections for 'Driver' and 'Driven machine' types are identical to those used in Table 7-1 in the text. A design value for center distance is selected by the user after being given nominal minimum and maximum values. Then the user is presented with a set of optional combination of sheave sizes from which one must be chosen. That design is then evaluated and output data show the results. Iterations can be done easily by restarting the calculation and making modified selections.

- **Comments: Synchronous Belts:** This module approximates the methodology described in Section 7-5 of the text. Belts of the styles shown in Figure 7-18, both metric and U.S. sizes, are selected and analyzed by the program. Most data are shown in metric units for either style, although pull-down menus permit some features to be shown in U.S. units. The program is quite powerful, allowing multiple pulleys to be driven by one belt in serpentine arrangement. Most applications in this book will include two and only two pulleys. Data are input in tabular style and some practice may be required to become familiar with the details. It is recommended that the center of the driver pulley (No. 1) be positioned at $x = 0$, $y = 0$ on the coordinate system shown in the graphic aids. Then position the center of the driven pulley (No. 2; called a 'jockey pulley' in the module) at $x$ = desired center distance and $y = 0$. Entering 0° for the 'Displacement angle' will place the driven pulley to the right of the driver pulley. Enter 0 values for the 'max effective $\phi$' for each pulley and select 'within' for the 'Location' because the pulleys are positioned within the belt. For loads, it is normal to specify the power input to pulley 1 and the power output from pulley 2 to be equal and positive numbers. For U.S. data, select 'hp US' as the unit for power. Then leave the 'Torque' and 'Tangential force' entries as zero. The 'Load Factor' should be obtained from Table 7-1 (the same table as used for V-Belt drives). Then start the calculation. You will be presented screens offering choices for belt style, belt length, and belt width and you would normally select the nominal value offered for initial trials. Other values can be tried for subsequent trials until a satisfactory design is achieved.

- **Comments: Chain Drives – Roller Chains ISO 10823:** This module is pertinent to Section 7-7 of the text. It uses an ISO standard approach to the selection of chain drives that produces recommend chain sizes from ISO 606 as shown in Table 7-6 of the text. These sizes are identical to standard U.S. sizes for chain pitch as shown in the table. The performance analysis may differ slightly from the methods shown in the text. When using U.S. units for input data, ensure that Power is in 'hp US' and that units for other pertinent input and output data are expressed in the desired units, typically rpm for rotational speed and inches for dimensions. Selecting first the input page option: 'Selection and calculation of one chain' will result in a selection table being presented with options for different chain pitches and number of strands. Each design will list the rated power of the drive and the 'Utilization' (the value of the required corrected power to the rated power of the design, expressed as a percentage). The 'Application factor to allow for the operating conditions, $f_1$, is similar to the 'Service factors' shown in the book in Table 7-10. It is recommended that users select the option 'Allow adjustment of factors $f_1$ and $f_2$' (Yes). Then manually enter the service factor for $f_1$ and set $f_2 = 0$. This will match most closely to the methods used in the text. Common U.S. practice generally does not use a 'Factor for number of teeth on drive sprocket' ($f_2$).

## *Keys and Keyseats* – Chapter 11

**Module group: Shafts-Hub Connections:** This module performs the calculations as described in Section 11-4 in the text, similar to Example Problem 11-1. Also included are calculations for the dimensioning variables *Y, S,* and *T* from Figure 11-2. Users enter the shaft diameter and either the torque of the combination of power and rotational speed. The yield strength of the key, shaft, and hub are entered or can be selected from a list of possible materials. After specifying the design factor to be used, the module produces the calculated results.

## *Rolling Contact Bearings* – Chapter 14

**Module group: Roller Bearings - Ball and Roller Bearings:** The primary features of this module and its use are described in Section 14-11 of the text. The module provides access to a prominent manufacturer's entire catalog for many types of bearings.

## *Plain Surface Bearings* – Chapter 16

**Module group: Journals – Plain Surface/Journals, US Standards:** This module is patterned after Section 16-5 of the text – Design of Boundary-Lubricated Bearings. Problems of the types shown in Example Problems 16-1 and 16-2 are solved using this module. After entering data for radial load on the bearing, the rotational speed of the shaft, the selected trial value for shaft diameter, and a trial value of the L/D ratio, the program calculates the actual length, *L*, the bearing pressure, *p*, the surface speed, *V*, and the *pV value*, and the design value of *pV*

(23calculated $pV$). It then searches a modest table of possible materials similar to that shown in Table 16-1 of the text for one that has a rated $pV$ value closest to but more than the design $pV$ value. The nominal diametral clearance for the bearing is also computed, using data shown in Figure 16-4 of the text.

- ***Please note that the label for the Diameter value on the Input page of this module uses the term, 'Nominal minimum diameter of the journal, $D_{min}$'. That is not the same as the minimum acceptable shaft diameter (based on shaft stress analysis). It should be the actual trial diameter for the shaft that is selected by the user and that must be graeater than the minimum acceptable value based on strength.***
- A recommended use for this module is as an aid in selecting commercially available sizes and materials for plain surface bearings from vendors such as those listed in the Internet sites for Chapter 16, particularly sites: *2 – Thomson Engineering & Polymers; 3 – Saint-Gobain Performance Plastics; 4 – GGB Bearing Technology; 5 – Graphite Metallizing Corporation; 6 – Beemer Precision, Inc.;* and *7 – Bunting Bearings Corporation.* These sites offer catalog data for their products and they list the design $pV$ values for the various materials from which the bearings are made.
- For use of catalog data, users should select a preliminary value for the internal diameter and length for a particular bearing and make note of the design $pV$ value for the selected material. Then compute the actual $L/D$ ratio. Then enter the dimensions into the MDESIGN module ($D$ and $L/D$), along with other given data for bearing load and rotational speed. The computed required "Design value of $pV$ factor" should then be compared with the catalog-listed value for the selected material. The suggested material given on the module output page should be ignored. Iterations are easily and quickly done by trying other sizes until an optimum design is identified.

**MACHINE ELEMENTS IN MECHANICAL DESIGN**
**Fifth Edition**

**Robert L. Mott**

**Prentice-Hall Publishing Company**

**Description of Spreadsheets Included with the Instructors Manual**

**Introduction**

The Instructors Manual for this book contains a set of 26 computational aids that are keyed to the book. The files are written as Microsoft Excel spreadsheets.

Many of the spreadsheets appear in the text. Others were prepared to produce solutions for the Solutions Manual. The given spreadsheets include data and results from certain figures in the text, from certain example problems, or for certain problems from the end of chapters containing the analysis and design procedures featured in the programs.

The following sections give brief descriptions of each spreadsheet. Many are discussed in the text in more extensive detail. It is expected that you will verify all of the elements of each spreadsheet before using them for solutions to specific problems.

**Using the Spreadsheets:**

- ***It is recommended that you maintain the given spreadsheets as they initially appear on the disk, considering them to be master copies.***
- ***To use a program for solving other problems, call it up in Excel and use the "Save as" command to give it a different name.***
- ***For instance, the original program called* Column Analysis *should be considered the master. Use "Save as" and call it, for example,* Column Analysis – Working. *Then use that version for general problem solving.***

You should study the concepts and the solution techniques for each type of problem before using the spreadsheets. You should work sample problems by hand first. Then enter the appropriate data into the spreadsheet to verify the solution. In most spreadsheets in the text, the data that need to be entered are identified by gray-shaded areas and by italic type.

**Descriptions of Spreadsheets**

The descriptions are given here in the order that the subjects for the spreadsheets are covered in the text.

***Column Analysis:*** Chapter 6. Analyzes straight columns of uniform cross section to determine the critical buckling load and the allowable load. The spreadsheet shows results for Example Problem 6-1. U.S. Customary units are used. A description is given in Section 6-8. The process is essentially the same as that shown in the flow chart of Figure 6-4. Note that a short macro program in Visual Basic is used to decide whether the column is *long* (Euler) or *short* (J. B. Johnson) and to complete the calculation of the critical buckling load. Be sure that your Excel program enables macros.

***Column Analysis SI:*** Chapter 6. Same as ***Column Analysis:*** except SI units are used. The solution to Example Problem 6-2 is given as an example for data entry.

***Circular Column Analysis:*** Chapter 6. Special version of ***Column Analysis*** in which the geometric properties of a column with a solid circular cross section are computed when the diameter is input. The spreadsheet can be used as an iterative design tool to determine the required diameter of a column with a circular cross section to carry a given load. See Figure 6-14.

***Crooked Column Analysis:*** Chapter 6. Section 6-11. Analyzes the allowable load on a column of constant cross section with a given amount of crookedness. Data from Example Problem 6-4 are used as shown in Figure 6-16 on page 252.

***Eccentric Column Analysis:*** Chapter 6. Section 6-12. Computes the required yield strength of the material and the resulting maximum deflection of the middle of a column that is loaded eccentrically. Data from Example Problem 6-6 are used as shown in Figure 6-18.

***Chain Drive Design:*** Chapter 7. Design of roller chain drives as described in Section 7-6. User must obtain rated power data from Tables 7-7, 7-8, or 7-9 to specify a suitable chain number and number of teeth in the smaller sprocket. A service factor must be selected from Table 7-10 in the text. Data from Example Problem 7-4 are shown in the master spreadsheet.

***Gear Geometry:*** Chapter 8. Computes the geometric features of spur and helical gears using the relationships in Sections 8-4 and 8-6. Can be used for Problems 1-9 and 41-44.

***Contact Ratio-Spur Gears:*** Chapter 8. Computes the contact ratio for spur gears using the procedure shown in Section 8-4.

***Bevel Gear Geometry:*** Chapter 8. Computes the geometric features of straight bevel gears using the formulas listed in Table 8-8 in Section 8-7 and illustrated in Example Problem 8-3. Two identical programs are shown side-by-side. One shows the results of Example Problem 8-3 and the other can be used to solve any given problem.

***Wormgearing Geometry, C, VR:*** Chapter 8. Computes essential geometric features of a worm and wormgear, the center distance (*C*) between their shafts, and the velocity ratio, *VR*. Uses procedure from Section 8-9 as illustrated in Example Problem 8-4. The spreadsheet was used to complete Problems 52-57 at the end of the chapter.

***Gears VR Design:*** Chapter 8. Aids in the specification of the number of teeth in a pinion and gear to produce a specified velocity ratio. Uses a procedure similar to that shown in Section 8-11 and illustrated in Table 8-10. An integer is entered for the number of teeth in the pinion. The program computes the required approximate number of teeth in the gear to produce the given velocity ratio. The user then enters an integer for the actual number of gear teeth. The program identifies the combination of numbers of teeth that produces the minimum differential between the desired ratio and the actual ratio. The spreadsheet was used to complete Problems 62-65 at the end of the chapter.

***Spur Gear Forces:*** Chapter 9. Computes the tangential, radial, and normal forces on spur gear teeth of a given design transmitting a given power at a given pinion speed. It uses the method of Section 9-3. The spreadsheet was used to complete Problems 1-6 at the end of Chapter 9. The results for Problems 1 and 2 are shown in the master.

***Spur Gears-Design-U.S.:*** Chapter 9. Performs a complete design analysis for a pair of spur gears, including the essential geometry, tangential force, required bending stress number, and required contact stress number. All modifying factors for stress calculations as described in Sections 9-8 to 9-11 are included. The data from Example Problem 9-4 are shown in the given spreadsheet as illustrated in Figure 9-25. An extensive discussion of the spreadsheet is given in Section 9-13. A feature of the spreadsheet is the computation of the required hardness (HB) for through-hardened Grade 1 steel using the equations in Figures 9-11 and 9-12. The user can then specify suitable materials and list them at the bottom of the spreadsheet.

***Geometry Factor-I-Pitting:*** Chapter 9. Computes the value of the geometry factor, *I*, used in the calculation of contact stress for spur gears in Equation 9-23. Program uses the algorithm from Appendix A18.

***Spur Gears-Design-U.S.-With I:*** Chapter 9. Same as ***Spur Gears-Design*** except the geometry factor, *I*, is computed within the program instead of being input by the user. The program ***Geometry Factor-I-Pitting*** is integrated within ***Spur Gears-Design.*** One additional input value is needed for the pressure angle, $\phi$.

***Spur Gears-Design-SI:*** Chapter 9. Similar to ***Spur Gears-Design:*** except SI metric data are used as described in Section 9-13 and illustrated in Example Problem 9-5. Data from Example Problem 9-5 are used in the given spreadsheet.

***Spur Gears-Capacity-U.S.:*** Chapter 9. Section 9-15. Determines the power transmitting capacity of a given set of spur gears considering both bending strength and pitting resistance. The user must input the allowable bending stress and allowable contact stress based on the material specified for the pinion and the gear using Figures 9-11 and 9-12 and Table 9-5. The spreadsheet includes the computation of the required bending stress number, $s_{at}$, and contact stress number, $s_{ac}$, based on user-entered hardness (HB) for through-hardened Grade 1 steel using the equations in Figures 9-11 and 9-12. The user must transcribe these values into the spreadsheet if, in fact, this kind of material is specified.

***Plastic Gears- Design:*** Chapter 9. Completes the design of plastic gears using the procedure from Section 9-16. Data are shown for Example Problem 9-7.

***Helical Gears-Design:*** Chapter 10. Computes the forces on helical gear teeth as described in Section 10-2 and illustrated in Example Problem 10-1. Completes the design analysis for a pair of helical gears as described in Sections 10-3 to 10-5 and illustrated in Example Problem 10-3. Used for the solutions to Problems 1-11 at the end of Chapter 10.

***Helical Gears-Capacity:*** Chapter 10. Similar to ***Spur Gears-Capacity:*** with modifications for the special geometry of helical gear teeth. Used for the solutions to Problems 12 and 13 at the end of Chapter 10. The user must input the allowable bending stress and allowable contact stress based on the material specified for the pinion and the gear using Figures 9-11 and 9-12 and Table 9-5. The spreadsheet includes the computation of the required bending stress number, $s_{at}$, and contact stress number, $s_{ac}$, based on user-entered hardness (HB) for through-hardened Grade 1 steel using the equations in Figures 9-11 and 9-12. The user must transcribe these values into the spreadsheet if, in fact, this kind of material is specified.

***Bevel Gears – Design:*** Chapter 10. Computes forces and stresses on bevel gears using the methods shown in Section 10-9.

***Wormgearing – Design:*** Computes worm and wormgear geometry values, forces, and stresses for wormgearing, using methods and data from Chapter 8 (Section 8-9) and Chapter 10 (Sections 10-10, 10-11, and 10-12. The master spreadsheet uses data from Example Problems 8-4, 10-9, and 10-10.

***Keyseat Data:*** Chapter 11. Computes the data required to dimension keyseats and keyways on shaft drawings according to the information in Figure 11-2.

***Shaft Design:*** Chapter 12. Computes the minimum acceptable diameter for shafts using Equation 12-24 when both bending and torsion are present and Equation 12-16 when only vertical shearing stress is present. Requires prior analysis for torques, forces, bending moments, pertinent material strengths, modifying factors on material strength, and stress concentration factor. The program is typically applied at several selected sections of the shaft as illustrated in Design Example 12-1 in Section 12-6. If the location being analyzed has a retaining ring installed, the computed minimum shaft diameter is considered to be for the base of the ring groove. The spreadsheet computes the nominal full shaft diameter by applying a factor of 1.06 as described at the end of Section 12-4. The data used in the master spreadsheet are for one location on the shaft in Design Example 12-1 as illustrated in Figure 12-19 in Section 12-9 where the spreadsheet and its use are described.

***Force Fits:*** Chapter 13, Section 13-8. Stresses for Force Fits. Computes the pressure at the interface between mating members assembled with an interference fit (See Section 13-6.) Also computes the resulting stresses and deformations for the mating members using the procedure in Section 13-8. Data from Example Problem 13-2 are shown in the example.

***Spring Design-Method 1:*** Chapter 18, Section 18-6. The given spreadsheet uses data and the method from Example Problem 18-2 to design a safe helical compression spring for a given loading and to fit given geometrical limitations. See Figure 18-16 and the accompanying discussion.

***Spring Design-Method 2:*** Chapter 19. Similar to ***Spring Design-Method 1*** without the restriction of designing to a set of geometrical limitations. See Example Problem 18-3, Figure 18-17, and the accompanying discussion.

# CHAPTER 1
# THE NATURE OF MECHANICAL DESIGN

Problems 1 - 14 require the specification of functions and design requirements for design projects and have no unique solution.

**15.** $D = 1.75\ \text{IN.} \times 25.4\ \text{mm/IN} = \underline{44.5\ \text{mm}}$

**16.** $L = 46\ \text{FT} \times 0.3048\ \text{m/FT} = \underline{14.0\ \text{m}}$

**17.** $T = 12\,550\ \text{LB·IN} \times 0.1130\ \text{N·m/LB·IN} = \underline{1418\ \text{N·m}}$

**18.** $A = 4.12\ \text{IN}^2 \times 645.2\ \text{mm}^2/\text{IN}^2 = \underline{2658\ \text{mm}^2}$

**19.** $Z = 14.8\ \text{IN}^3 \times 1.639 \times 10^4\ \text{mm}^3/\text{IN}^3 = \underline{2.43 \times 10^5\ \text{mm}^3}$

**20.** $I = 88.0\ \text{IN}^4 \times 4.162 \times 10^5\ \text{mm}^4/\text{IN}^4 = \underline{3.66 \times 10^7\ \text{mm}^4}$

**21.** GIVEN $A_{MIN} = 750\ \text{mm}^2$; IN U.S. UNITS: $A_{MIN} = 1.162\ \text{IN}^2$
APP. 15-1; $L\ 2 \times 2 \times 3/8$, $A = 1.36\ \text{IN}^2 = 890\ \text{mm}^2$
APP 15-3; ANGLES $50 \times 100 \times 6$ AND $75 \times 75 \times 5$ HAVE $A = 864\ \text{mm}^2$

**22.** $P = 7.5\ \text{hp} \times 745.7\ \text{W/hp} = 5.59 \times 10^3\ \text{W} = \underline{5.59\ \text{kW}}$

**23.** $S_u = 127\ \text{ksi} \times 6.895\ \text{MPa/ksi} = \underline{876\ \text{MPa}}$

**24.** LET $D = 0.035\ \text{m}$; $L = 0.675\ \text{m}$; VOLUME $= V = A \times L = (\pi D^2/4) \times L$

$$V = \frac{\pi (0.035\ \text{m})^2}{4} \times 0.675\ \text{m} = 6.49 \times 10^{-4}\ \text{m}^3$$

$$\text{MASS} = \text{DENSITY} \times V = 7680\ \text{kg/m}^3 \times 6.49 \times 10^{-4}\ \text{m}^3 = 4.98\ \text{kg}$$

$$\text{WEIGHT} = m \times g = 4.98\ \text{kg} \times 9.81\ \text{m/s}^2 = 48.9\ \text{kg·m/s}^2 = \underline{48.9\ \text{N}}$$

$\boxed{25.}$ $T = 180 \text{ LB·IN} \times 0.1130 \text{ N·m/LB·IN} = \underline{20.3 \text{ N·m}}$

$\theta = 35° \times \pi \text{ RAD}/180° = \underline{0.611 \text{ RAD.}}$

$\text{SCALE} = T/\theta = 180 \text{ LB·IN}/35° = \underline{5.14 \text{ LB·IN/DEGREE}}$

$\text{SCALE} = T/\theta = 20.3 \text{ N·m}/0.611 \text{ RAD.} = \underline{33.3 \text{ N·m/RAD.}}$

$\boxed{26.}$ ENERGY = POWER × TIME

$$E = 12.5 \text{ hp} \times \frac{16 \text{ h}}{\text{DAY}} \times \frac{5 \text{ DAYS}}{\text{WEEK}} \times \frac{52 \text{ WKS}}{\text{YEAR}} \times \frac{550 \text{ FT·LB}}{\text{S·hp}} \times \frac{3600 \text{ S}}{\text{h}}$$

$E = \underline{1.03 \times 10^{11} \text{ FT·LB/YEAR}}$

$$E = 1.03 \times 10^{11} \frac{\text{FT·LB}}{\text{YEAR}} \times \frac{1.356 \text{ J}}{\text{FT·LB}} \times \frac{1.0 \text{ N·m}}{\text{J}} \times \frac{1.0 \text{ W}}{\text{N·m/S}} \times \frac{1 \text{ h}}{3600 \text{ S}}$$

$E = 38.8 \times 10^{6} \text{ W·h/YEAR} = \underline{38.8 \text{ MW·h/YEAR}}$

$\boxed{27.}$ $\text{VISCOSITY} = \mu = 3.75 \text{ REYN} \times \frac{1.0 \text{ LB·S}}{\text{IN}^2 \text{·REYN}} \times \frac{144 \text{ IN}^2}{\text{FT}^2} = \underline{540 \frac{\text{LB·S}}{\text{FT}^2}}$

$$\mu = 3.75 \frac{\text{LB·S}}{\text{IN}^2} \times \frac{4.448 \text{ N}}{\text{LB}} \times \frac{1.0 \text{ IN}^2}{645.2 \text{ mm}^2} \times \frac{10^6 \text{ mm}^2}{\text{m}^2} = \underline{25.9 \times 10^3 \frac{\text{N·S}}{\text{m}^2}}$$

$\boxed{28.}$ $\text{LIFE} = \frac{1750 \text{ REV}}{\text{MIN}} \times \frac{24 \text{ h}}{\text{DAY}} \times \frac{60 \text{ MIN.}}{\text{h}} \times \frac{365 \text{ DAYS}}{\text{YEAR}} \times 5 \text{ YEARS}$

$\text{LIFE} = \underline{4.60 \times 10^9 \text{ REVOLUTIONS}}$

# CHAPTER 2
# MATERIALS IN MECHANICAL DESIGN

1. Ultimate tensile strength is the apparent stress at the peak of the stress-strain curve.

2. Yield point is the value of the apparent stress from the stress-strain curve at which there is a large increase in strain with no increase in stress. It is the point where the stress-strain curve exhibits a horizontal slope.

3. Yield strength is the apparent stress from the stress-strain curve at which there is a large increase in strain with little increase in stress for materials that do not exhibit a yield point. The offset method is used by drawing a line parallel to the straight part of the stress-strain curve through a value of 0.2% on the strain axis.

4. Many low alloy steels exhibit a yield point.

5. The proportional limit is the apparent stress on the stress-strain curve at which the curve deviates from a straight line. At this value, the material is usually still elastic. The elastic limit is the apparent stress at which the material is deformed plastically and will not return to its original size and shape.

6. Hooke's law applies to that portion of the stress-strain curve that is a straight line for which stress is proportional to strain.

7. The modulus of elasticity is a measure of the stiffness of a material.

8. The percent elongation is a measure of the ductility of a material.

9. The material is not ductile. Materials having a percent elongation greater than 5% are considered to be ductile.

10. Poisson's ratio is the ratio of the lateral strain in a material to the axial strain when subjected to a tensile load.

11. From EQ. 2-5 $G = E/[2(1+\nu)] = (114 \text{ GPa})/[2(1+0.33)]$

$G = 42.9 \text{ GPa}$

12. Hardness = 52.8 HRC (Approximate; Appendix 17)

13. Tensile strength = 235 ksi (Approximate; Appendix 17)

**14.-17. Errors in given statements:**

14. A hardness of HB 750 is extremely hard, characteristic of the hardest steels in the as-quenched or surface hardened condition. Appendix 3 shows annealed steels to have hardness values in the approximate range of HB 120 to 230.

15. Hardness on the HRB scale is normally limited to HRB 100.

16. Hardness on the HRC scale is normally no lower than HRC 20.

17. The relationship between hardness and tensile strength is only valid for steels.

18. Charpy and Izod tests measure impact strength.

19. Iron and carbon. Other elements are often present.

20. In addition to iron and carbon SAE 4340 steel contains nickel, chromium, and molybdenum. (Table 2-8)

21. Approximately 0.40% carbon in SAE 4340 steel.

22. Low-carbon: Less than 0.3%
Medium-carbon: 0.30% to 0.50%
High-carbon: 0.50% to 0.95%

23. Typically a bearing steel contains 1.0% carbon.

24. Lead is added to SAE 12L13 steel to improve machinability.

25. Shafts are often made from SAE 1040, 4140, 4640, 5150, 6150, and 8650 steels. (Table 2-9)

26. Gears are often made from SAE 1040, 4140, 4340, 4640, 5150, 6150, and 8650 steels. (Table 2-9)

27. The blades of a post hole digger should have good wear resistance, high strength, and good ductility. SAE 1080 steel is a reasonable choice.

28. SAE 5160 OQT 1000 is a high-carbon, chromium steel, containing approximately 0.60% carbon and 0.80% chromium. It was heat treated by heating above its upper critical temperature, quenched in oil, and then tempered at 1000 degrees Fahrenheit. It has fairly high strength ($s_y$ = 151 ksi or 1040 MPa) and good ductility (14% elongation).

29. In general, a high hardness with good ductility are desirable for machine parts and tools subjected to impact loads as seen by a shovel. A hardness of HRC 40 corresponds to approximately HB 375 and is considered moderately hard. While this is a good level, even a higher value up to HRC 50 (HB 475) would be better, provided ductility is fairly high, say about 15% elongation. Appendix 3 shows some forms of oil-quenched SAE 1040 and none listed have sufficiently high hardness. Appendix 4-1 shows the same material quenched in water and tempered. SAE 1040 WQT 700 has a hardness of HB 401 (HRC 43) with approximately 20% elongation and a yield point of 92 ksi.

30. Through hardening involves heating the entire part followed by quenching to achieve the hardened condition. Except for some variation in thick sections, the part is hardened throughout. But no chemical composition changes occur. In carburizing, the chemical composition of the surface is changed by the infusion of carbon. Thus, carburizing results in a hard surface while the core is softer.

31. Induction hardening is a heat treating process in which the area to be hardened is subjected to a high-frequency electric current created by a coil, inducing current flow near the surface of the part and causing local heating. After sufficient time to bring the surface to a temperature above the upper critical temperature of the material, the part is quenched to harden the surface.

32. Some carburizing grades of steels are SAE 1015, 1020, 1022, 1117, 1118, 4118, 4320, 4620, 4820, 8620 and 9310. The carbon content ranges from 0.10% to 0.20%. App. A-5.

33. The AISI 200 and 300 series of stainless steels are nonmagnetic.

34. Chromium gives stainless steels good corrosion resistance.

35. ASTM A992 structural steel is used for most wide-flange beams.

36. HSLA structural steels are high-strength, low-alloy steels having yield strengths in the range of 42 - 100 ksi (290 - 700 MPa.

37. Three types of cast iron are gray iron, ductile iron, and malleable iron.

38. ASTM A48 , Grade 30 is a gray iron with a tensile strength of 30 ksi (207 MPa); no yield strength; less than 1% elongation (brittle); modulus of elasticity (stiffness) of 16.9 x10$^6$ psi ( 117 GPa).

Problem 38. (continued)

ASTM A536 Grade 100-70-03 is a ductile iron with a tensile strength of 100 ksi (689 MPa); a yield strength of 70 ksi (483 MPa); 3% elongation (brittle); modulus of elasticity (stiffness) of 24 $x10^6$ psi (165 GPa).

ASTM A47 , Grade 32510 is a malleable iron with a tensile strength of 50 ksi (345 MPa); a yield strength of 32.5 ksi (224 MPa); 10% elongation (ductile); modulus of elasticity (stiffness) of $25x10^6$ psi (172 GPa).

ASTM A220, Grade 70003 is a malleable iron with a tensile strength of 85 ksi (586 MPa); a yield strength of 70 ksi (483 MPa); 3% elongation (brittle); modulus of elasticity (stiffness) of $26x10^6$ psi (179 GPa).

39. Powdered metals are preformed in a die under high pressure and sintered at a high temperature to fuse the particles. Re-pressing after sintering is sometimes used.

40. Parts made from Zamak 3 zinc casting alloy typically have good dimensional accuracy and smooth surfaces, a tensile strength of approximately 41 Ksi (283 MPa), a yield strength of 32 Ksi (221 MPa), 10% elongation, and a modulus of elasticity of $12.4x10^6$ psi (85 GPa). (Appendix 10)

41. Type D tool steels are typically used for stamping dies, punches, and gages. (Table 2-11)

42. The suffix O on aluminum 6061-O indicates the annealed condition.

43. The suffix H on aluminum 3003-H14 indicates that it was strain hardened.

44. The suffix T on aluminum 6061-T6 indicates that it was heat treated.

45. Aluminum 7178-T6 has the highest strength; tensile strength = 88 ksi (607 MPa); yield strength = 78 ksi (538 MPa).

46. Aluminum alloy 6061 is one of the most versatile.

47. Three typical uses of titanium alloys are aerospace structures, chemical processing equipment, and marine hardware.

48. Bronze is an alloy of copper with tin, aluminum, lead, phosphorus, nickel, zinc, manganese, or silicon.

49. Bronze C86200 is a manganese bronze casting alloy with a tensile strength of 95 ksi (655MPa); yield strength of 48 ksi (331 MPa); 20% elongation (ductile); modulus of elasticity of $15x10^6$ psi (103 GPa).

50. Bronze is used for gears and bearings.

51. Thermosetting plastics undergo a chemical change during forming resulting in a structure of cross-linked molecules. The process cannot be reversed or repeated. Thermoplastic materials can be formed repeatedly by reheating because the molecular structure is essentially unchanged during processing.

52. a) Gears: Nylon, polycarbonate, acetal, PET, polyurethane elastomer, phelolic. b) Helmets: ABS and polycarbonates. c) Transparent shield: Acrylic. d) Structural housing: PET, ABS, polycarbonate, acrylic, PVC, phenolic, polyester/glass composite. e) Pipe: ABS, PVC. f) Wheels: Polyurethane elastomer. g) Switch parts: polyimide, phenolic, PET.

53. Designers of parts to be made from composite materials can control 1) base resin, 2) reinforcing fibers, 3) amount of fibers, 4) orientation of fibers, 5) number of layers, 6) overall thickness, 7) orientation of layers, 8) combinations of types of materials.

54. Composite materials are comprised of two or more different materials, typically a resin reinforced by fibers.

55. Resins used for composites include polyesters, epoxies, polyimides, PHENOLICS (ALL THERMOSETS), THERMO PLASTICS: PE, PA, PEEK, PPS, PVC.

56. Reinforcing fibers used for composites are glass, boron, aramid, and carbon/graphite.

57. Sporting equipment is made from glass/epoxy, boron/epoxy, and graphite/epoxy composites.

58. Aerospace structures are made from glass/epoxy, boron/epoxy, graphite/epoxy, and aramid/epoxy composites.

59. Sheet molding compound is typically a glass/polyester composite.

60. SMC's are used for auto and truck body panels and large housings.

61. Reinforcing fibers are produced as continuous filaments, chopped fibers, roving, fabric, yarn, and mats.

62. Wet processing of composites involves the layup of fabric reinforcing sheets on a form, saturation of the sheets with the resin, and curing under heat and pressure.

63. Preimpregnated composite materials are produced with the resin already on the fibers in a convenient form, called a prepreg. The prepreg is layered onto the form and cured.

64. SMC's are preimpregnated fabric sheets formed in a mold and cured simultaneously under heat and pressure.

65. Pultrusion is a process of coating the fiber reinforcement as it is pulled through a heated die to produce a continuous form such as tubing, structural shapes, rod, and hat sections used to stiffen aircraft structures.

66. In the filament winding process, continuous filaments are placed around a mandrel in a controlled pattern and then cured. The process is used for pipe, pressure vessels, rocket motor cases, containers and enclosures.

67. Specific strength is the ratio of the strength of a material to its specific weight.

68. Specific stiffness is the ratio of the modulus of elasticity of a material to its specific weight.

69. Many composites have significantly higher values of specific strength and specific stiffness than metals.

**70 - 73 refer to Figure 2-23 and Table 2-17.**

General conclusions from Questions 70 - 73: The specific strengths of the metals listed range from $0.194 \times 10^6$ to $1.00 \times 10^6$ in, approximately a factor of 5.0. The specific stiffnesses are very nearly equal for all metals listed, approximately $1.0 \times 10^8$ in. The specific strengths of the composites listed range 1.87 to $4.8 \times 10^6$ in, much higher than any of the metals. Glass/epoxy has a specific stiffness about 2/3 that of the metals. The other composites listed range from 2.2 to 8.3 times as stiff as the metals.

**See Section 2-17 for answers to Questions 74 to 100.**

***Supplementary Proplems – Chapter 2***

1. Poisson's ratio: a) Carbon steel – 0.29; c) Lead – 0.43; e) Concrete – 0.10 to 0.25
2. See Section 2-2, subsection: Flexural Strength and Modulus, and Figure 2-5.
3. Erosive, abrasive, adhesive, fretting, surface fatigue
4. From Table 2-6: 14 alloys listed, Examples: ASTM A36, SAE 1018 HR or CD, SAE 1045 HR or CD, SAE 8620 CD.
5. From Table 2-6: SAE 304 and SAE 316
6. From Table 2-6: Six alloys listed, Examples: 2024-T4, 3003-H14, 6061-T6, 6063-T6
7. From Section 2-3: ASTM International, AISI, SAE
8. From Section 2-3: Aluminum Association
9. From Table 2-7: a) DIN 42CrMo4 or W-1.7225; b) BS 708A42; c) EN 42CrMo4; d) GB ML42CrMo4; e) JIS SCM 440H
10. From Table 2-7: a) DIN C45 or W-1.0503; b) BS 060A47; c) EN C45; d) GB 699-45; e) JIS S45C
11. From Table 2-7: a) DIN X6Cr17 or W-1.4016; b) BS 430S17; c) EN X6Cr17; d) GB ML1Cr17; e) JIS SUS430
12. From Table 2-7: a) DIN AlZnMgCu1.5 or W-3.4365; b) BS L.95, L.96; c) EN AlZn6MgCu
13. Water, brine, mineral oil, water-soluble polyalkylene glycol (PAG)
14. From Section 2-6: Fine steel or cast iron shot is projected at high velocity on critical surfaces to produce residual compressive stress that tends to improve the fatigue strength.
15. From Table 2-10: ASTM A27/A27M; A915/A915M; A128/A128M; A148/A148M
16. From Table 2-10: ASTM A757; ASTM A351; ASTM A216; ASTM A389
17. Carbidic austempered ductile iron – used for: railroad rolling stock, earthmoving equipment, agricultural machinery, crushers
18. From Section 2-10: White iron is made by rapidly chilling a casting of gray iron or ductile iron during solidification. ASTM Standard A532 describes the process. Used to improve wear resistance for ball mills, crushers, mixing equipment, and material handling devices.
19. From Section 2-11: Powders are pressed to their basic form and then heated to sinter the powder particles into a strong solid.
20. From Section 2-11: Powders are compressed by a flexible membrane in a hermetic chamber to produce a high density; may be done cold or at elevated temperatures.
21. From Section 2-11: Metal powders are fed into an injection molding machine to form a green part that is then sintered to complete the solidification and bonding processes.
22. From Section 2-11: Metal powders are first pressed and sintered, then forged in a closed-die press to achieve final form and properties.
23. From Table 2-12: Carbon steel F-0008-HT, $s_u$ = 85 ksi (590 MPa);

Low-alloy steel FL-4405-HT, $s_u$ = 160 ksi (1100 MPa);

Diffusion-alloyed steel FD-0205-HT, $s_u$ = 130 ksi (900 MPa);

Sinter-hardened steel FLC-4608-HT, $s_u$ = 100 ksi (690 MPa)

24. From Table 2-12: a) Nickel silver – CNZ-1818; $s_u$ = 20 ksi (118 MPa)

b) Bronze – CTG-1001; (no strength listed; used for bearings)

c) Copper – C-0000; No strength listed; used for electrical applications

d) Aluminum - $s_u$ = 32 ksi (221 MPa)

25. From Section 2-11: Projected surface area less than 50 $in^2$ (32 000 $mm^2$)

26. From Section 2-12: Aluminum casting alloys: 202, 222, 319, 360, 413, 444, 512, 535, 712, 771, 850, 852. Others available.

27. From Section 2-12: Aluminum 2014, 2024, 6061

28. From Section 2-13: Zinc *alloy No. 3* or *Zamak 3*.

29. From Section 2-13 and Appendix A10-1: Zinc ZA-8, $s_u$ = 54 ksi (374 MPa)

ZA-12, $s_u$ = 59 ksi (404 MPa); ZA-27, $s_u$ = 61 ksi (421 MPa)

30. From Section 2-14: Nickel-based alloys have good corrosion resistance and retain good levels of strength at high temperatures.

31. From Section 2-15: a) Bearing bronze C93200l; b) Phosphor bronze C54400;

c) Muntz metal C37000; d) Manganese bronze C86200;

e) Copper-nickel-zinc alloy C96200; f) Manganese bronze C67500

32. From Section 2-15: H-numbers indicate the degree of hardening by strain hardening methods; a) H04 – Full hard; b) H02 – ½ hard; c) H01 – 1/8 hard; d)H08 – Spring hard

33. From Section 2-15: TD temper indicates – solution heat treated and cold worked

34. From Section 2-15 and Figure 2-18: As the percent cold reduction increases, tensile and yield strengths increase and ductility as indicated by percent elongation decreases.

10% cold work: $s_u$ = 133 ksi (917 MPa), $s_y$ = 121 ksi (834 MPa), 17% elongation

40% cold work: $s_u$ = 154 ksi (1062 MPa), $s_y$ = 142 ksi (979 MPa), 1% elongation

35. From Section 2-18: Metals, polymers, ceramics, glasses, elastomers, hybrids

36. From Section 2-18: Foams, sandwich structures, honeycomb structures

37. From Fig. 2-31: d) Metals, b) ceramics, g) composites, c) polymers, a) wood,

h) rubbers/elastomers, f) foams

38. From Fig. 2-32: b) ceramics, d) metals, g) composites, a) wood, c) polymers, f) foams,

h) elastomers

39. From Figure 3-32: (lightest to heaviest) e) foams, a) wood, h) elastomers, g) composites,

d and b) Metals and ceramics (about equal)

# CHAPTER 3
# STRESS AND DEFORMATION ANALYSIS

## Direct Tension and Compression

$\boxed{1.}$ $\sigma = F/A$; $A = \pi(18^2 - 12^2)/4 = 141.4\ mm^2$

$\sigma = 4500N/141.4\ mm^2 = 31.8\ N/mm^2 = \underline{31.8\ MPa}$

$$\delta = \frac{PL}{EA} = \frac{(4500\ N)(750\ mm)}{(207 \times 10^9\ N/m^2)(141.4\ mm^2)} \times \frac{10^6\ mm^2}{m^2} = \underline{0.12\ mm}$$

$\boxed{2.}$ $\sigma = F/A = 3500\ N/(\pi(10)^2/4)\ mm^2 = \underline{44.6\ MPa}$

$\boxed{3.}$ $\sigma = F/A = 20 \times 10^3 N/(10 \cdot 30)\ mm^2 = \underline{66.7\ MPa}$

$\boxed{4.}$ $\sigma = F/A = 860LB/(0.40\ IN)^2 = \underline{5375\ PSI}$

$\boxed{5.}$ $\sigma = F/A = 1900LB/\pi(0.375IN)^2/4 = \underline{17,200\ PSI}$

$\boxed{6.}$ $\sigma = P/A$; $A = (12mm)^2 = 144\ mm^2$; $\sigma = 5000N/144mm^2 = \underline{34.7MPa}$ (ALL)

$$\delta = \frac{PL}{EA} = \frac{(5000\ N)(1650\ mm)}{(E\ N/mm^2)(144mm^2)} = \frac{57\,292}{E}\ mm$$

a) AISI 1020; $E = 207GPa = 207 \times 10^3\ N/mm^2$; $\underline{\delta = 0.277\ mm}$

b) AISI 8650; $E = 207GPa$; $\underline{\delta = 0.277\ mm}$

c) DUCTILE IRON; $E = 165\ GPa$; $\underline{\delta = 0.347\ mm}$

d) ALUMINUM 6061-T6; $E = 69GPa$; $\underline{\delta = 0.830\ mm}$

e) TITANIUM Ti-6AL-4V; $E = 114GPa$; $\underline{\delta = 0.503\ mm}$

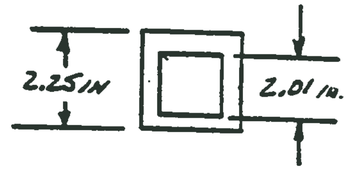
f) PVC; $E = 2410MPa = 2070\ N/mm^2$; $\underline{\delta = 27.7\ mm}$

g) PHENOLIC; $E = 7580MPa$; $\underline{\delta = 7.56\ mm}$

<u>NOTE:</u> STRESS IS CLOSE TO THE ULTIMATE FOR f AND g.

$\boxed{7.}$

$\delta = PL/EA\ ;\quad P = \dfrac{\delta EA}{L}$



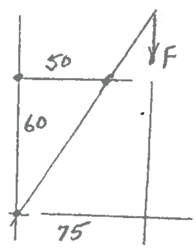

$A = (2.25^2 - 2.01^2)\,IN^2 = 1.02\,IN^2$

$$P = \frac{(0.004\,IN)(10\times10^6\,LB/IN^2)(1.02\,IN^2)}{16.0\,IN.} = 2556\,LB$$

$\sigma = P/A = 2556\,LB/1.02\,IN^2 = 2506\,PSI$

$\boxed{8.}$

$\Sigma M_B = 0 = 2500(75) - F_c(60)$

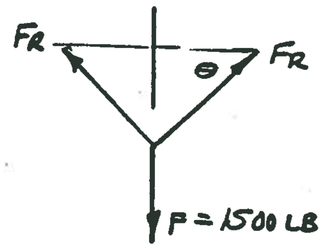



$F_c = 2500\,(75/60) = 3125\,LB$ = TENSILE FORCE IN AC

$\sigma = \dfrac{P}{A} = \dfrac{3125\,LB}{(0.50)(3.50)\,IN^2} = \underline{595\,PSI}$

$\boxed{9.}$

$2 F_R \sin\theta = 1500\,LB$




$F_R = 1500/(2\sin(45°)) = \underline{1061\,LB}$

$\boxed{10.}$

$\sigma = P/A\ ;\quad$ REQ'D. $A = P/\sigma_{ALLOW}$

$$A = \frac{1061\,LB}{18\,000\,LB/IN^2} = 0.0589\,IN^2 = \pi D^2/4$$

$$D = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4(0.0589)}{\pi}} = \underline{0.274\,IN.}\ \underline{\text{MINIMUM}}$$

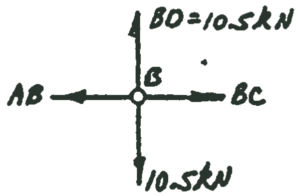
$\boxed{11.}$

$$\theta = 15°\ ;\quad F_R = \frac{1500\,LB}{2\sin\theta} = \frac{1500\,LB}{2\sin(15)} = \underline{2898\,LB}$$

$$A = 2898/18000 = 0.161\,IN^2\ ;\ D = \sqrt{\frac{4(.161)}{\pi}} = \underline{0.453\,IN}$$

12.

JOINT B



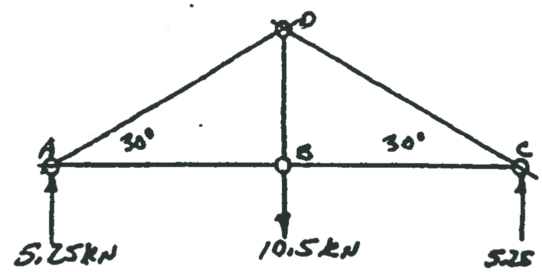
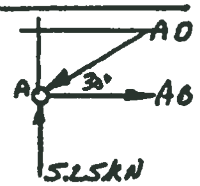




FIG. P3-12

JOINT A




$AD \sin 30° = 5.25\ kN$

$AD = 10.5\ kN = CD$

$AB = AD \cos 30° = 9.09\ kN = BC$

STRESSES:

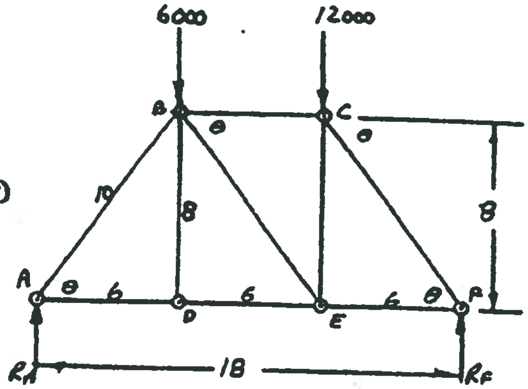
AB, BC: $\sigma_{AB} = \sigma_{BC} = \dfrac{9.09 \times 10^3\ N}{(12)(30)\ mm^2} = \underline{25.3\ MPa}$ TENSION

BD: $\sigma_{BD} = \dfrac{10.5 \times 10^3\ N}{(2)(10)(30)\ mm^2} = \underline{17.5\ MPa}$ TENSION

AD, CD: $A = (30)^2 - (20)^2 = 500\ mm^2$

$\sigma_{AD} = \sigma_{CD} = \dfrac{-10.5 \times 10^3\ N}{500\ mm^2} = \underline{-21.0\ MPa}$ COMPRESSION

**13.**



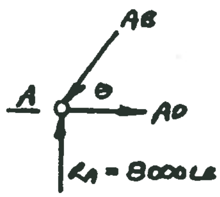

<u>FIGURE P3-26</u>

$\Sigma M_A = 0 = 6000(6) + 12000(12) - R_F(18)$

$R_F = 10000$ LB

$\Sigma M_F = 0 = 12000(6) + 6000(12) - R_A(18)$

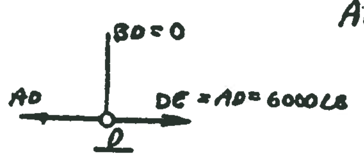
$R_A = 8000$ LB

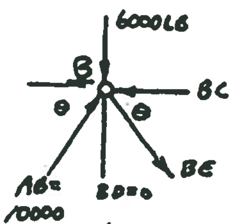



$R_A = AB \sin\theta = AB(0.8)$

$AB = R_A/0.8 = 8000/0.8 = 10000$ LB COMP.

$AD = AB \cos\theta = 10000(0.6) = 6000$ LB TENS.



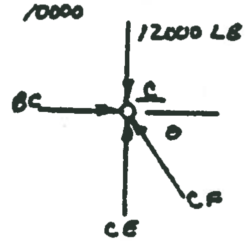




$BE \sin\theta + 6000 - AB \sin\theta = 0$

$BE = \dfrac{AB \sin\theta - 6000}{\sin\theta} = \dfrac{10000(0.8) - 6000}{0.8} = 2500$ LB TENS.

$BC = AB \cos\theta + BE \cos\theta = 10000(0.6) + 2500(0.6) =$

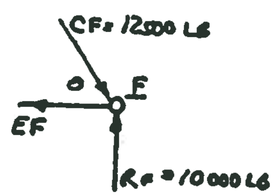
$BC = 7500$ LB COMP.




$BC = CF \cos\theta$

$CF = BC/\cos\theta = 7500/0.6 = 12500$ LB COMP.

$CE = 12000 - CF \sin\theta = 12000 - 12500(0.8) = 2000$ LB C




$EF = CF \cos\theta = 12500(0.6) = 7500$ LB TENS.

<u>AREAS OF MEMBERS</u>: (APP. A5, A6)

AD, DE, EF – $2(0.484) = 0.968\ \text{in}^2$

BD, BE, CE – $0.484\ \text{in}^2$

AB, BC, CF – $2(1.21) = 2.42\ \text{in}^2$

NOTE: COMPRESSION MEMBERS MUST BE CHECKED FOR COLUMN BUCKLING

<u>STRESSES</u>:

$\sigma_{AD} = \sigma_{DE} = 6000/0.968 = +6198$ psi

$\sigma_{EF} = 7500/0.968 = +7748$ psi

$\sigma_{BD} = 0$

$\sigma_{BE} = 2500/0.484 = +5165$ psi

$\sigma_{CE} = -2000/0.484 = -4132$ psi

$\sigma_{AB} = -10000/2.42 = -4132$ psi

$\sigma_{BC} = -7500/2.42 = -3099$ psi

$\sigma_{CF} = -12500/2.42 = -5165$ psi

**14.**

$A = (2.65)(1.40) + 2[(1.40)(0.5)(\tfrac{1}{2})] = 4.41\ \text{in}^2$

$\sigma = F/A = (52000\ \text{LB}/4.41\ \text{in}^2) = \underline{11791\ \text{psi}}$

**15.** $A = (80)(40) + \pi(40)^2/4 = 4457\ mm^2$

$\sigma = F/A = 640\times10^3 N/4457\ mm^2 = \underline{143.6\ MPa}$

## Direct Shear Stress

**16.** PIN DIA = 0.50 IN.; DOUBLE SHEAR

$A_s = 2(\pi D^2/4) = 2\pi(0.50)^2/4 = 0.3927\ IN^2$

$\tau = F_s/A_s$

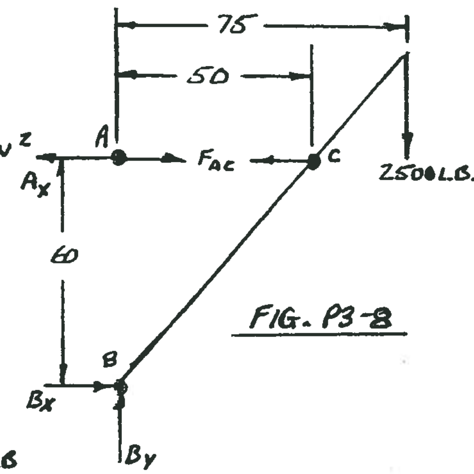



FIG. P3-8

MEMBER BC:

$\Sigma M_B = 0 = (2500)(75) - F_{AC}(60)$

$F_{AC} = 2500(75/60) = 3125\ LB$

AND $B_x = F_{AC} = 3125\ LB$

$B_y = 2500\ LB$

RESULTANT AT B: $B = \sqrt{3125^2 + 2500^2} = 4002\ LB$

PINS A AND C: $\tau = F_s/A_s = 3125\ LB/0.3927\ IN^2 = \underline{7958\ PSI}$

PIN B: $\tau = F_s/A_s = 4002\ LB/0.3927\ IN^2 = \underline{10{,}190\ PSI}$

**17.** FROM PROB. 9: FORCE IN EACH ROD = $F_R = 1500\ LB/2\sin\theta$

FOR $\theta = 40°$; $F_R = 1167\ LB$ = SHEAR FORCE ON UPPER PINS

ASSUME DOUBLE SHEAR: $A_s = 2\pi D^2/4 = 2\pi(0.75)^2/4 = 0.8836\ IN^2$

$\tau = F_s/A_s = 1167\ LB/0.8836\ IN^2 = \underline{1321\ PSI}$

LOWER PIN: $F_s = 1500\ LB$

$\tau = F_s/A_s = 1500\ LB/0.8836\ IN^2 = \underline{1698\ PSI}$

**18.** ANALYSIS FROM PROBLEMS 9 AND 17. LET $\theta = 15°$

$F_R = 1500\ LB/2\sin\theta = 1500\ LB/2\sin 15° = 2898\ LB$

$\tau = F_s/A_s = 2898\ LB/0.8836\ IN^2 = \underline{3280\ PSI}$ IN ALL PINS

**19.** FIGURE 3-7 KEY IN SHEAR. $A_s = b\cdot L = (12)(45) = 540\ mm^2$

$F_s$ = TORQUE/RADIUS $= 1600\ N\cdot m/30\ mm \times 10^3 mm/m = 53\ 333\ N$

$\tau = F_s/A_s = \dfrac{53\ 333\ N}{540\ mm^2} = 98.8\ N/mm^2 = \underline{98.8\ MPa}$

**20.** PUNCH – FIG P3-20 $A_s = (\text{PERIM.})\, t = [2.50 + 2.00 + 1.50 + \sqrt{0.5^2 + 2.5^2}](0.060)$

$A_s = (8.55\ \text{IN})(0.060\ \text{IN}) = 0.513\ \text{IN}^2$

$\tau = F_s/A_s = 52\,000\ \text{LB}/0.513\ \text{IN}^2 = \underline{101{,}400\ \text{psi}}$

**21.** PUNCH – FIG. P3-21. $\text{PERIM} = 60 + 2(30) + 2(7.5) + 3\left[\frac{\pi(15)}{2}\right] = 205.7\ \text{mm}$

$A_s = (\text{PERIM.})\, t = (205.7)(2.0) = 411.4\ \text{mm}^2$

$\tau = F_s/A_s = \dfrac{225\,000\ \text{N}}{411.4\ \text{mm}^2} = 547\ \text{N/mm}^2 = \underline{547\ \text{MPa}}$

**Torsion**

**22.** $\tau = \dfrac{T}{Z_p} = \dfrac{T}{\pi D^3/16} = \dfrac{800\ \text{N}\cdot\text{m}}{\pi(50)^3/16\ \text{mm}^3} \times \dfrac{10^3\ \text{mm}}{1\ \text{m}} = 32.6\ \dfrac{\text{N}}{\text{mm}^2} = \underline{32.6\ \text{MPa}}$

**23.** $\theta = \dfrac{TL}{GJ}$ : $T = 800\ \text{N}\cdot\text{m} = 800\times10^3\ \text{N}\cdot\text{mm}$ ; $G = 80\ \text{GPa} = 80\times10^3\ \text{N/mm}^2$

$J = \dfrac{\pi D^4}{32} = \dfrac{\pi(50)^4}{32}\ \text{mm}^4 = 6.14\times10^5\ \text{mm}^4$

$\theta = \dfrac{(800\times10^3\ \text{N}\cdot\text{mm})(850\ \text{mm})}{(80\times10^3\ \text{N/mm}^2)(6.14\times10^5\ \text{mm}^4)} = 0.0138\ \text{RAD} \times \dfrac{180\ \text{DEG}}{\pi\ \text{RAD}} = \underline{0.79\ \text{DEG.}}$

**24.** $\tau = \dfrac{T}{Z_p} = \dfrac{T}{\pi D^3/16} = \dfrac{88.0\ \text{LB-IN.}}{\pi(0.40\ \text{IN})^3/16} = \underline{7003\ \text{psi}}$

**25.** $T = 63\,000\,(P)/n = 63\,000\,(110\ \text{hp})/560\ \text{RPM} = 12\,375\ \text{LB-IN.}$

$\tau = \dfrac{T}{Z_p} = \dfrac{12\,375\ \text{LB-IN}}{\pi(1.25\ \text{IN})^3/16} = \underline{32\,270\ \text{psi}}$

**26.** $T = P/n = 28\times10^3\ \text{N}\cdot\text{m/s} \big/ 45\ \text{RAD/s} = 622\ \text{N}\cdot\text{m}$

$Z_p = \dfrac{\pi(D^4 - d^4)}{16D} = \dfrac{\pi[40^4 - 30^4]\ \text{mm}^4}{16\ (40\ \text{mm})} = 8590\ \text{mm}^3$

$\tau = \dfrac{T}{Z_p} = \dfrac{622\ \text{N}\cdot\text{m}}{8590\ \text{mm}^3} \times \dfrac{10^3\ \text{mm}}{\text{m}} = 72.4\ \text{N/mm}^2 = \underline{72.4\ \text{MPa}}$

**27.** $\theta = \dfrac{TL}{GJ}$ : $J = \dfrac{\pi(D^4 - d^4)}{32} = \dfrac{\pi[40^4 - 30^4]}{32}\ \text{mm}^4 = 1.718\times10^5\ \text{mm}^4$

$\theta = \dfrac{(622\ \text{N}\cdot\text{m})(400\ \text{mm})}{(80\times10^3\ \text{N/mm}^2)(1.718\times10^5\ \text{mm}^4)} \times \dfrac{10^3\ \text{mm}}{\text{m}} = 0.018\ \text{RAD} \times \dfrac{180°}{\pi\ \text{RAD}} = \underline{1.04°}$

## Noncircular Members in Torsion

**28.**

SQUARE: $a = 25\,mm$; $Q = 0.208\,a^3 = 3250\,mm^3$ } FIG. 3-6

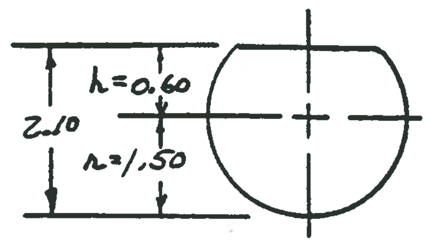
$K = 0.141\,a^4 = 5.51\times10^4\,mm^4$

$$\tau = T/Q = \left(230\,N\cdot m / 3250\,mm^3\right)\frac{10^3\,mm}{m} = 70.8\,N/mm^2 = \underline{70.8\,MPa}$$

$$\theta = \frac{TL}{GK} = \frac{(230\times10^3\,N\cdot mm)(650\,mm)}{(80\times10^3\,N/mm^2)(5.51\times10^4\,mm^4)} = 0.0339\,RAD\times\frac{180^\circ}{\pi\,RAD} = \underline{1.94^\circ}$$

**29.**

$h/r = 0.60/1.50 = 0.40$ (FIG. P3-29)




$C_1 = 0.78$ ; $C_2 = 0.70$

$K = C_1 r^4 = 0.78(1.50)^4 = 3.95\,IN^4$

$Q = C_2 r^3 = 0.70(1.50)^3 = 2.36\,IN^3$

$$\tau = \frac{T}{Q} = \frac{10\,600\,LB\text{-}IN.}{2.36\,IN^3} = \underline{4487\,psi}$$

$$\theta = \frac{TL}{GK} = \frac{(10\,600\,LB\cdot IN)(44.0\,IN)}{(11.5\times10^6\,LB/IN^2)\,3.95\,IN^4} = 0.0103\,RAD\times\frac{180^\circ}{\pi\,RAD} = \underline{0.59^\circ}$$

**30.**

$a = 2.0\,IN$ ; $b = 4.0\,IN$ ; $t = 0.109\,IN$ ; $(a-t) = 1.891\,IN$ ; $(b-t) = 3.891\,IN$ ; $L = 6.5\,FT$
$L = 78\,IN$

$$K = \frac{2t(a-t)^2(b-t)^2}{a+b-2t} = \frac{2(0.109)(1.891)^2(3.891)^2}{[2.0+4.0-2(0.109)]} = 2.041\,IN^4$$

$Q = 2t(a-t)(b-t) = 2(0.109)(1.891)(3.891) = 1.604\,IN^3$

$T = \tau Q = (6000\,LB/IN^2)(1.604\,IN^3) = \underline{9624\,LB\text{-}IN}$

$$\theta = \frac{TL}{GK} = \frac{(9624\,LB\cdot IN)(78\,IN.)}{(11.5\times10^6\,LB/IN^2)(2.041\,IN^4)} = 0.032\,RAD\times\frac{180^\circ}{\pi\,RAD} = \underline{1.83^\circ}$$

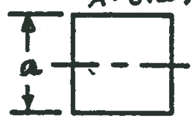
**31.** $\sigma = M/S$ : REQ'D. $S = M/\sigma_{ALLOW}$

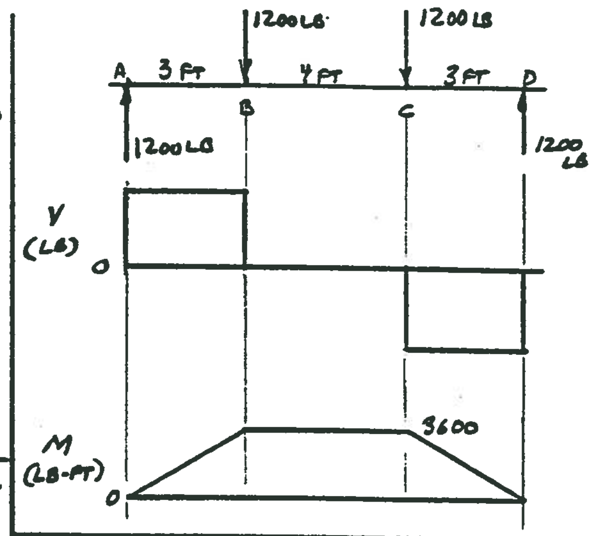
$S = \dfrac{3600 \text{ LB-FT}}{18000 \text{ LB/IN}^2} \times \dfrac{12 \text{ IN}}{\text{FT}} = 2.40 \text{ IN}^3$

a) SQUARE; $S = a^3/6$

$a = \sqrt[3]{6S} = 2.43 \text{ IN}$

USE $a = 2.50$ IN



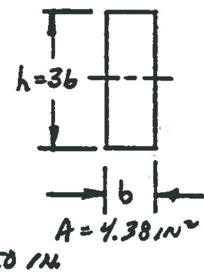

b) RECTANGLE

$S = \dfrac{bh^2}{6} = \dfrac{b(3b)^2}{6} = 1.5b^3$

$b = \sqrt[3]{S/1.5} = 1.17 \text{ IN}$

$h = 3b = 3.50$ IN

USE $b = 1.25$ IN, $h = 3.50$ IN

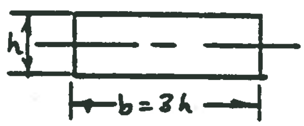



c) RECTANGLE

$S = \dfrac{bh^2}{6} = \dfrac{3h(h)^2}{6} = \dfrac{h^3}{2}$

$h = \sqrt[3]{2S} = 1.69 \text{ IN}; b = 5.06 \text{ IN}$

USE $h = 1.75$ IN; $b = 5.00$ IN  $A = 8.75 \text{ IN}^2$

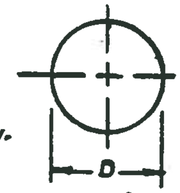



d) CIRCLE

$S = \pi D^3/32$

$D = \sqrt[3]{\dfrac{32S}{\pi}} = 2.90 \text{ IN.}$

USE $D = 3.00$ IN.




$A = 7.07 \text{ IN}^2$

e) S4 x 7.7  $S = 3.04 \text{ IN}^3$  $A = 2.26 \text{ IN}^2$

f) C15 x 40  $S_y = 3.37 \text{ IN}^3$  $A = 11.8 \text{ IN}^2$

g) 4-IN SCHEDULE 40 PIPE  $S = 3.215 \text{ IN}^3$  $A = 3.17 \text{ IN}^2$

**32.** a) VOLUME $= A \times L = (2.5 \text{ IN})^2 \times 120 \text{ IN} = 750 \text{ IN}^3$; $w = (.283 \text{ LB/IN}^3)(750 \text{ IN}^3)$

$w = 212$ LB.

b) $V = A \times L = (1.25)(3.50)(120) = 525 \text{ IN}^3$; $w = (.283)(525) = 149$ LB

c) $V = (1.75)(5.00)(120) = 1050 \text{ IN}^3$; $w = (.283)(1050) = 297$ LB

d) $V = \dfrac{\pi D^2}{4} \times L = \dfrac{\pi(3.00 \text{ IN})^2}{4} \times 120 \text{ IN} = 848 \text{ IN}^3$; $w = (0.283)(848) = 240$ LB

e) 7.7 LB/FT (10 FT) = 77.0 LB  (f) 40 LB/FT (10 FT) = 400 LB

g) 10.78 LB/FT (10 FT) = 107.8 LB  $V = A \cdot L = (3.174 \text{ IN}^2)(12.0 \text{ IN}) = 38.09 \text{ IN}^3$ FOR 1.0 FT

$w = 0.283(38.09) = 10.78$ LB/FT

**33.** FROM CASE C; APPENDIX 14: $a = 36$ IN; $L = 120$ IN; $P = 1200$ LB; $E = 30 \times 10^6$ PSI

$$y_{MAX} = \frac{Pa}{24EI}(3L^2 - 4a^2) = \frac{1200(36)[3(120)^2 - 4(36)^2]}{24(30 \times 10^6)\, I} = \frac{2.281}{I} \text{ IN}$$

$$y_B = y_C = \frac{Pa^2(3L - 4a)}{6EI} = \frac{1200(36)^2[3(120) - 4(36)]}{6(30 \times 10^6)\, I} = \frac{1.866}{I} \text{ IN}$$

(SEE NEXT PAGE)

**33.** (CONT)

a) $I = a^4/12 = (2.50)^4/12 = 3.26\ IN^4$ ; $y_{MAX} = \dfrac{2.281}{3.26} = \underline{0.701\ IN}$

$y_B = y_C = \dfrac{1.866}{3.26} = \underline{0.572\ IN.}$

b) $I = bh^3/12 = (1.25)(3.50)^3/12 = 4.47\ IN^4$ : $\underline{y_{MAX} = 0.511\ IN}$; $\underline{y_B = 0.418\ IN}$

c) $I = bh^3/12 = 5.00(1.75)^3/12 = 2.23\ IN^4$ : $\underline{y_{MAX} = 1.021\ IN}$; $\underline{y_B = 0.836\ IN.}$

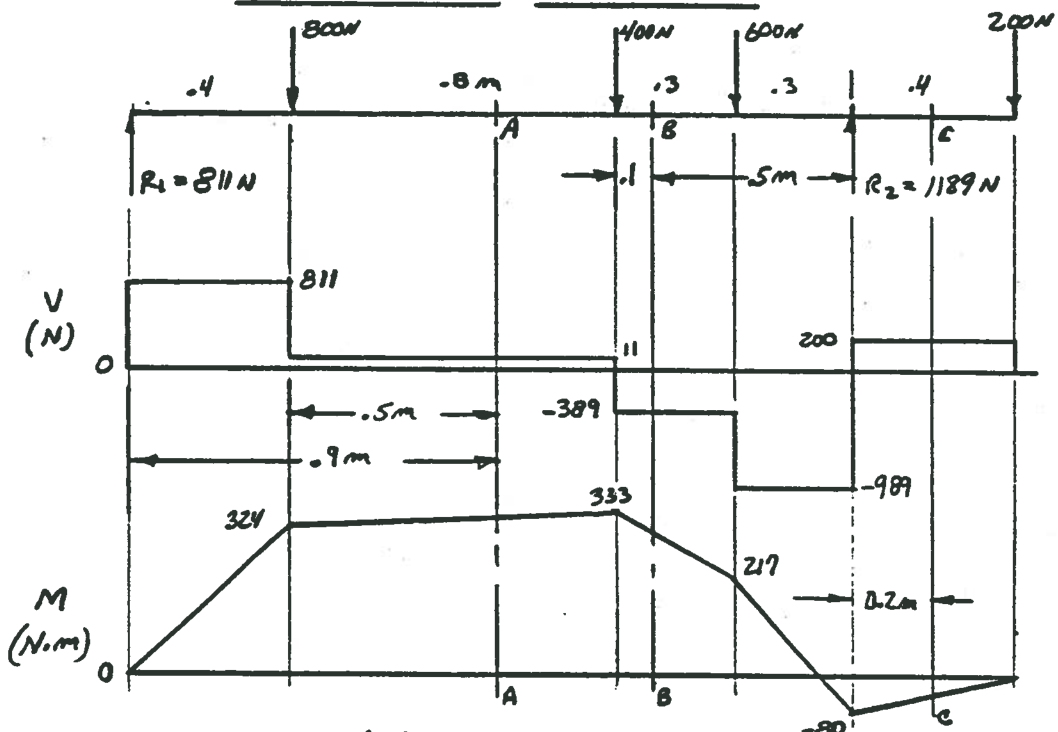
d) $I = \dfrac{\pi D^4}{64} = \dfrac{\pi(3.00)^4}{64} = 3.98\ IN^4$ : $\underline{y_{MAX} = 0.574\ IN}$; $\underline{y_B = 0.469\ IN.}$

e) $I = 6.08\ IN^4$ ; $\underline{y_{MAX} = 0.375\ IN}$ ; $\underline{y_B = 0.307\ IN.}$

f) $I_y = 9.23\ IN^4$ ; $\underline{y_{MAX} = 0.247\ IN}$ ; $\underline{y_B = 0.202\ IN}$

g) $I = 7.23\ IN^4$ ; $\underline{y_{MAX} = 0.315\ IN}$ ; $\underline{y_B = 0.258\ IN.}$

**34.**




$M_A = 324 + 11(0.5) = \underline{330\ N.m}$

$M_B = 333 - 389(0.1) = \underline{294\ N.m}$

$M_C = -80 + 200(.2) = \underline{-40\ N.m}$

**35.** FOR STRENGTH:

$$\text{REQ'D } S = \frac{M_{MAX}}{\sigma_{ALLOW}} = \frac{333\ N.m}{100\ N/mm^2} \times \frac{10^3 mm}{m} = 3330\ mm^3 \text{ OR } 0.203\ IN^3$$

METRIC SHAPE WITH SMALLEST $\underline{A}$ AND $S \geq 3330\ mm^2$: $\underline{\text{APP. 16-19}}$

RES. g: MECH. TUBING - ROUND: $D_o = 45$ mm, $D_i = 40$ mm, $A = 333.8\ mm^2$

$S = 3361\ mm^3$

**36.** a) SIMPLE CANTILEVER – CASE b – APPENDIX 14-2; $\underline{I = 7.23\ IN^4 \quad APP. 15\text{-}17}$

$$y_A = \frac{-Px^2}{6EI}(3a - x) = \frac{-(800)(48)^2}{6(30\times10^6)(7.23)}[3(72) - 48] = \underline{-0.238\ IN.} = y_A$$

$x_1 = 4\ ft = 48\ IN;\ a = 6\ ft = 72\ IN;\ x_2 = 8\ ft = 96\ IN.$

$$y_B = \frac{-Pa^2}{6EI}(3x_2 - a) = \frac{-800(72)^2}{6(30\times10^6)(7.23)}[3(96) - 72] = \underline{-0.688\ IN} = y_B$$

b) SUPPORTED CANTILEVER – CASE b – APPENDIX 14-3

$$y_A = \frac{-Px_1^2 b}{12EIL^3}(3C_1 - C_2 x_1)$$

$x_1 = 4\ ft = 48\ IN;\ a = 6\ ft = 72\ IN;\ b = 4\ ft = 48\ IN;\ L = 10\ ft = 120\ IN;\ v = 2\ ft = 24\ IN$

$C_1 = aL(L+b) = 72(120)(168) = 1.452\times10^6\ IN^3$

$C_2 = (L+a)(L+b) + aL = (192)(168) + 72(120) = 4.090\times10^4\ IN^2$

$$y_A = \frac{-(800)(48)^2(48)}{12(30\times10^6)(7.23)(120)^3}\left[3(1.452\times10^6) - 4.09\times10^4(48)\right] = \underline{-0.047\ IN} = y_A$$

$$y_B = \frac{-Pa^2 v}{12EIL^3}\left[3L^2 b - v^2(3L - a)\right]$$

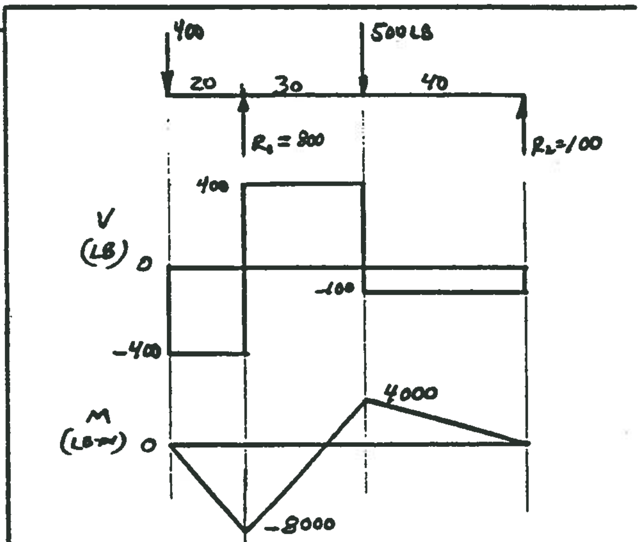
$$y_B = \frac{-(800)(72)^2(24)}{12(30\times10^6)(7.23)(120)^3}\left[3(120)^2(48) - (24)^2(3(120) - 72)\right] = \underline{-0.042\ IN} = y_B$$

---

**37.** $\sigma = M/S;\ S = M/\sigma_{ALLOW}$

$$S = \frac{8000\ LB\text{-}IN}{12\,000\ LB/IN^2} = 0.667\ IN^3$$

SMALLEST BEAM OK.
$3I \times 1.637;\ S = 1.49\ IN^3;\ I = 2.24\ IN^4$



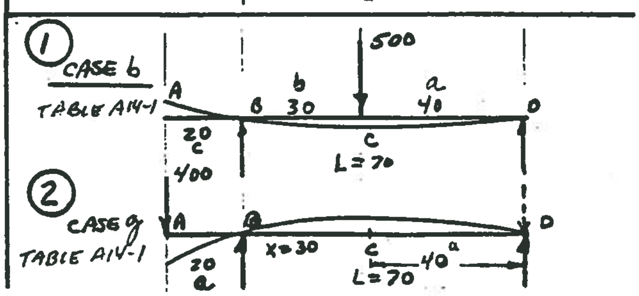

$$y_{A1} = \frac{+Pabc}{6EIL}(L + a)$$

$$= \frac{(+500)(30)(40)(20)[70+40]}{6(10\times10^6)(2.24)(70)} = +0.140\ IN \text{ UP}$$

$$y_{A2} = \frac{-Pa^2}{3EI}(a + L)$$

$$= \frac{400(20)^2(20+70)}{3(10\times10^6)(2.24)} = -0.214\ IN \text{ DOWN}$$

$$y_A = y_{A1} + y_{A2} = +.140 - .214 = \underline{-0.074\ IN}\ \text{DOWN}$$




SEE NEXT PAGE FOR $y_C$

**37.** (CONT.)

$$y_{C1} = \frac{-P\,a^2 b^2}{3EIL} = \frac{-(500)(30)^2(40)^2}{3(10\times10^6)(2.24)(70)} = -0.153 \text{ IN DOWN}$$

$$y_{C2} = \frac{PaL^2}{EI}(0.06415) = \frac{(400)(20)(70)^2}{(10\times10^6)(2.24)}(0.06415) \doteq 0.112 \text{ IN UP}$$

NOTE: $a/L = 40/70 = 0.571$. THEN POINT C IS CLOSE TO $y_{MAX}$ IN CASE g.

$$y_C = y_{C1} + y_{C2} = -0.153 + 0.112 = \underline{-0.041 \text{ IN DOWN}}$$

**38.**

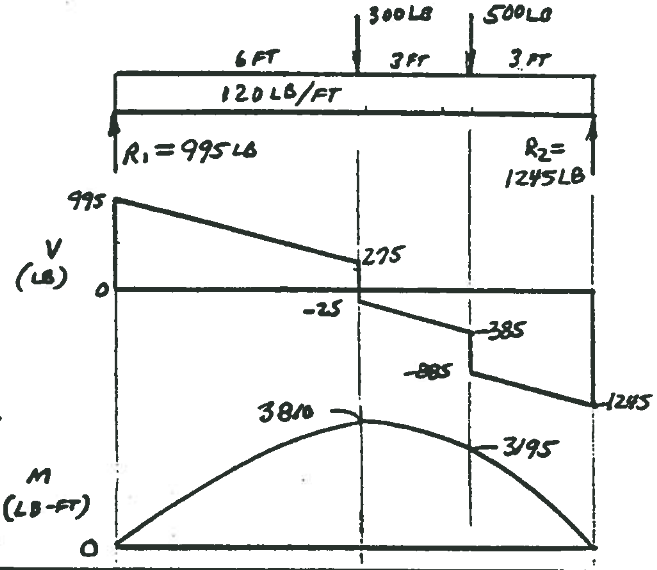
$$M_{MAX} = 3810 \text{ LB-FT}(12 \text{ IN/FT}) = 45720 \text{ LB-IN}$$

$$I = \frac{bh^3}{12} = \frac{(1.50)(7.25)^3}{12} = 47.6 \text{ IN}^4$$

$$S = \frac{bh^2}{6} = \frac{(1.50)(7.25)^2}{6} = 13.1 \text{ IN}^3$$

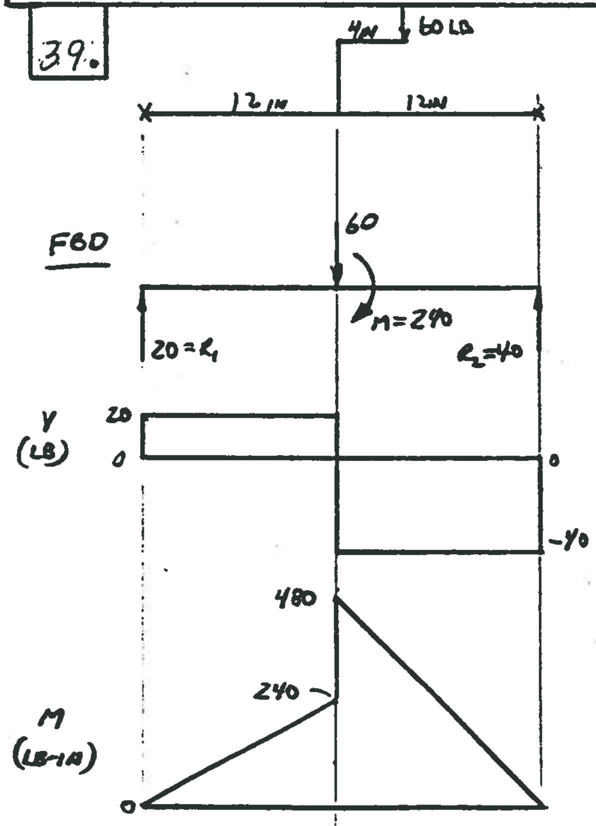
$$\sigma = \frac{M}{S} = \frac{45720}{13.1} = \underline{3480 \text{ PSI}}$$

$$\tau = \frac{3V}{2A} = \frac{3(1245)}{2(1.50)(7.25)} = \underline{172 \text{ PSI}}$$

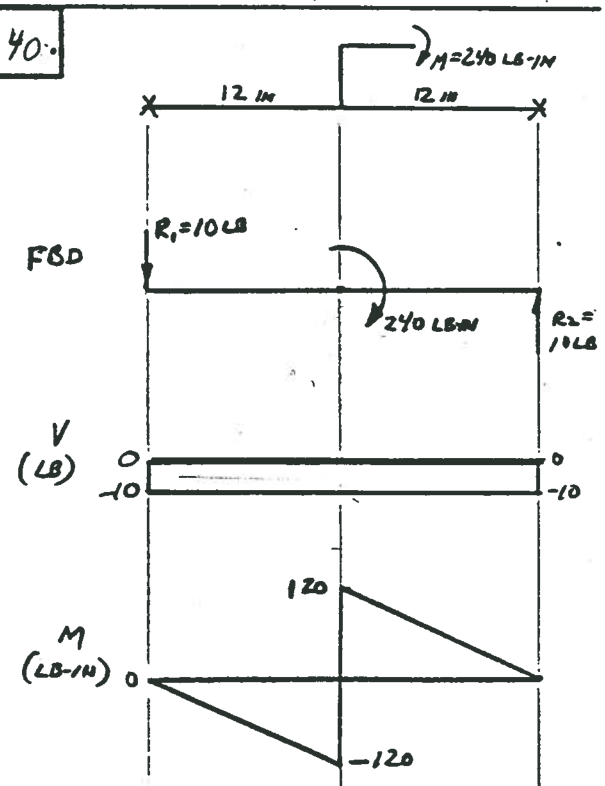



## Beams with Concentrated Bending Moments

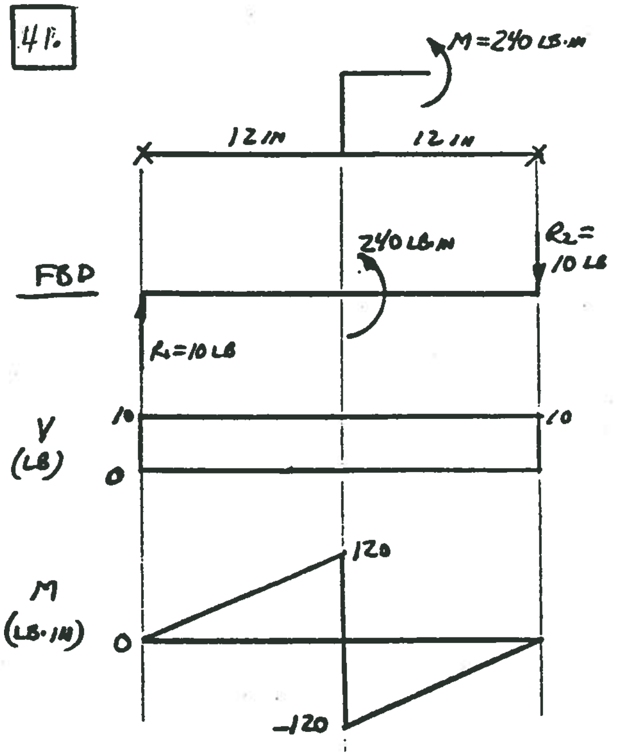
**39.**



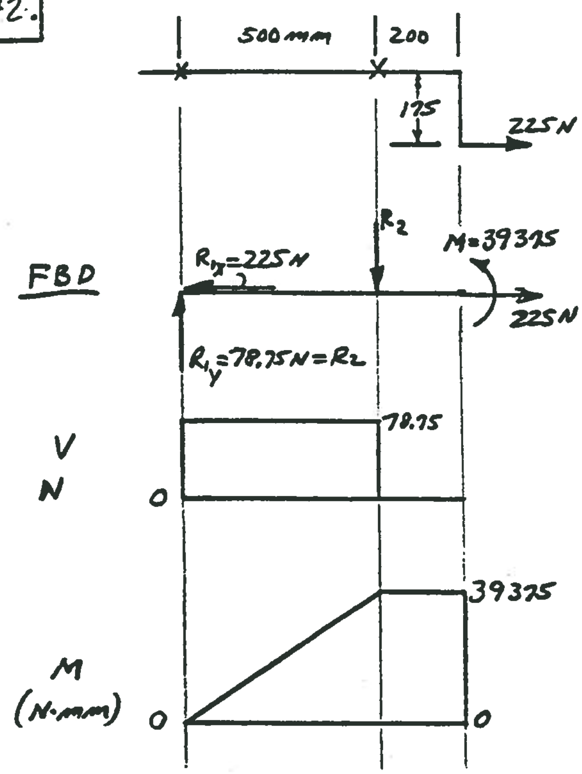

**40.**



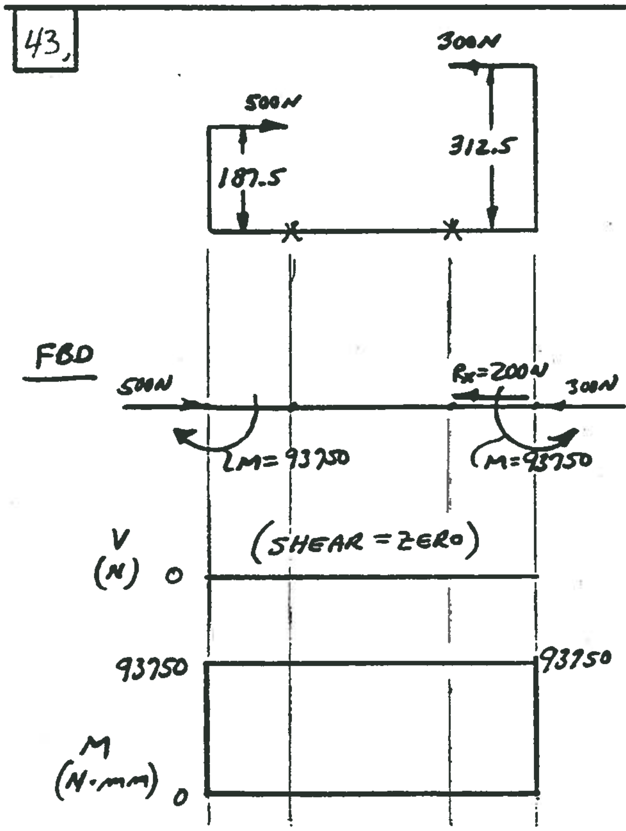
41.


M=240 LB·IN
12 IN
12 IN
FBD
240 LB·IN
R2= 10 LB
R1=10 LB
V (LB)
10
10
0
M (LB·IN)
120
0
-120

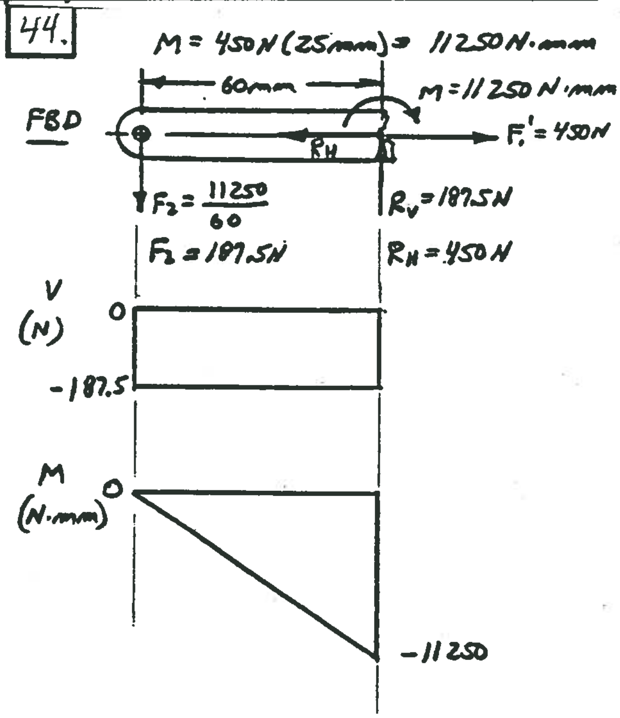
42.


500 mm
200
175
225 N
FBD
R2
M=39375
R1x=225 N
225 N
R1y=78.75 N=R2
V N
78.75
0
M (N·mm)
39375
0
0

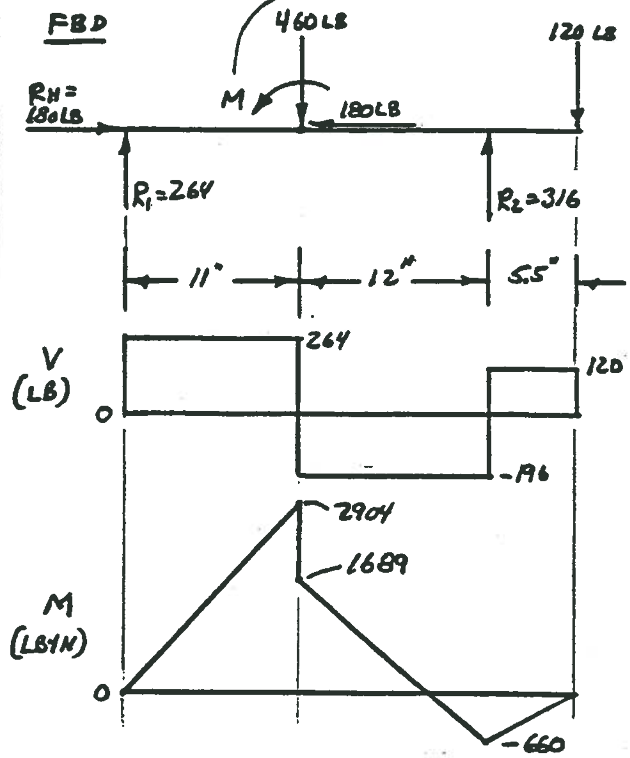
43.


300 N
500 N
312.5
187.5
FBD
500 N
Rx=200 N
300 N
M=93750
M=93750
V (N)
(SHEAR = ZERO)
0
93750
93750
M (N·mm)
0

44.


M = 450 N (25 mm) = 11250 N·mm
60 mm
M=11250 N·mm
FBD
RH
F1'=450 N
F2= 11250/60
F2=187.5 N
Rv=187.5 N
RH=450 N
V (N)
0
-187.5
M (N·mm)
0
-11250

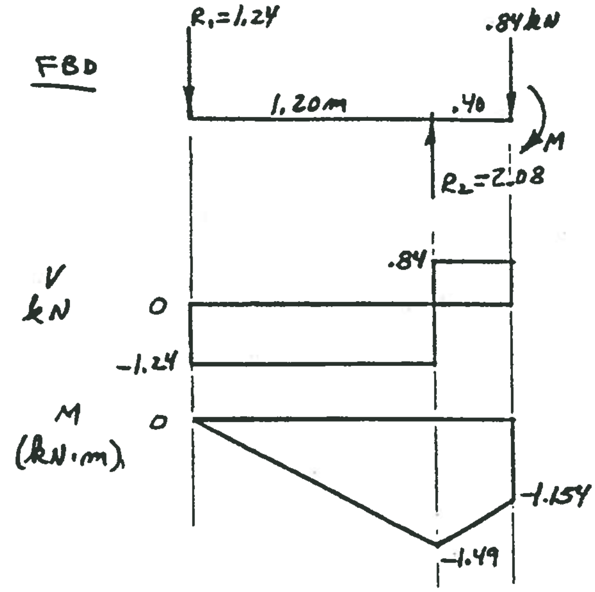
**45** $M = 180(6.75) = 1215 \text{ LB·IN}$



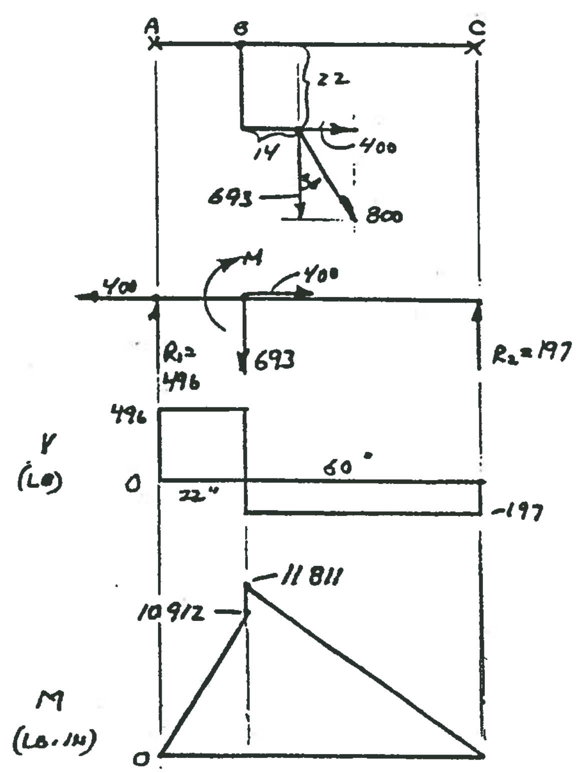

**46** AT RIGHT END:

NET VERTICAL FORCE $= 2.64 - 1.80 = 0.84 \text{ kN}$

$M = 2.64(2.60) + 1.80(.260) = 1.154 \text{ kN·m}$

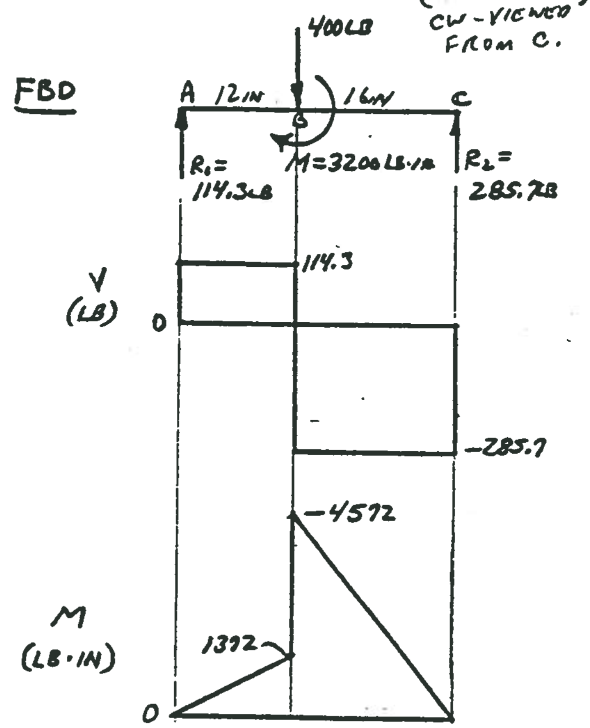



**47** $M = 693(14) - 400(22) = 899 \text{ LB·IN}$




**48.** $M = 200(4) + 600(4) = 3200 \text{ LB-IN}$

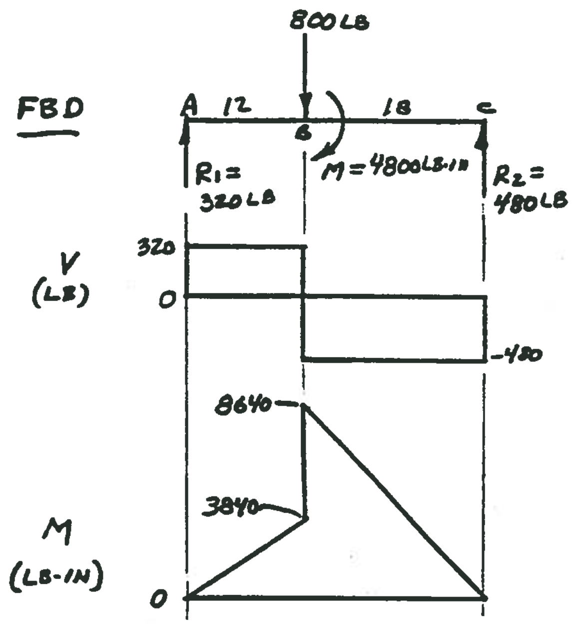
TORQUE $= T = 200(8) + 600(8) = 6400 \text{ LB·IN}$ (NOT SHOWN) CW-VIEWED FROM C.



**49**

$M = 600(6) + 200(6) = 4800$ LB-IN

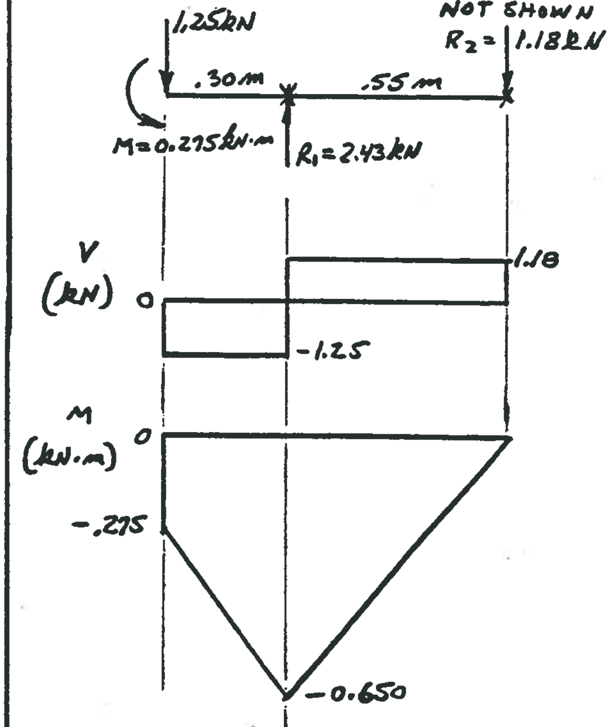
NET TORQUE $= 600(4) - 200(10) = 400$ LB-IN NOT SHOWN




**50**

$M = 1.25\,kN(.22\,m) = 0.275\,kN{\cdot}m$

$T = 1.25\,kN(.30\,m) = 0.375\,kN{\cdot}m$ NOT SHOWN




## Combined Normal Stresses

**51.**

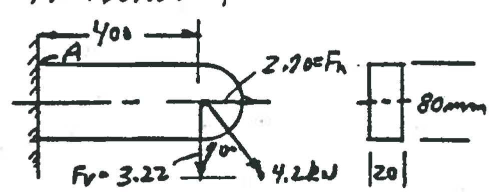
$A = 20(80) = 1600\,mm^2$

$S = bh^2/6 = 20(80)^2/6 = 21333\,mm^3$

$\sigma_A = \dfrac{F_h}{A} + \dfrac{F_v(400)}{S} = \dfrac{2700\,N}{1600\,mm^2} + \dfrac{(3220\,N)\,400\,mm}{21333\,mm^3}$

$\sigma_A = 1.69 + 60.38 = \underline{62.07\,MPa}$

$M = F_v(400\,mm)$




**52.**

$\tan\theta = 6/12 = 0.5 : \theta = 26.6°$

$\Sigma M_C = 0 = 1800(8) - A_y(12)$

$A_y = 1200$ LB ; $C_y = 1800 - A_y = 600$ LB

$A_x = 2A_y = 2400$ LB $= C_x$

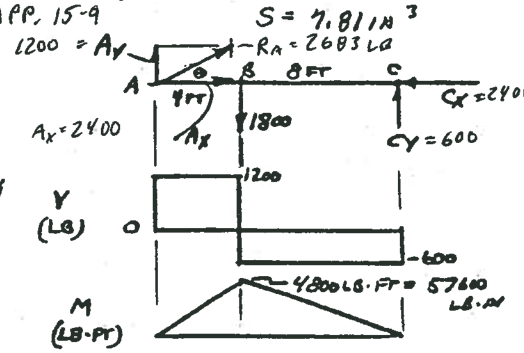
AT B ON TOP OF BEAM

$\sigma = -\dfrac{A_x}{A} - \dfrac{M}{S} = \dfrac{-2400}{2.96} - \dfrac{57600}{7.81} = -8186$ PSI (COMPR.)

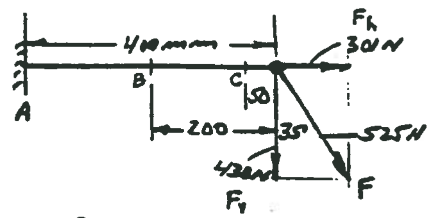
AT B ON BOTTOM OF BEAM

$\sigma = \dfrac{-A_x}{A} + \dfrac{M}{S} = \dfrac{-2400}{2.96} + \dfrac{57600}{7.81} = +6564$ PSI (TENSION)

FOR W 8X10 BEAM: $A = 2.96\,IN^2$
APP. 15-9 $S = 7.81\,IN^3$



**53.**




AT A (TOP)

$$\sigma = \frac{F_h}{A} + \frac{M}{S} = \frac{301N}{50(20)mm^2} + \frac{(430N)(400mm)}{(20)(50)^2/6\ mm^3}$$

$\sigma_A = 0.301 + 20.64 = \underline{20.94 MPa\ \text{TENSION}}$

AT B, $M = 430(200) = 86000\ N\cdot mm$

ASSUME AXIAL STRESS IS SMALL: $\sigma \approx M/S$

REQ'D $S = \frac{M}{\sigma} = \frac{86000\ N\cdot mm}{20.94\ N/mm^2} = 4107 mm^3 = th^2/6$

$h = \sqrt{\frac{6S}{t}} = \sqrt{\frac{6(4107)}{20}} = 35.1\ mm$  LET $h = 36mm$; $S = \frac{th^2}{6} = \frac{20(36)^2}{6} = 4320 mm^3$

$A = th = 720\ mm^2$

$$\sigma_B = \frac{301N}{720mm^2} + \frac{86000\ N\cdot mm}{4320\ mm^3} = \underline{20.33 MPa;\ OK}$$

AT C, $M = 430(50) = 21500\ N\cdot mm$

$S = \frac{M}{\sigma} = \frac{21500}{20.94} = 1027\ mm^3$; $h = \sqrt{\frac{6S}{t}} = \sqrt{\frac{6(1027)}{20}} = 17.6mm$

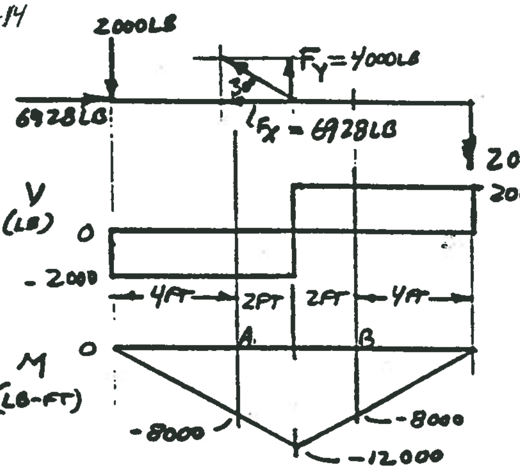
LET $h = 18mm$; $t = 20mm$; $S = 20(18)^2/6 = 1080mm^3$; $A = 18(20) = 360mm^2$

$$\sigma_C = \frac{301N}{360mm^2} + \frac{21500\ N\cdot mm}{1080\ mm^3} = \underline{20.74 MPa\ OK}$$

**54.** $6 \times 2 \times 1/4$ HOLLOW RECT. TUBE: APP. 15-14 FOR CROSS SECTION OF BEAM

$A = 3.59\ IN^2$

$S = 4.60\ IN^3$




AT A: $\sigma = \frac{-F_x}{A} + \frac{M}{S}$ TENSION ON TOP SURFACE

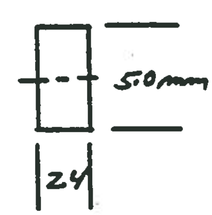
$$\sigma_A = \frac{-6928\ LB}{3.59 IN^2} + \frac{8000(12)\ LB\text{-}IN}{4.60 IN^3}$$

$\sigma_A = -1930\ psi + 20870\ psi = 18940\ psi$

AT B: $\sigma = \frac{M}{S} = \underline{20870\ psi}$

**55.**

FROM FIG 3-22; $M = 875\ N\cdot mm$; $F_x = 35.0N$




$A = 5.0(2.4) = 12.0\ mm$

$S = 2.4(5.0)^2/6 = 10.0\ mm^3$

$$\sigma = \frac{-F}{A} + \frac{M}{S} = \frac{-35.0N}{12.0mm^2} + \frac{875N\cdot mm}{10.0mm^3} = \underline{84.58\ MPa\ \text{TENSION}}$$

ON BOTTOM OF SECTION

$\boxed{56.}$

$BC = \sqrt{50^2+60^2} = 78.1\ IN$

$CF = 25IN/\sin\theta = 39.05\ IN$

$\left.\right\} BF = 117.15$

$\Sigma M_B = 0 = -C_Y(78.1) + F_Y(117.15)$

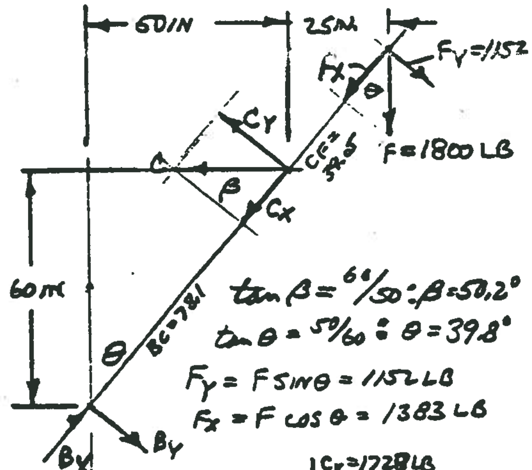
$C_Y = F_Y(117.15/78.1) = 1728\ LB$

$C_X = C_Y/\tan\beta = 1439\ LB$

$\Sigma M_C = 0 = 1152(39.05) - B_Y(78.1)$

$B_Y = 576\ LB$

$B_X = C_X + F_X = 1439 + 1383 = 2822\ LB$




$\tan\beta = 60/50 : \beta = 50.2°$

$\tan\theta = 50/60 : \theta = 39.8°$

$F_Y = F\sin\theta = 1152\ LB$

$F_X = F\cos\theta = 1383\ LB$

FOR 6x4x1/4 TUBE: (APP. 15-14)

$A = 4.59\ IN^2;\ S_X = 7.36\ IN^3$

JUST ABOVE C:

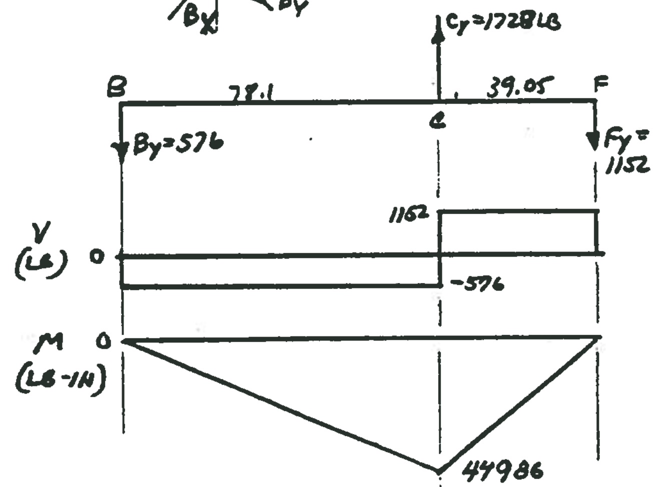
$$\sigma = \frac{-F_X}{A} + \frac{M}{S} = \frac{-1383}{4.59} + \frac{44986}{7.36}$$

$\sigma = \underline{5811\ PSI}$ TENSION

JUST BELOW C:

$$\sigma = \frac{-B_X}{A} - \frac{M}{S} = \frac{-2822}{4.59} - \frac{44986}{7.36}$$

$\sigma = \underline{-6727\ PSI}$ COMPRESSION



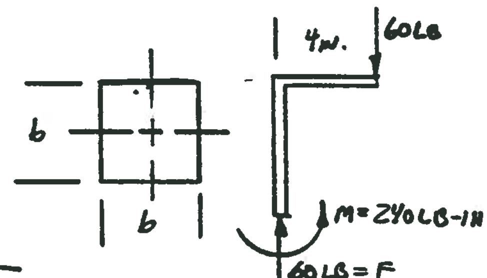

$\boxed{57.}$

$A = b^2;\ S = b^3/6$

$$\sigma = -\frac{F}{A} - \frac{M}{S} = \frac{-60}{b^2} - \frac{240}{b^3/6}$$

$$\sigma = -\frac{60}{b^2} - \frac{1440}{b^3}$$ BUT ASSUME F/A IS SMALL

$$\sigma \approx -1440/b^3 : b = \sqrt[3]{\frac{-1440}{\sigma_{ALLOW}}} = \sqrt[3]{\frac{1440\ LB\cdot IN}{-12\,000\ LB/IN^2}} = 0.493\ IN$$

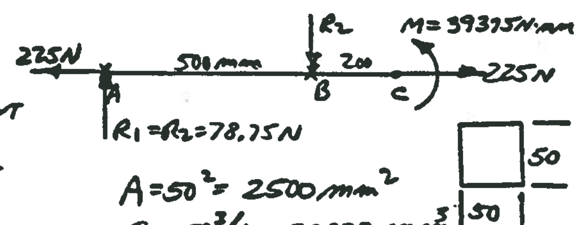



TRY $\underline{b = 0.500\ IN} = 1/2\ IN : A = 0.25\ IN^2;\ S = b^3/6 = 0.0208\ IN^3$

$$\sigma = \frac{-60\ LB}{0.25\ IN^2} - \frac{240\ LB\text{-}IN}{0.0208\ IN^3} = -240 - 11520 = -11760\ PSI\ \underline{OK}$$

$\boxed{58.}$ FBD FROM PROBLEM 3-42:

PART FROM B-C SEES 39375 N·mm MOMENT




$$\sigma_{MAX} = \frac{F}{A} + \frac{M}{S} = \frac{225\ N}{2500\ mm^2} + \frac{39375\ N\cdot mm}{20833\ mm^3}$$

$A = 50^2 = 2500\ mm^2$

$S = 50^3/6 = 20833\ mm^3$

$\sigma_{MAX} = 0.09 + 1.89 = \underline{1.98\ MPa}$ TENSION ON BOTTOM SURFACE BETWEEN B AND C.

**59.** FROM PROB. 3-43: $M = \text{CONSTANT} = 93750\ N \cdot mm$

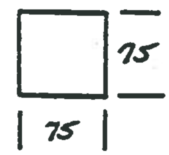
$F_x = 500N$ FROM A TO C


$$\sigma_{MAX} = -\frac{F_x}{A} - \frac{M}{S}$$

$$= \frac{-500N}{5625\ mm^2} - \frac{93750\ N \cdot mm}{70\,313\ mm^2}$$

$A = 75^2 = 5625\ mm^2$

$S = 75^3/6 = 70313\ mm^3$




$\sigma_{MAX} = -0.089 - 1.333 = \underline{7.422\ MPa}$ COMPRESSION ON TOP SURFACE BETWEEN A AND C

**60.**

FROM PROB 3-47: POINT B IS WHERE BRACKET ATTACHES

TO RIGHT OF B; $M = 11811\ LB \cdot IN$; $\sigma = M/S$; REQ'D $S = \frac{M}{\sigma} = \frac{11811\ LB \cdot IN}{25000\ LB/IN^2} = 0.472\ IN^3$

$S = \pi D^3/32$; $D = \sqrt[3]{32\,S/\pi} = \sqrt[3]{32(0.472)/\pi} = 1.69\ IN$ ∴ USE $D = 1.75\ IN$

CHECK TO LEFT OF B: $M = 10912\ LB \cdot IN$; $F_x = 400\ LB$ $\quad A = \pi D^2/4 = 2.41\ IN^2$

$$\sigma = \frac{F_x}{A} + \frac{M}{S} = \frac{400}{2.41} + \frac{10912}{0.526} = 166 + 20739 = 20900\ PSI \quad S = \pi D^3/32 = 0.526\ IN^3$$

OK

**Stress Concentrations: $K_t$ factors obtained from the website:**

[www.efatigue.com/constantamplitude/stress-concentration /](http://www.efatigue.com/constantamplitude/stress-concentration/) **[Note for a flat plate with a circular hole in bending: For $d/W < 0.50$, use $K_t = 1.0$. *The eFatigue site does not show this.***

**61.** $\left.\begin{array}{l} D/d = 9/6 = 1.50 \\ r/d = 0.5/6 = 0.083 \end{array}\right\} K_t = 2.03$

$$\sigma_{MAX} = \frac{K_t F}{A} = \frac{(2.03)(1250N)}{\frac{\pi(6)^2}{4}\ mm^2} = \underline{89.7\ MPa}$$

**62.** $\sigma_{MAX} = K_t F/A$

BOTTOM: $\left.\begin{array}{l} D/d = 2.0/0.5 = 4.0 \\ r/d = 0.08/0.5 = 0.16 \end{array}\right\} K_t = 1.86$

$$\sigma_m = \frac{(1.86)(1200)}{\pi(.5)^2/4} = \underline{11{,}367\ PSI}\ \ MAX$$

MIDDLE: $\left.\begin{array}{l} D/d = 2.0/0.75 = 2.67 \\ r/d = .08/.75 = 0.11 \end{array}\right\} K_t = 2.08$

$$\sigma_m = \frac{(2.08)2400}{\pi(.75)^2/4} = 11{,}300\ psi$$

TOP: $\left.\begin{array}{l} D/d = 2.0/1.0 = 2.0 \\ r/d = .08/1.0 = .08 \end{array}\right\} K_t = 2.22$

$$\sigma_m = \frac{(2.22)(3600)}{\pi(1.0)^2/4} = 10\,176\ psi$$

**63.** LEFT HOLE: $d/w = .72/1.40 = 0.51$

$K_t = 2.15$

$$\sigma_m = \frac{(2.15)(6200)}{(1.40 - .72)0.5} = \underline{39\,206\ PSI\ \ MAX}$$

MIDDLE: $d/w = 0.40/1.40 = 0.29 \rightarrow K_t = 2.38$

$$\sigma_m = \frac{(2.38)(6200)}{(1.40 - .40)(.5)} = \underline{29\,512\ PSI}$$

RIGHT: $d/w = .50/1.40 = 0.36 \rightarrow K_t = 2.29$

$$\sigma_m = \frac{(2.29)(6200)}{(1.40 - .50)(.5)} = \underline{31{,}551\ PSI}$$

**64.** $\left.\begin{array}{l} H/h = 1.50/0.80 = 1.875 \\ r/h = 0.12/0.80 = 0.15 \end{array}\right\} K_t = 2.25$

$$\sigma_m = \frac{2.25(1625)}{(.25)(0.80)} = \underline{18281\ PSI}$$

**65.** $\left.\begin{array}{l} D/d = 42/30 = 1.40 \\ r/d = 1.50/30 = 0.05 \end{array}\right\} K_t = 2.30$

$$\sigma_m = \frac{(2.30)(30300)\ N}{\pi(30)^2/4\ mm^2} = \underline{98.6\ MPa}$$

**66.** $\left.\begin{array}{l} D/d = 2.00/1.25 = 1.60 \\ r/d = 0.10/1.25 = 0.08 \end{array}\right\} K_t = 1.43$

$$\tau_{MAX} = \frac{(1.43)(2200)\ LB \cdot IN}{\pi(1.25)^3/16\ IN^3} = \underline{8203\ PSI}$$

**67.**

$D/d = 2.00/1.25 = 1.60$
$r/d = 0.06/1.25 = 0.048$ $\Big\}$ $K_t = 2.23$ $\therefore$ $\sigma_M = \dfrac{2.23\,(2800)\text{ LB·IN}}{\pi(1.25)^3/32\text{ IN}^3} = \underline{32\,564\text{ PSI}}$

**68.**

$d/w = 1.38/2.00 = 0.69 \rightarrow K_t = 1.38$

$\sigma_M = K_t\,\sigma_{NOM} = \dfrac{K_t\,6MW}{(W^3-d^3)t} = \dfrac{(1.38)(6)(12000)(2.00)}{[(2.00)^3-(1.38)^3]\,0.75} = \underline{49\,323\text{ PSI}}$

## Problems of a General Nature

**69.**

$\Sigma M_C = 0 = 12.5\text{ kN}(4.0\text{ m}) - R_B(2.5\text{ m})$

$R_B = (12.5)(4.0)/2.5 = 20.0\text{ kN}\uparrow$

$R_C = 20\text{ kN} - 12.5\text{ kN} = 7.5\text{ kN}\downarrow$

$M_{MAX} = 18.75\text{ kN·m} \times \dfrac{10^3\text{ N}}{\text{kN}} \cdot \dfrac{10^3\text{ mm}}{\text{m}}$

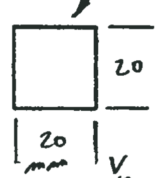
$M_{MAX} = 18.75 \times 10^6\text{ N·mm}$

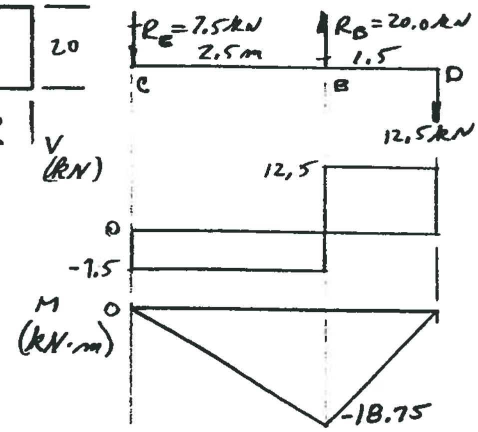
SECTION MODULUS $= S = a^3/6 = (20.0\text{ mm})^3/6$

$S = 1333\text{ mm}^3$

AREA OF $\overline{AB} = (20\text{ mm})^2 = 400\text{ mm}^2$

CROSS SECTION OF $\overline{AB}$ & $\overline{CD}$




SHEAR AREA OF PIN - DOUBLE SHEAR

$A_S = \dfrac{2\pi D^2}{4} = \dfrac{\pi D^2}{2} = \dfrac{\pi(8.0\text{ mm})^2}{2} = 100.5\text{ mm}^2$

TENSION IN $\overline{AB}$: $\sigma = \dfrac{R_B}{A} = \dfrac{20\,000\text{ N}}{400\text{ mm}^2} = \underline{50\text{ MPa}}$

SHEAR IN PIN: $\tau = \dfrac{R_B}{A_S} = \dfrac{20\,000\text{ N}}{100.5\text{ mm}^2} = \underline{199\text{ MPa}}$

BENDING IN $\overline{CD}$; AT B: $\sigma_B = M/S = \dfrac{18.75\times10^6\text{ N·mm}}{1333\text{ mm}^3} = \underline{14\,063\text{ MPa}}$

VERY HIGH

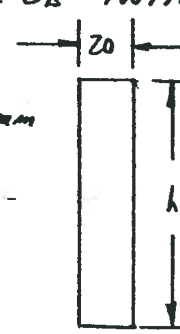
SUGGEST CHANGE IN CROSS SECTION OF $\overline{CD}$ TO MAKE $\sigma_B < 400\text{ MPa}$

REQ'D $S = M/\sigma = \dfrac{18.75\times10^6\text{ N·mm}}{400\text{ N/mm}^2} = 46\,875\text{ mm}^3$

$S = (20)(h)^2/6$ : REQ'D $h = \sqrt{6S/20} = \sqrt{\dfrac{6(46\,875\text{ mm}^3)}{20\text{ mm}}} = 118\text{ mm}$

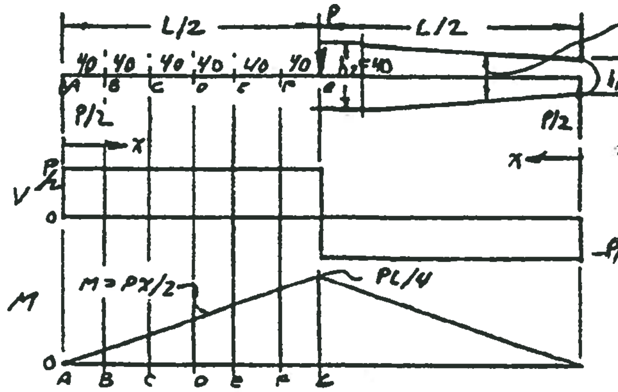
LET $h = 120$ mm : THEN $S = \dfrac{(20)(120)^2\text{ mm}^3}{6} = 48000\text{ mm}^3$

$\sigma = \dfrac{M}{S} = \dfrac{18.75\times10^3\text{ N·mm}}{48000\text{ mm}^3} = \underline{391\text{ MPa}}$ $\underline{\text{OK}}$




PROPOSED CROSS SECTION OF $\overline{CD}$

**70.**




$h_x = 20 + 0.1x$ FOR $x \leq 200$

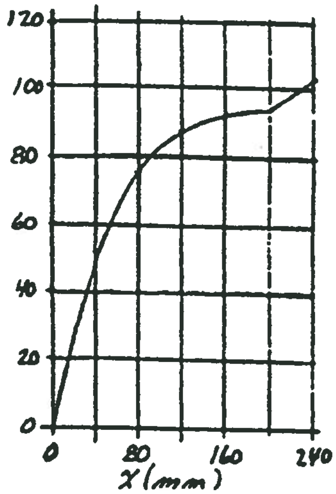
$\sigma = M/S$

$S = th^2/6$

$P = 5.0$ kN

$t = 20$ mm

| | x (m) | M (kN·m) | h (mm) | S mm³ | σ MPa |
|---|---|---|---|---|---|
| A | 0 | 0 | 20 | 1333 | 0 |
| B | 0.040 | .020P = 0.10 | 24 | 1920 | 52.1 |
| C | 0.080 | .040P = 0.20 | 28 | 2613 | 76.5 |
| D | 0.120 | .060P = 0.30 | 32 | 3413 | 87.9 |
| E | 0.160 | .080P = 0.40 | 36 | 4320 | 92.6 |
| F | 0.200 | .100P = 0.50 | 40 | 5333 | 93.8 |
| G | 0.240 | .120P = 0.60 | 40 | 5333 | 112.5 |



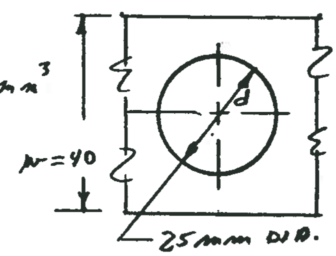

**71.**

FROM PROB. 70, $M = 0.60$ kN·m UNDER LOAD.

$$\sigma = \frac{K_t M}{S_{NET}} \; ; \; S_{NET} = \frac{(w^3 - d^3)t}{6w} = \frac{(40^3 - 25^3)(20)}{6(40)} = 4031 \text{ mm}^3$$

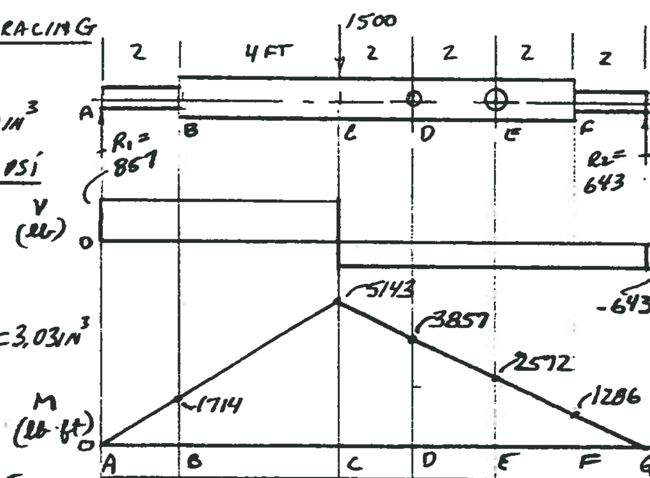
$d/w = 25/40 = 0.625$; $K_t = 1.25$

$$\sigma = \frac{(1.25)(0.60 \text{ kN·m})}{4031 \text{ mm}^2} \cdot \frac{10^3 \text{ N}}{\text{kN}} \cdot \frac{10^3 \text{ mm}}{\text{m}} = \underline{186 \text{ MPa}}$$




**72.**

$\sigma = \dfrac{K_t M}{S}$ ASSUME LATERAL BRACING




(a) AT C AT LOAD; $M_C = 5143$ LB·FT

$K_t = 1.0$; $S = tH^2/6 = \dfrac{(1.20)(4.0)^2}{6} = 3.20 \text{ IN}^3$

$$\sigma_C = \frac{(1.0)(5143 \text{ LB·FT})(12 \text{ IN/FT})}{3.20 \text{ IN}^3} = \underline{19286 \text{ PSI}}$$

(b) AT D AT 1.50 IN DIA. HOLE

$M_D = 3857$ LB·FT; $d/w = \dfrac{1.50}{4.00} = 0.375$

$K_t = 1.00$

$$S_{NET} = \frac{(w^3 - d^3)(t)}{6w} = \frac{(4.0^3 - 1.50^3)(1.20)}{6(4.0)} = 3.03 \text{ IN}^3$$

$$\sigma_D = \frac{(1.0)(3857)(12)}{3.03} = \underline{15270 \text{ PSI}}$$

(c) AT E AT 2.50 IN DIA. HOLE

$M_E = 2572$ LB·FT; $d/w = \dfrac{2.50}{4.00} = 0.625$

$K_t = 1.25$

$$S_{NET} = \frac{(4.0^3 - 2.50^3)(1.20)}{6(4.00)} = 2.419 \text{ IN}^3$$

$$\sigma_E = \frac{(1.25)(2572)(12)}{(2.419)} = \underline{15948 \text{ PSI}}$$

(d) AT B AT STEP; $H = 4.00$, $h = 2.80$, $r = 0.15$

$H/h = 1.43$; $r/h = 0.054$; $K_t = 2.2$

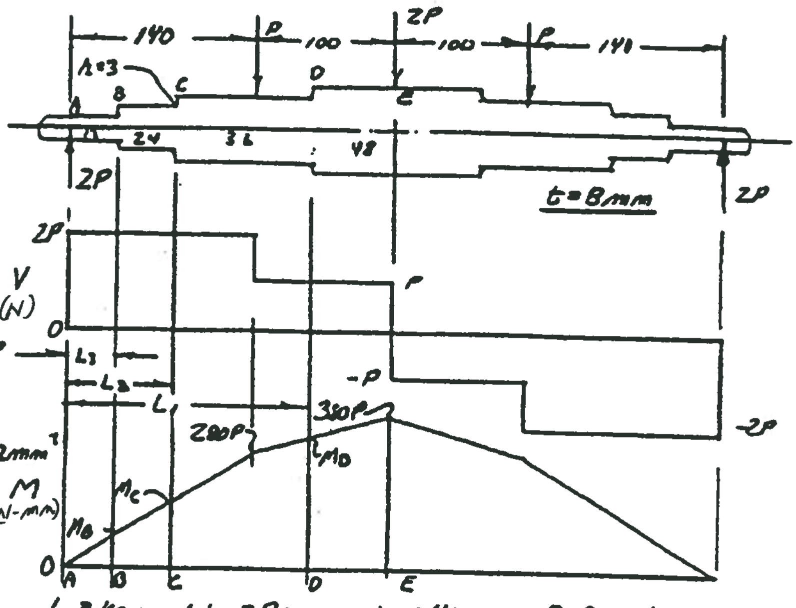
$M_B = 1714$ LB·FT; $S = \dfrac{th^2}{6} = \dfrac{(1.2)(2.8)^2}{6} = 1.57 \text{ IN}^3$

$$\sigma_B = \frac{(2.2)(1714)(12)}{1.57} = \underline{29952 \text{ PSI}}$$

MAXIMUM

73.

$$\sigma = \frac{K_t M}{S}$$

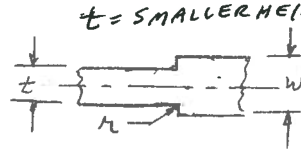



$M_B = 2PL_3$
$M_C = 2PL_2$
$M_D = 280P + (L_1 - 140)P$
$M_D = 140P + L_1 P$
$S = th^2/6$
$S_B = 8(12)^2/6 = 192\ mm^3$
$S_C = \frac{8(24)^2}{6} = 768$
$S_D = \frac{8(36)^2}{6} = 1728$
$S_E = \frac{8(48)^2}{6} = 3072$

$L_1 = 180\ mm$; $L_2 = 80\ mm$; $L_3 = 40\ mm$; $P = 200\ N$

| POINT | M (N-mm) | S (mm³) | r/t | t/w | Kt | σ (MPa) |
|---|---|---|---|---|---|---|
| B | 16000 | 192 | .25 | 0.50 | 1.46 | 121.7 MAXIMUM |
| C | 32000 | 768 | .125 | 0.67 | 1.72 | 71.67 |
| D | 64000 | 1728 | .083 | 0.75 | 1.88 | 69.63 |
| E | 76000 | 3072 | — | — | 1.0 | 24.7 |

FOR $K_t$:
r = FILLET RADIUS
w = LARGER HEIGHT
t = SMALLER HEIGHT




74.

$M = F(52 + 25/2) = (2500\ N)(64.5\ mm) = 161250\ N\cdot mm$ ALONG UPPER PART

$$\sigma = \frac{K_t M}{S_{NET}}\ ;\quad S_{NET} = \frac{(w^3 - d^3)(t)}{6w}$$

AT B-B: $d/w = 15/25 = 0.6 \rightarrow K_t = 1.20$

$$\sigma = \frac{(1.20)(6)(161250)(25)}{(25^3 - 15^3)(16)} = \underline{148.1\ MPa}$$

NOTE: For $K_t$ at hole in flat plate in bending: If $d/W < 0.50$, use $K_t = 1.0$

75.

SEE ALSO PROBLEM 74.

AT B-B: $d/w = 12/25 = 0.48 \rightarrow K_t = 1.0$

$$\sigma = \frac{K_t 6 M w}{(w^3 - d^3)(t)} = \frac{(1.0)(6)(161250)(25)}{(25^3 - 12^3)(16)} = \underline{108.8\ MPa}$$

76.

AT FULCRUM: $S = \frac{th^2}{6} = \frac{(0.75)(2.00)^2}{6} = 0.50\ IN^3$

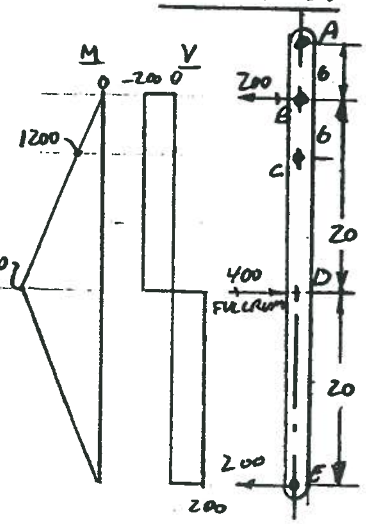
$\sigma_D = M/S = 4000\ LB\cdot IN. / 0.50\ IN^3 = \underline{8000\ PSI}$

AT HOLE C: APP 15-2: $d/w = 1.25/2.0 = 0.625$; $K_t = 1.25$

$$S_{NET} = \frac{(w^3 - d^3)t}{6w} = \frac{(2.00^3 - 1.25^3)(0.75)}{6(2.00)} = 0.378\ IN^3$$

$$\sigma_C = \frac{K_t M_C}{S} = \frac{1.25(1200)}{0.378} = \underline{3969\ PSI}$$

PIVOT AT B.



$\boxed{77.}$ PIVOT AT A.

AT FULCRUM: $S = \frac{th^2}{6} = \frac{(0.75)(2.00)^2}{6} = 0.50\ IN^3$

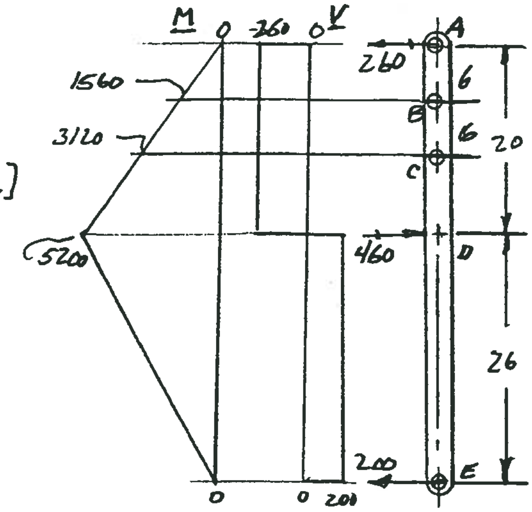
$\sigma_D = \frac{M}{S} = \frac{5200\ LB\cdot IN}{0.50\ IN^3} = \underline{10{,}400\ PSI}$

AT B: $K_t = 1.25$; $S_{NET} = 0.378\ IN^3$ [PROB 76.]

$\sigma_B = \frac{K_t M}{S} = \frac{1.25(1560)}{0.378} = \underline{5160\ PSI}$

AT C:

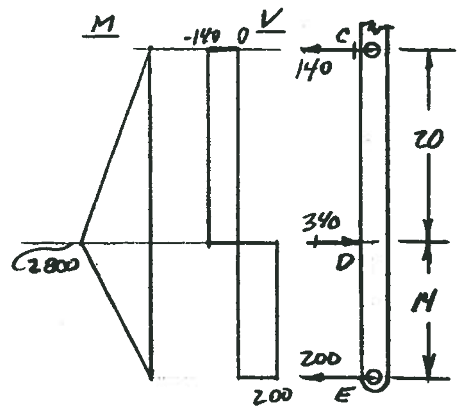
$\sigma_C = \frac{K_t M}{S} = \frac{1.25(3120)}{0.378} = \underline{10320\ PSI}$



PIVOT AT C.

AT FULCRUM: $S = 0.50\ IN^3$

$\sigma_D = \frac{M_D}{S} = \frac{2800\ LB\cdot IN}{0.50\ IN^3} = \underline{5600\ PSI}$



$\boxed{78.}$

$M = 0$ AT A, E

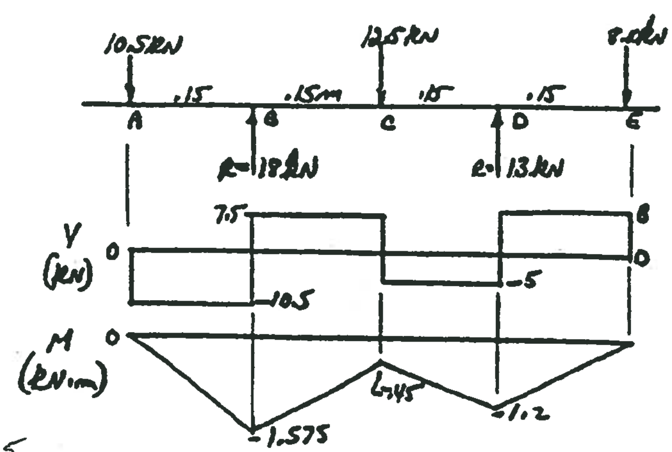
POINT B IS CRITICAL

$\sigma = \frac{M K_t}{S}$

$S = \frac{\pi (45)^3}{32} = 8946\ mm^3$

FOR FILLET

$\left.\begin{aligned} \frac{r}{d} &= \frac{2}{45} = 0.044 \\ \frac{D}{d} &= \frac{55}{45} = 1.22 \end{aligned}\right\} K_t = 2.05$



$\sigma = \frac{M K_t}{S} = \frac{1.575 \times 10^3\ N\cdot m\ (2.05)}{8946\ mm^3} \times \frac{10^3\ mm}{m} = \underline{360.9\ MPa}$

ONE POSSIBLE DESIGN

79. LUG JOINT: SEE FIG. 3-69; $d = 8.0$ mm, $w = 20.0$ mm
FIND: THICKNESS $t$, MATERIALS FOR LUG AND PIN. $N = 5$.
FROM SOLUTION FOR PROB. 3-69, $F = 20.0$ kN.
LET $t = 0.5d = 0.5(8.0\text{ mm}) = 4.0$ mm
LET $w = d/0.40 = 8.0\text{ mm}/0.40 = 20.0$ mm
LET END DISTANCE $= h = w = 20.0$ mm
HOLE DIAMETER $= d_{hole} = d_{pin}(1.002) = 8.016$ mm

$$\sigma_{max} = K_t\sigma_{nom};\ \sigma_{nom} = \frac{F}{(w-d)t} = \frac{20000\text{ N}}{(20-8)(4.0)\text{ mm}^2} = 416.7\text{ MPa}$$

IN FIG 3-29: $d/w = 8/20 = 0.40$; $K_t = 3.00$

$$\sigma_{max} = K_t\sigma_{nom} = (3.00)(416.7\text{ MPa}) = 1250\text{ MPa}$$

LET $\sigma_{max} = \sigma_d = \frac{S_u}{N}$; REQ. $S_u = N\sigma_{max} = 5(1250) = 6250$ MPa
TOO HIGH FOR TYPICAL STEELS IN APP. 3.

RE-DESIGN FOR USE OF SAE 4340 OQT 800, $S_u = 1450$ MPa

$$\sigma_d = \frac{S_u}{N} = \frac{1450\text{ MPa}}{5} = 290\text{ MPa}$$

LET $\sigma_d = \sigma_{max} = K_t\sigma_{nom} = 3.0\,\sigma_{nom}$ FOR $d/w = 0.40$

$$\sigma_{nom} = \frac{\sigma_d}{3.0} = \frac{290\text{ MPa}}{3} = 96.67\text{ MPa} = \frac{F}{A_{net}} = \frac{F}{(w-d)(t)}$$

$$\text{REQ'D } A_{net} = \frac{F}{96.67\text{ MPa}} = \frac{20000\text{ N}}{96.67\text{ N/mm}^2} = 206.9\text{ mm}^2$$

FOR $w = \frac{d}{0.40} = 2.5d$; $t = 0.5d$:

$$A_{net} = (w-d)(t) = (2.5d - d)(0.5d) = 0.75d^2 = 206.9\text{ mm}^2$$

$$d = \sqrt{206.9\text{ mm}^2/0.75} = 16.6\text{ mm}.$$

SPECIFY PREFERRED SIZE: $d = 18$ mm (APP. 2)

THEN: $w = \frac{d}{0.4} = \frac{18}{0.4} = 45.0$ mm; $t = 0.5d = 9.0$ mm

$$\sigma_{nom} = \frac{F}{(w-d)(t)} = \frac{20000\text{ N}}{(45-18)(9)} = \frac{20000\text{ N}}{243\text{ mm}^2} = 82.3\text{ MPa}$$

(GIVEN DESIGN PARAMETERS WERE NOT FEASIBLE.)

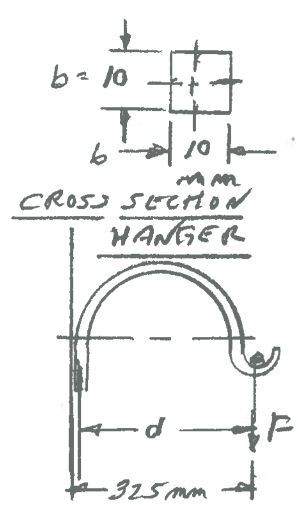
CHECK SHEAR STRESS IN PIN: $\tau = \frac{F}{A_s} = \frac{F}{2A} = \frac{F}{2\pi D^2/4} = \frac{F}{\pi D^2/2}$

$$\tau = \frac{20000\text{ N}}{\pi(18\text{ mm})^2/2} = 39.3\text{ MPa}$$

CHECK $\tau_d = \frac{S_{su}}{N} = \frac{0.75S_u}{5} = \frac{0.75(1450\text{ MPa})}{5} = 217.5\text{ MPa} > 39.3\text{ MPa}$
OK FOR PIN SHEAR

HOLE DIA. $= d_{HOLE} = d_{PIN}(1.002) = 18.0\text{ mm}(1.002) = 18.036$ mm

81 CURVED BEAMS: FIND F FOR YIELDING OF STEEL




ASTM A36 STEEL $S_y = 36\,ksi = 248\,MPa$

$\sigma_o = \dfrac{M(R - r_o)}{A\,r_o(r_c - R)}$ $\quad\sigma_i = \dfrac{M(R - r_i)}{A\,r_i(r_c - R)}$

$R = A/ASF;\ A = b^2 = 10^2 = 100\,mm^2$

GIVEN: $r_i = 150\,mm;\ r_o = r_i + 10 = 160\,mm$

$r_c = r_i + b/2 = 150 + 5 = 155\,mm$

$ASF = b\ln(r_o/r_i) = 10\ln(160/150) = 0.64539\,mm$

$R = A/ASF = 100/0.64539 = 154.946\,mm$

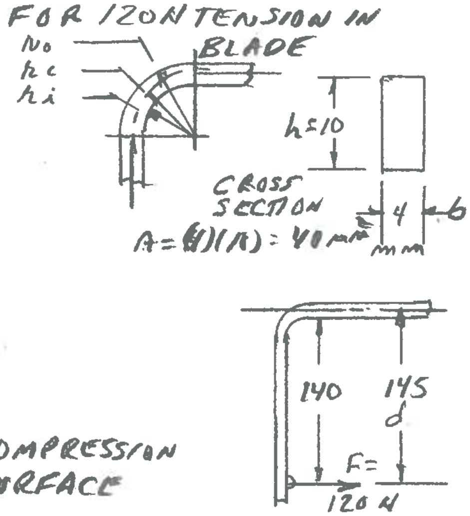
$M = F\cdot d = F(325 - b/2) = 320F$ (NEGATIVE)

$\sigma_o = \dfrac{(-320F)(154.946 - 160)}{(100)(160)(155 - 154.946)} = \dfrac{1.87948F}{mm^2}$

$\sigma_i = \dfrac{(-320F)(154.946 - 150)}{(100)(150)(155 - 154.946)} = \dfrac{-1.962F}{mm^2}$ MAXIMUM AT INSIDE SURFACE

LET $\sigma_{MAX} = \dfrac{-1.962F}{mm^2} = -248\,N/mm^2$; COMPRESSIVE YIELD STRENGTH $\ F = \dfrac{-248\,N}{-1.962} = 126.4\,N$ ANSWER

82 CURVED BEAM: COPING SAW FIND N FOR 120N TENSION IN BLADE




SAE 1020 CD STEEL; $S_y = 352\,MPa$

$M = F\cdot d = (120N)(145\,mm) = -17400\,N\cdot mm$ (NEG.)

$r_i = 22\,mm$ GIVEN; $r_o = 22 + 10 = 32\,mm$

$r_c = 22 + 5 = 27\,mm$

$ASF = b\ln(r_o/r_i) = 4\ln(32/22) = 1.4987\,mm$

$R = A/ASF = 40/1.4987 = 26.688\,mm$

SEE PROB. 81 FOR EQUATIONS.

$(R - r_i) = 26.688 - 22 = 4.688\,mm$

$(r_c - R) = 27.0 - 26.688 = 0.3115\,mm$

$(R - r_o) = 26.688 - 32.0 = -5.3115\,mm$

$\sigma_i = \dfrac{(-17400)(4.688)}{(40)(22)(0.3115)} = -297.6\,MPa$ COMPRESSION ON INSIDE SURFACE

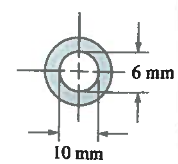
$\sigma_o = \dfrac{(-17400)(-5.3115)}{(40)(32)(0.3115)} = 231.8\,MPa$ TENSION ON OUTSIDE SURFACE

$N_i = \dfrac{S_y}{\sigma_i} = \dfrac{-352\,MPa}{-297.6\,MPa} = 1.18$ MINIMUM

$N_o = \dfrac{S_y}{\sigma_o} = \dfrac{352\,MPa}{231.8\,MPa} = 1.52$

BOTH LOW

83 CURVED BEAM HACKSAW FIG. P3-83

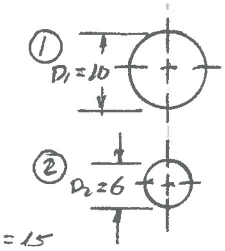


Section A–A

GIVEN: $F = 480N$; FIND N.

SAE 1020 CD; $S_y = 352\ MPa$

CONSIDER TUBE TO BE A COMPOSITE SECTION: ① − ②




$A_1 = \pi(10)^2/4 = 78.54\ mm^2$; $A_2 = \pi(6)^2/4 = 28.27\ mm^2$

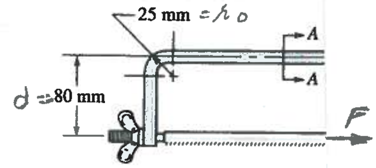
$A = A_1 - A_2 = 50.265\ mm^2$

$ASF = ASF_1 - ASF_2$

$ASF_1 = 2\pi\left[r_c - (r_c^2 - D_1^2/4)^{1/2}\right] = 3.9903\ mm$

$ASF_2 = 2\pi\left[r_c - (r_c^2 - D_2^2/4)^{1/2}\right] = 1.4218\ mm$

$r_c = 20$; $r_i = 15$




$ASF = 3.9903 - 1.4218 = 2.5685\ mm$

$$R = \frac{A}{ASF} = \frac{50.265\ mm^2}{2.5685\ mm} = 19.569\ mm$$

SEE PROBLEM 81 FOR EQUATIONS:

$(R - r_o) = (19.569 - 25) = -5.4307\ mm$

$(r_c - R) = (20 - 19.569) = 0.4307\ mm$

$(R - r_i) = (19.569 - 15) = 4.569\ mm$

$M = F \cdot d = -480N(80\ mm) = 38400\ N \cdot mm$

$$\sigma_i = \frac{(-38400)(4.569)}{(50.265)(15)(0.4307)} = -540.3\ MPa$$ COMPRESSION ON INSIDE SURFACE

$$\sigma_o = \frac{(-38400)(-5.4307)}{(50.265)(25)(0.4307)} = 385.3\ MPa$$ TENSION ON OUTSIDE SURFACE

$$N_i = \frac{S_Y}{\sigma_i} = \frac{-352\ MPa}{-540.3\ MPa} = 0.651$$

$$N_o = \frac{S_Y}{\sigma_o} = \frac{352\ MPa}{385.3\ MPa} = 0.913$$

BOTH INDICATE FAILURE.

ADDITION TO PROBLEM: FIND $F_{ALLOW}$ TO ACHIEVE $N \geq 2.0$

STRESSES ARE PROPORTIONAL TO MOMENT AND TO F.

STRESS REDUCTION REQD.: $\sigma_{ALL} = \frac{S_Y}{2} = \frac{352\ MPa}{2} = 176\ MPa$

$$\frac{\sigma_{ALL}}{\sigma_i} = \frac{-176\ MPa}{-540.3\ MPa} = 0.326 = \frac{M_{ALL}}{M_1}$$

$M_{ALL} = 0.326\ M_1 = 0.326(-38400) = -12509\ N \cdot mm = F_{ALL} \cdot d$

$$F_{ALL} = \frac{12509\ N \cdot mm}{80\ mm} = 156.4\ N$$

84 CURVED BEAM GARDEN TOOL

FIND F FOR YIELDING.

CAST ALUMINUM: 356.0-T6, $S_y = 207$ MPa

$ASF = 2\pi\left[r_c - (r_c^2 - D^2/4)^{1/2}\right]$

$ASF = 2\pi\left[12 - (12^2 - 8^2/4)^{1/2}\right] = 4.312$ mm

$r_c = 12$ mm ; $r_o = 16$ mm
$r_i = 8$ mm

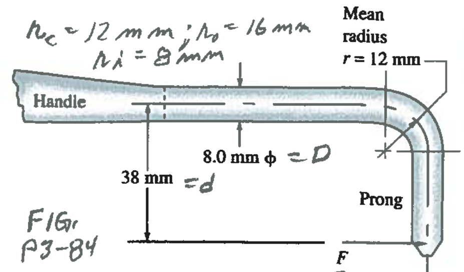



FIG. P3-84

$A = \frac{\pi D^2}{4} = \frac{\pi (8)^2}{4} = 50.265$ mm²

$R = \frac{A}{ASF} = \frac{50.265\text{ mm}^2}{4.312\text{ mm}} = 11.6568$ mm

$(r_c - R) = (12 - 11.6568) = 0.3431$ mm

$(R - r_o) = (11.6568 - 16) = -4.343$ mm

$(R - r_i) = (11.6568 - 8) = 3.6568$ mm

LET $M = F \cdot d$, $\sigma = S_y = 207$ MPa (POS.)

$\sigma_o = S_y = \frac{(F_o \cdot d)(R - r_o)}{A r_o (r_c - R)}$

$F_{o_{ALL}} = \frac{S_y (A) r_o (r_c - R)}{d(R - r_o)}$

LET $S_y = -207$ MPa COMPRESSION

$F_{o_{ALL}} = \frac{(-207\text{ N/mm}^2)(50.265\text{ mm}^2)(16\text{ mm})(0.3431\text{ mm})}{(38\text{ mm})(-4.343\text{ mm})} = 346.1$ N

SIMILARLY:

$F_{i_{ALL}} = \frac{S_y A r_i (r_c - R)}{d(R - r_i)} = \frac{(207)(50.265)(8)(0.3431)}{(38)(3.6568)} = 205.6$ N

GOVERNS

85 CURVED BEAM HOOP SUPPORT FIG. P3-85: $r_i = r_o - D_1$

STEEL: ASTM A53-GR B $S_y = 35$ ksi

$r_i = 9.125$ in

$M = F \cdot d = (230\text{ lb})(48\text{ in}) = -11040$ lb·in (NEG.)

$r_c = r_o - D_1/2$

ANALYSIS AS IN PROB 3-83

$r_c = 10.563$ in

STRESS EQNS. IN PROB 3-81

$ASF = ASF_1 - ASF_2 = 0.61748 - 0.45484 = 0.1626$ in

$R = \frac{A}{ASF} = \frac{1.704\text{ in}^2}{0.1626\text{ in}} = 10.4769$ in

$(R - r_o) = -1.5231$ in ; $(R - r_i) = 1.35185$ in ; $(r_c - R) = 0.08564$ in

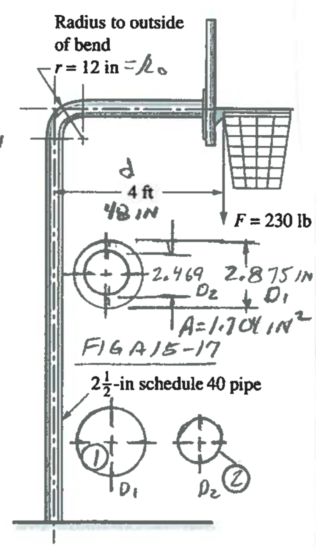



FIG A15-17

$\sigma_i = \frac{(-11040\text{ lb·in})(1.35185\text{ in})}{(1.704\text{ in}^2)(9.125\text{ in})(0.08564\text{ in})} = -11208$ psi COMPR.

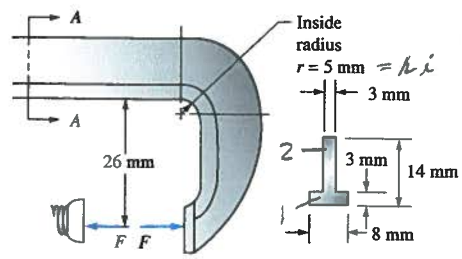
$\sigma_o = \frac{(-11040\text{ lb·in})(-1.5231)}{(1.704\text{ in}^2)(12)(0.08564\text{ in})} = 9602.4$ psi TENSION

$N_i = \frac{S_y}{\sigma_i} = \frac{-35000\text{ psi}}{-11208\text{ psi}} = 3.123$ LOWEST - GOVERNS

SATISFACTORY N

$N_o = \frac{S_y}{\sigma_o} = \frac{35000\text{ psi}}{9602.4\text{ psi}} = 3.645$

$\boxed{86}$ **CURVED BEAM** **C-CLAMP FIG. P3-86**



Section A–A

CAST ZINC ZA-12; $S_{uT} = 404\ MPa$

$S_{uC} = 269\ MPa$

FIND: F FOR N=3

$$\bar{y} = \frac{1}{A}\left[(b_1 t_1)\, t_1/2 + (b_2 t_2)(t_1 + t_2/2)\right]$$

$$\bar{y} = \frac{1}{57}\left[(8\cdot 3)(1.5) + (3\cdot 11)(3+5.5)\right] = 5.553\ mm$$

$$ASF = b_1 \ln(r_1/r_i) + b_2 \ln(r_o/r_1)\ ;\ r_1 = r_i + t_1 = 5+3 = 8\ mm$$

$$ASF = 8\ln(8/5) + 3\ln(19/8) = 6.355\ mm$$

$$R = A/ASF = 57/6.355 = 8.969\ mm$$

$$M = F(26+\bar{y}) = F(26+5.553) = 31.553\,F\ \text{(POS.)}$$

$$\sigma_i = \frac{M(R-r_i)}{A r_i (r_c - R)} = \frac{31.553F(3.969)}{(57)(5)(1.583)} = 0.2776\,F_i \quad \text{TENSION}$$

$$\text{LET } \sigma_i = \frac{S_{uT}}{3} = \frac{401\ MPa}{3} = 134.67\ MPa$$

$$F_i = \frac{134.67\ N/mm^2}{0.2776\ mm^2} = 485.1\ N$$

$$\sigma_o = \frac{M(R-r_o)}{A r_o (r_c - R)} = \frac{31.553F(-10.0307)}{(57)(19)(1.583)} = -0.1846\,F_o \quad \text{COMPRESSION}$$

$$\text{LET } \sigma_o = \frac{S_{uC}}{3} = \frac{-269\ MPa}{3} = -89.67\ N/mm^2$$

$$F_o = \frac{-89.67\ N/mm^2}{-0.1846\ mm^2} = 485.7\ N$$

PART 1: $A_1 = 3\cdot 8 = 24\ mm^2$

PART 2: $A_2 = 3\cdot 11 = 33\ mm^2$

$A = A_1 + A_2 = 57\ mm^2$

$r_o = r_i + 14 = 5+14 = 19\ mm$

$r_c = r_i + \bar{y} = 5+5.553$

$r_c = 10.553\ mm$

$(R - r_i) = 8.969 - 5 = 3.969\ mm$

$(R - r_o) = 8.969 - 19 = -10.0307\ mm$

$(r_c - R) = 10.553 - 8.969 = 1.583\ mm$

BECAUSE $F_o \approx F_i$, THE DESIGN OF THE INVERTED T-SECTION USES THE MATERIAL VERY EFFICIENTLY.

# CHAPTER 4
# COMBINED STRESSES AND MOHR'S CIRCLE

***NOTE:*** **The solutions to Chapter 4 problems 1 – 30 are shown on the following pages as images of the output from the MDESIGN – MOTT software that is included in the text. Each problem produces a solution in line with the procedure shown for manual solution in Section 4-4 in Chapter 4 of the text and as shown in the four Example Problems in Section 4-5 of the text. Problem 4-1 is shown worked out in manual form below and the MDESIGN – MOTT solution is shown on the following page. Solutions for all other problems are shown only as the results from the MDESIGN – MOTT solutions. *Note that in the MDESIGN-MOTT output, the graphic view of Mohr's circle and the stress elements show the stress values only in psi.***

$\boxed{1}$ $\sigma_x = 20$ KSI; $\sigma_y = 0$; $\tau_{xy} = 10$ KSI; X-AXIS IN 1ST QUADRANT

$\sigma_{AVG} = (\sigma_x + \sigma_y)/2 = (20+0)/2 = 10.0$ KSI

$a = \sigma_x - \sigma_{AVG} = 20 - 10 = 10$ KSI; $b = \tau_{xy} = 10$ KSI

$R = \sqrt{a^2+b^2} = \sqrt{10^2+10^2} = 14.14$ KSI $= \tau_{MAX}$

$\alpha = \tan^{-1}(b/a) = \tan^{-1}(10/10) = 45° = 2\phi_\sigma$ CW FROM X-AXIS TO $\sigma_1$.

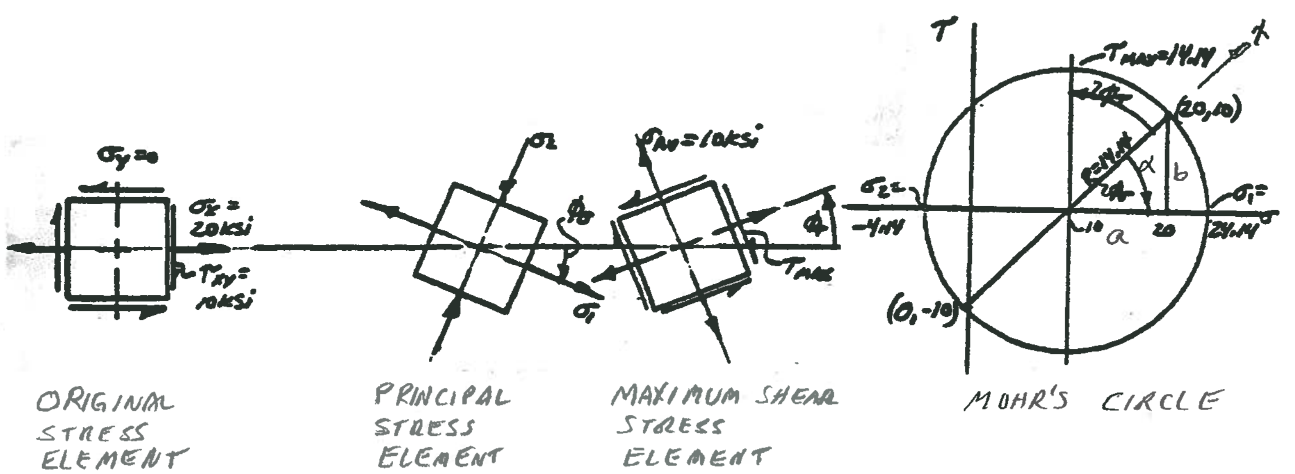
$\phi_\sigma = 2\phi_\sigma/2 = 45.0/2 = 22.5°$

$2\phi_\tau = 90° - \alpha = 90 - 45 = 45°$ CCW FROM X-AXIS TO $\tau_{MAX}$

$\phi_\tau = 2\phi_\tau/2 = 45/2 = 22.5°$

$\sigma_1 = \sigma_{AVG} + R = 10 + 14.14 = 24.14$ KSI

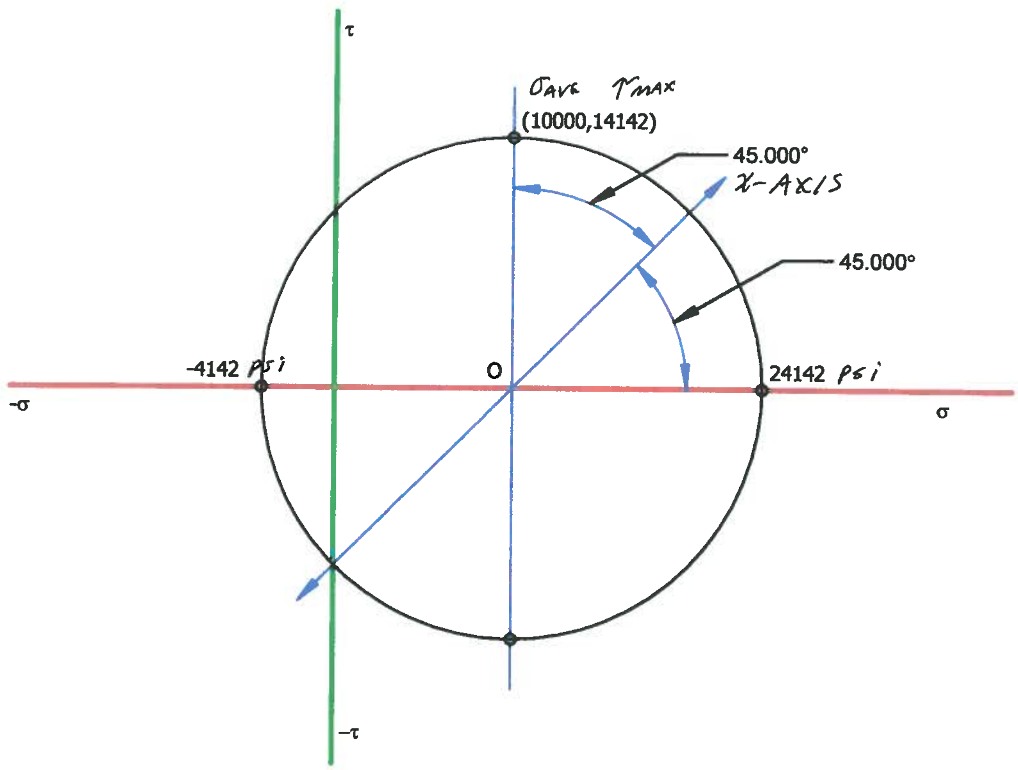
$\sigma_2 = \sigma_{AVG} - R = 10 - 14.14 = -4.14$ KSI

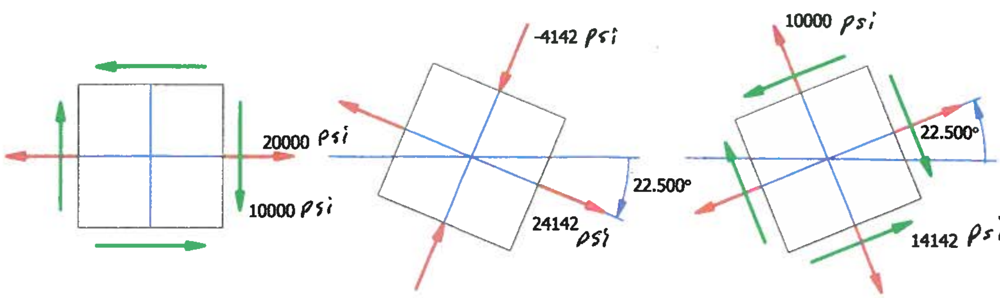


1

| | | |
|---|---|---|
| | $\sigma x$ = 20 | ksi |
| | $\sigma y$ = 0 | ksi |
| **Results:** | $\tau xy$ = 10 | ksi |

| | | | | |
|---|---|---|---|---|
| Maximum principal stress | $\sigma 1$ = | 24.142 | ksi | |
| Minimum principal stress | $\sigma 2$ = | -4.142 | ksi | |
| Maximum shear stress | $\tau max$ = | 14.142 | ksi | |
| Average normal stress | $\sigma avg$ = | 10.000 | ksi | |
| Principal planes | $\phi\sigma$ = | 22.500 | ° | CW |
| Angle of maximum shear stress | $\phi\tau$ = | 22.500 | ° | CCW |




Original stress element

Principal stress element

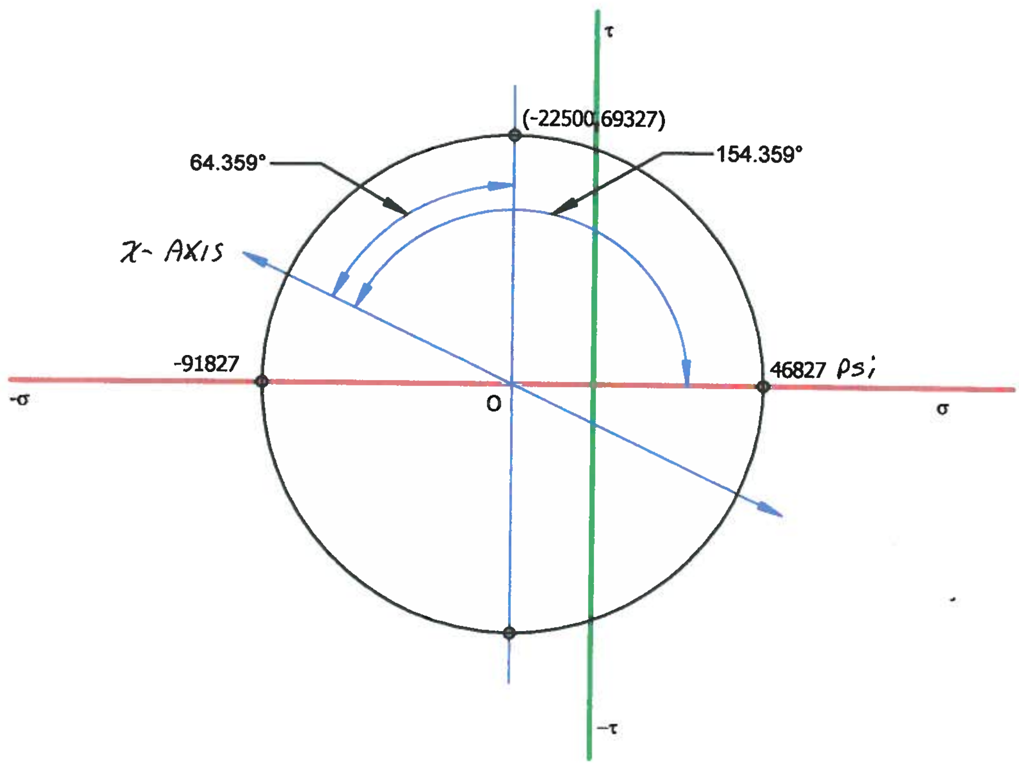
Maximum shear stress element

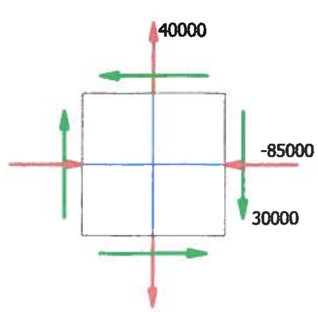
2

| | | |
|---|---|---|
| σx = | -85000 | psi |
| σy = | 40000 | psi |
| τxy = | 30000 | psi |

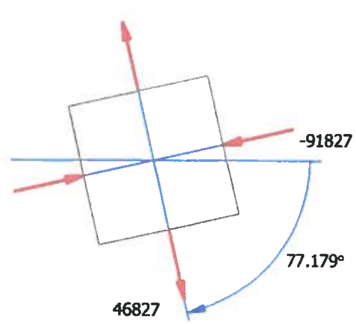
**Results:**

| | | | | |
|---|---|---|---|---|
| Maximum principal stress | σ1 = | 46827.123 | psi | |
| Minimum principal stress | σ2 = | -91827.123 | psi | |
| Maximum shear stress | τmax = | 69327.123 | psi | |
| Average normal stress | σavg = | -22500.000 | psi | |
| Principal planes | φσ = | 77.179 | ° | CW |
| Angle of maximum shear stress | φτ = | 32.179 | ° | CW |

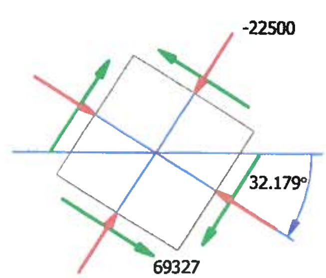



Original stress element




Principal stress element



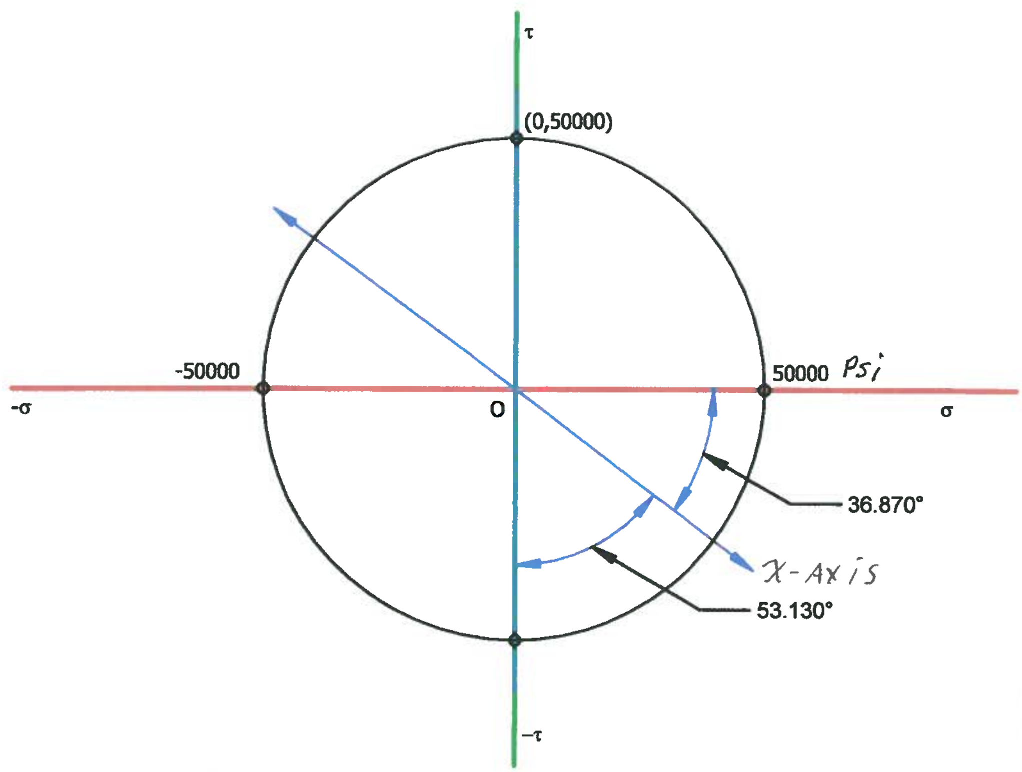

Maximum shear stress element

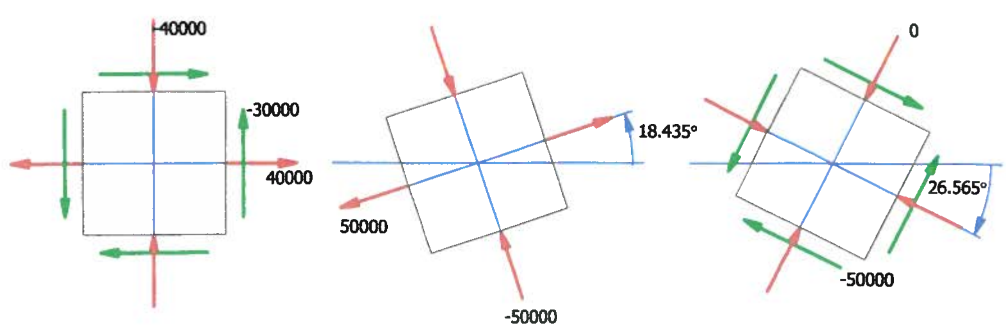
3

**Results:**

$\sigma x = 40$ ksi
$\sigma y = -40$ ksi
$\tau xy = -30$ ksi

| | | | | |
|---|---|---|---|---|
| Maximum principal stress | $\sigma 1$ = | 50.000 | ksi | |
| Minimum principal stress | $\sigma 2$ = | -50.000 | ksi | |
| Maximum shear stress | $\tau max$ = | 50.000 | ksi | |
| Average normal stress | $\sigma avg$ = | 0.000 | ksi | |
| Principal planes | $\phi\sigma$ = | 18.435 | ° | ccw |
| Angle of maximum shear stress | $\phi\tau$ = | 26.565 | ° | cw to $-\tau max$ |




Original stress element

Principal stress element

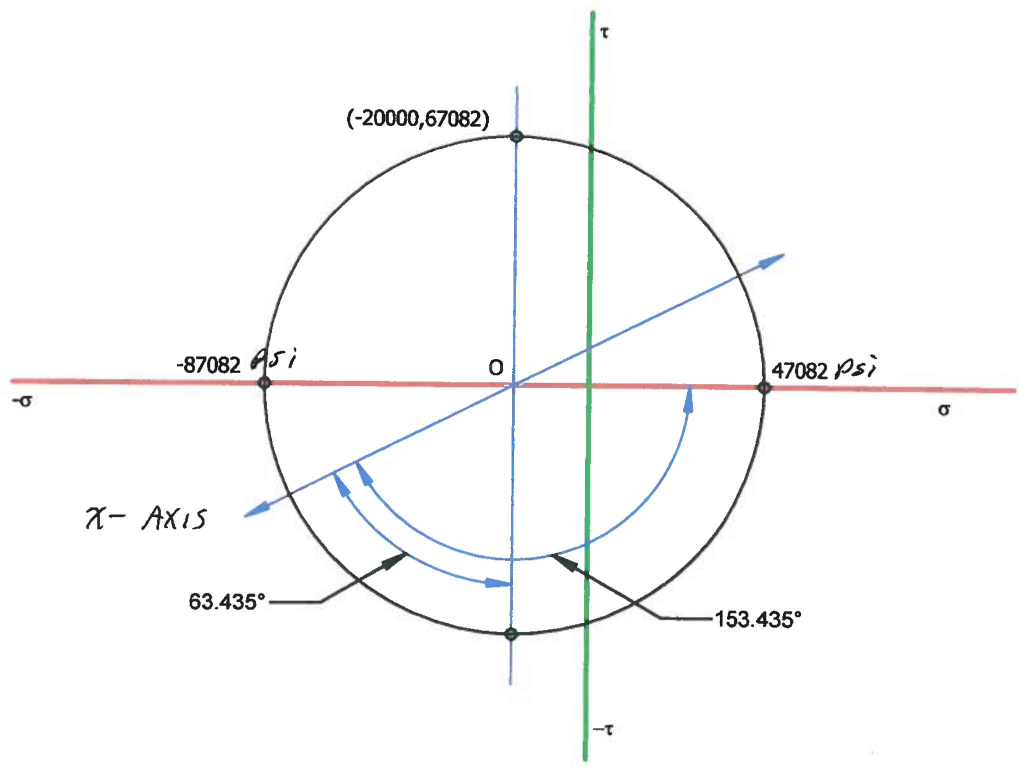
Maximum shear stress element

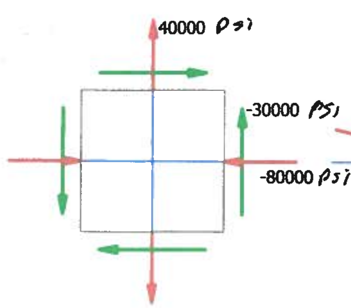
$\boxed{4}$

$\sigma x$ = -80 ksi
$\sigma y$ = 40 ksi
$\tau xy$ = -30 ksi

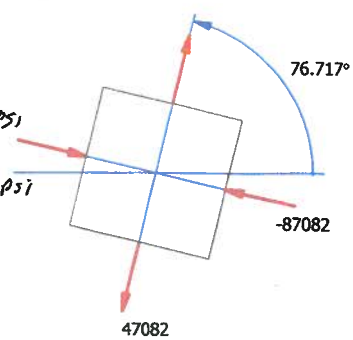
**Results:**

| | | | | | |
|---|---|---|---|---|---|
| Maximum principal stress | $\sigma 1$ | = | 47.082 | ksi | |
| Minimum principal stress | $\sigma 2$ | = | -87.082 | ksi | |
| Maximum shear stress | $\tau max$ | = | 67.082 | ksi | |
| Average normal stress | $\sigma avg$ | = | -20.000 | ksi | |
| Principal planes | $\phi\sigma$ | = | 76.717 | ° | ccw |
| Angle of maximum shear stress | $\phi\tau$ | = | 31.717 | ° | ccw to $-\tau max$ |

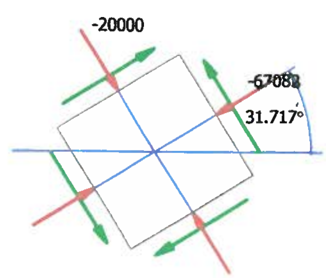



Original stress element




Principal stress element



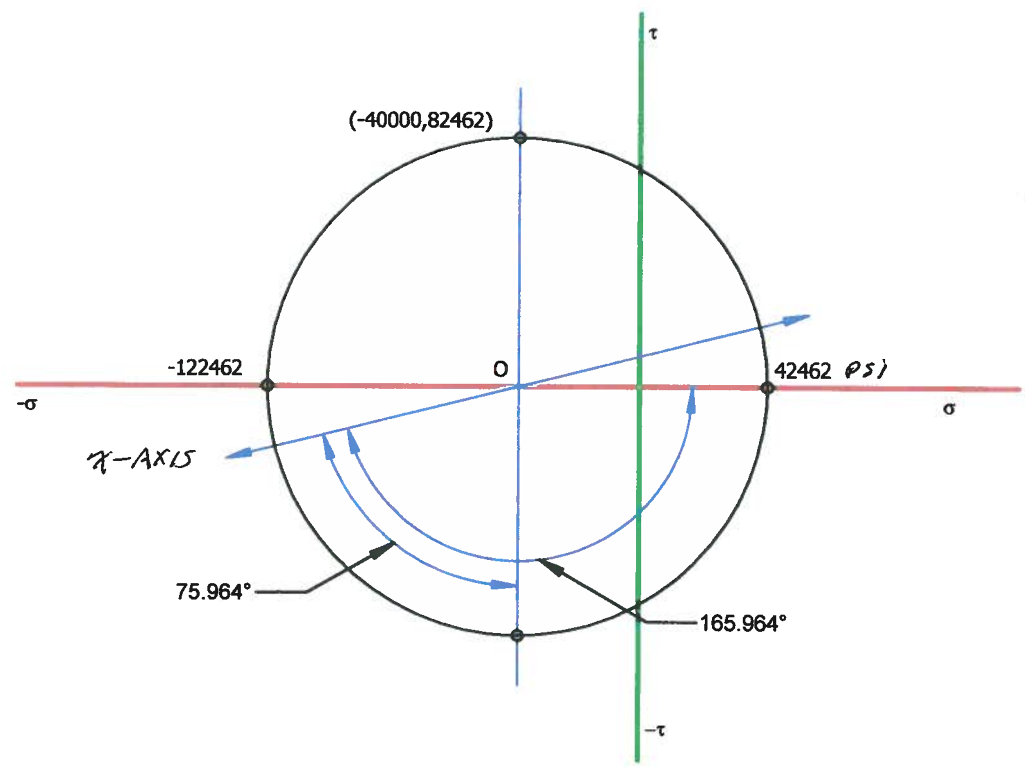

Maximum shear stress element

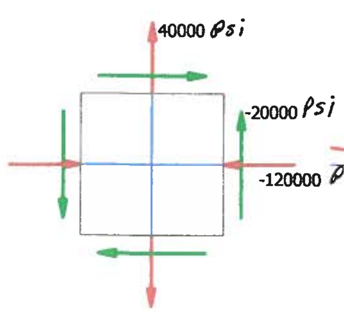
5

$\sigma x$ = -120000 psi
$\sigma y$ = 40000 psi
$\tau xy$ = -20000 psi

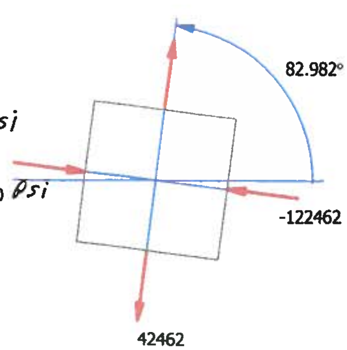
**Results:**

| | | | | |
|---|---|---|---|---|
| Maximum principal stress | $\sigma 1$ = | 42462.113 | psi | |
| Minimum principal stress | $\sigma 2$ = | -122462.113 | psi | |
| Maximum shear stress | $\tau max$ = | 82462.113 | psi | |
| Average normal stress | $\sigma avg$ = | -40000.000 | psi | |
| Principal planes | $\phi\sigma$ = | 82.982 | ° | ccw |
| Angle of maximum shear stress | $\phi\tau$ = | 37.982 | ° | ccw to $-\tau max$ |

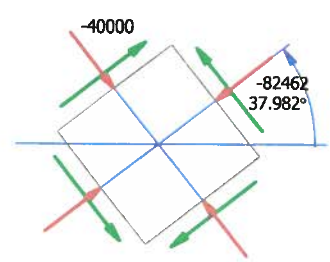



Original stress element




Principal stress element



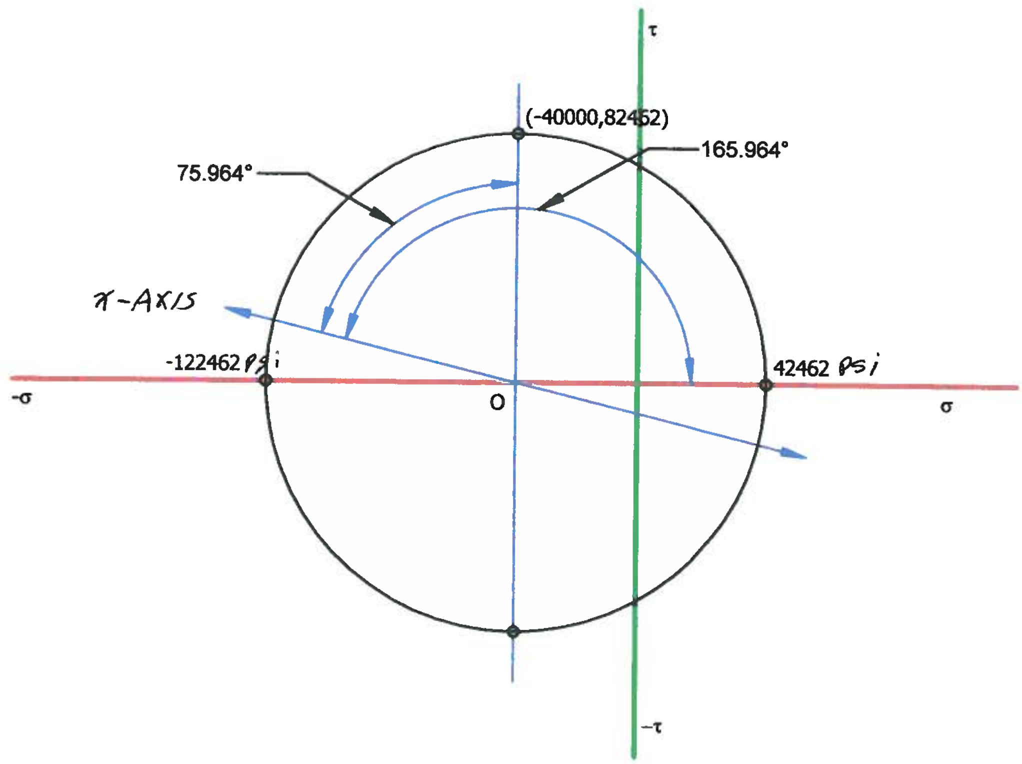

Maximum shear stress element

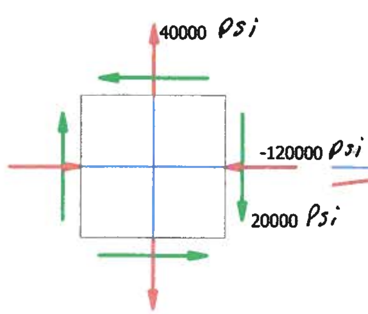
$\boxed{6}$

| | | |
|---|---|---|
| $\sigma x$ = | -120 | ksi |
| $\sigma y$ = | 40 | ksi |
| $\tau xy$ = | 20 | ksi |

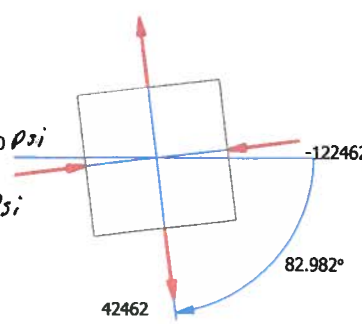
**Results:**

| | | | |
|---|---|---|---|
| Maximum principal stress | $\sigma 1$ = | 42.462 | ksi |
| Minimum principal stress | $\sigma 2$ = | -122.462 | ksi |
| Maximum shear stress | $\tau max$ = | 82.462 | ksi |
| Average normal stress | $\sigma avg$ = | -40.000 | ksi |
| Principal planes | $\phi\sigma$ = | 82.982 | ° cw |
| Angle of maximum shear stress | $\phi\tau$ = | 37.982 | ° cw |

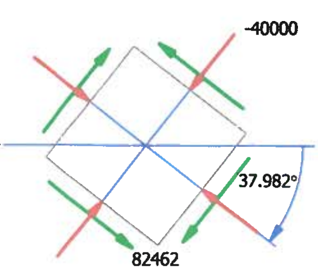



Original stress element




Principal stress element



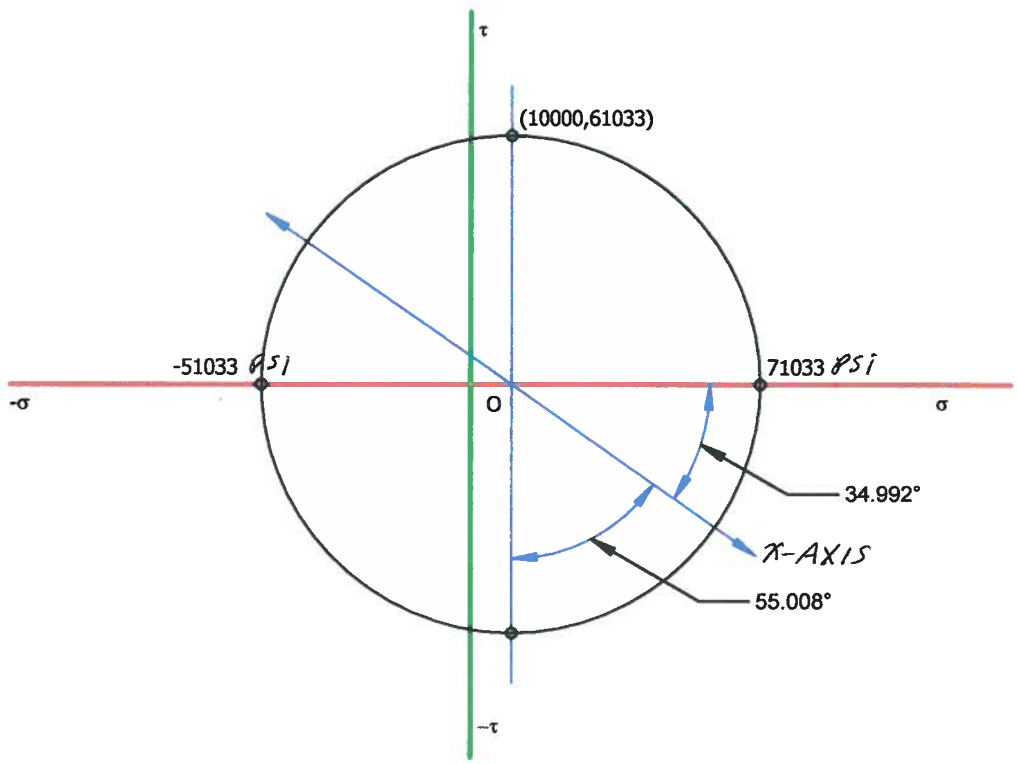

Maximum shear stress element

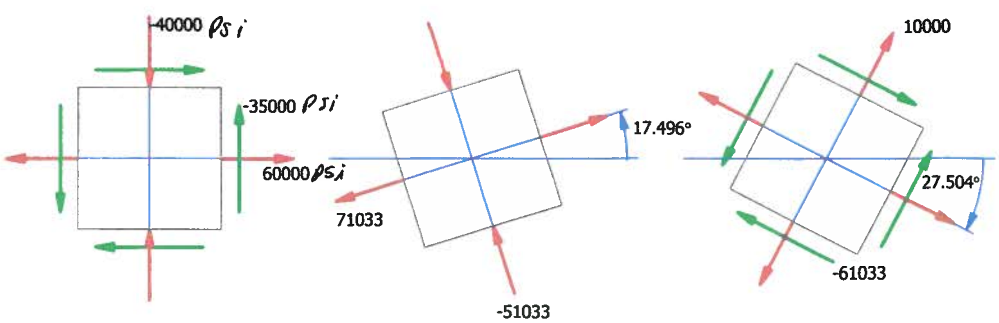
7

$\sigma x$ = 60000 psi
$\sigma y$ = -40000 psi
$\tau xy$ = -35000 psi

**Results:**

| | | | | |
|---|---|---|---|---|
| Maximum principal stress | $\sigma 1$ = | 71032.778 | psi | |
| Minimum principal stress | $\sigma 2$ = | -51032.778 | psi | |
| Maximum shear stress | $\tau max$ = | 61032.778 | psi | |
| Average normal stress | $\sigma avg$ = | 10000.000 | psi | |
| Principal planes | $\phi\sigma$ = | 17.496 | ° | ccw |
| Angle of maximum shear stress | $\phi\tau$ = | 27.504 | ° | cw to $-\tau max$ |




Original stress element

Principal stress element

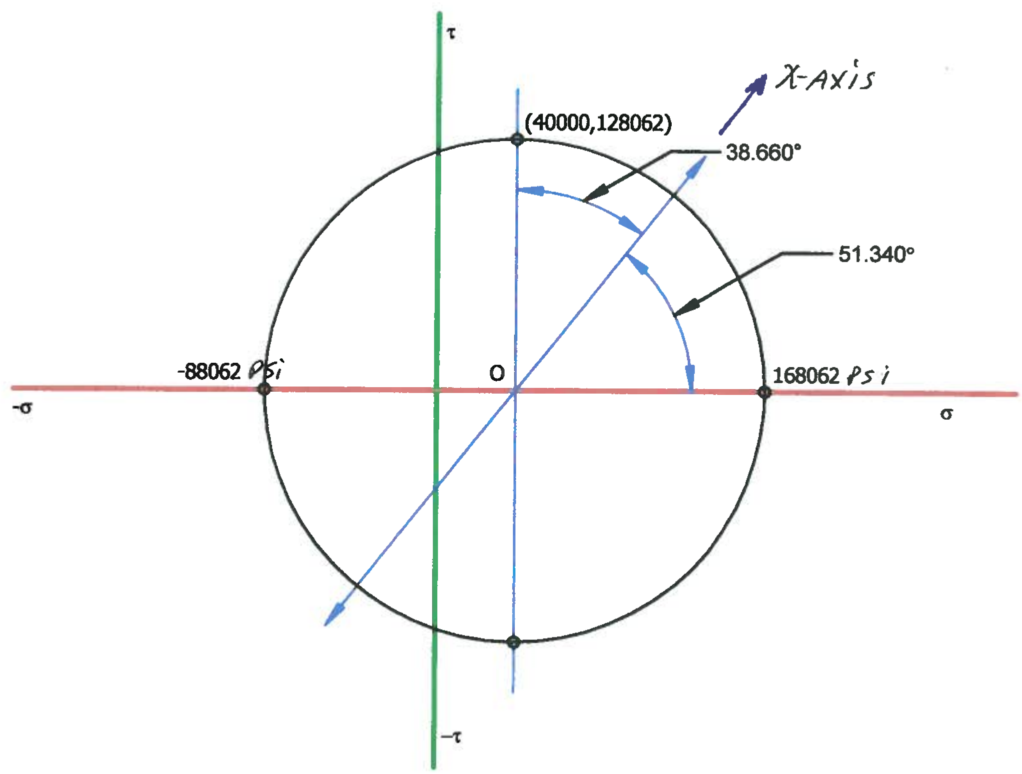
Maximum shear stress element

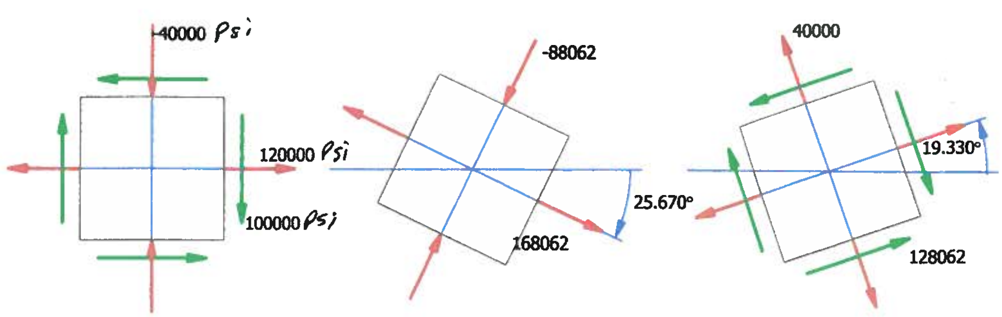
8

$\sigma x$ = 120 ksi
$\sigma y$ = -40 ksi
$\tau xy$ = 100 ksi

**Results:**

| | | | | |
|---|---|---|---|---|
| Maximum principal stress | $\sigma 1$ | = | 168.062 | ksi |
| Minimum principal stress | $\sigma 2$ | = | -88.062 | ksi |
| Maximum shear stress | $\tau max$ | = | 128.062 | ksi |
| Average normal stress | $\sigma avg$ | = | 40.000 | ksi |
| Principal planes | $\phi\sigma$ | = | 25.670 | ° CW |
| Angle of maximum shear stress | $\phi\tau$ | = | 19.330 | ° CCW |




Original stress element

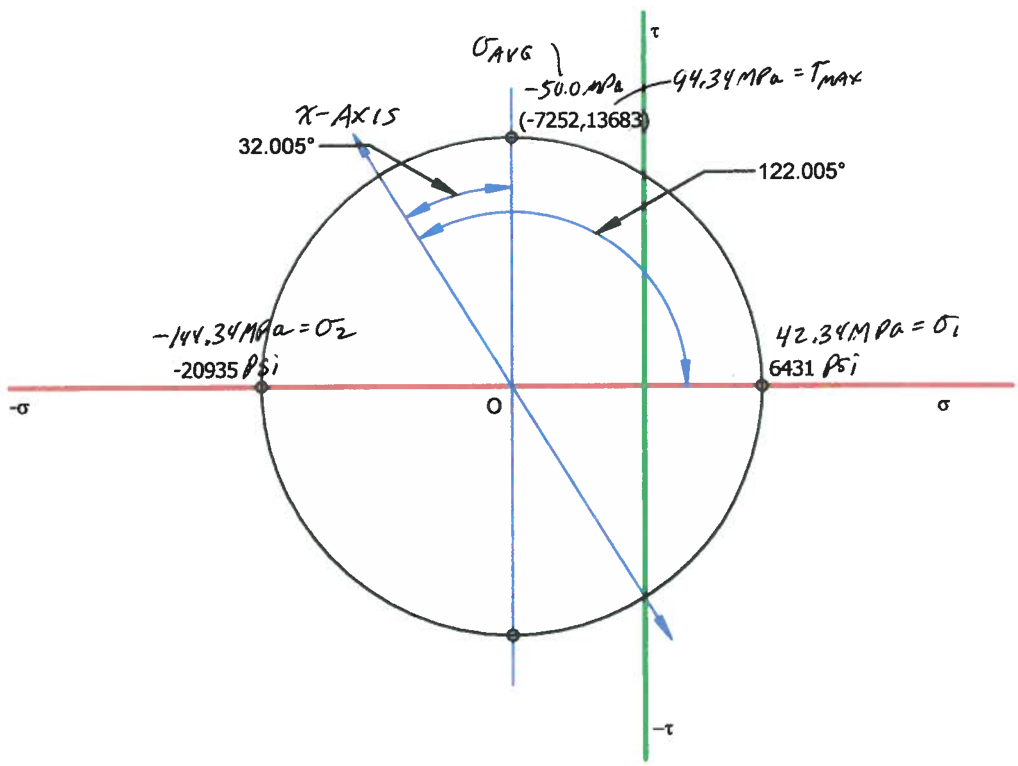
Principal stress element

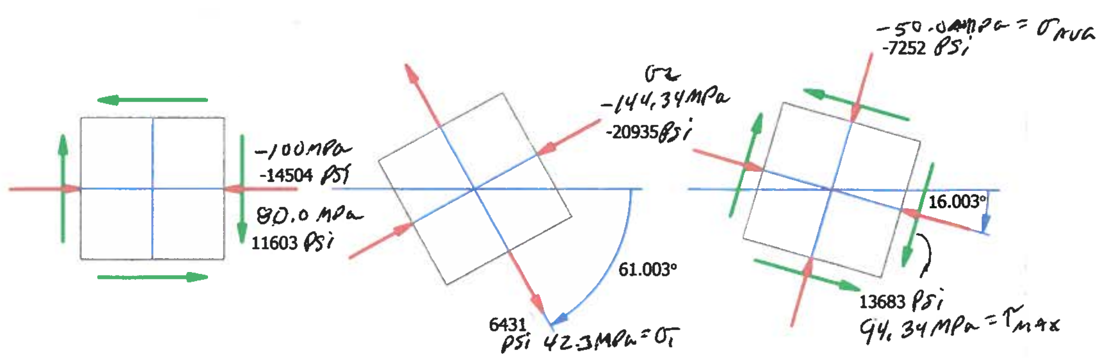
Maximum shear stress element

9

$\sigma x = -100$ MPa
$\sigma y = 0$ MPa
**Results:** $\tau xy = 80$ MPa

| | | | | |
|---|---|---|---|---|
| Maximum principal stress | $\sigma 1$ = | 44.340 | MPa | |
| Minimum principal stress | $\sigma 2$ = | -144.340 | MPa | |
| Maximum shear stress | $\tau max$ = | 94.340 | MPa | |
| Average normal stress | $\sigma avg$ = | -50.000 | MPa | |
| Principal planes | $\phi\sigma$ = | 61.003 | ° | CW |
| Angle of maximum shear stress | $\phi\tau$ = | 16.003 | ° | CW |




Original stress element

Principal stress element

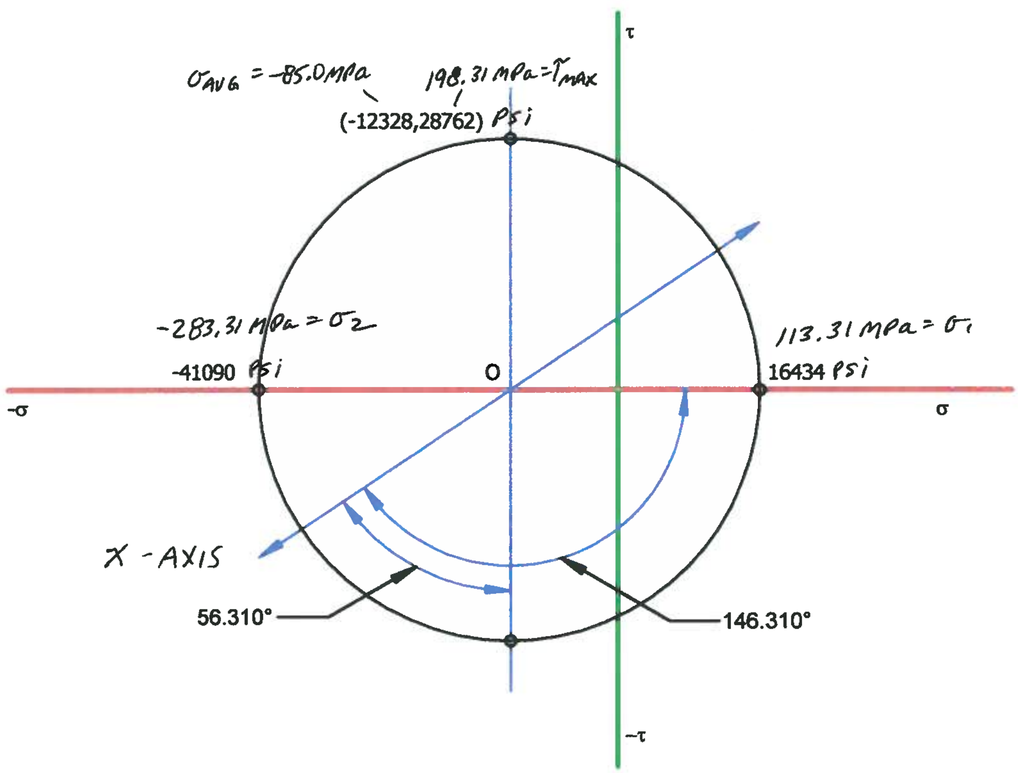
Maximum shear stress element

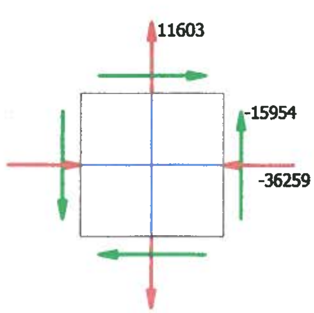
10

$\sigma x$ = -250 MPa
$\sigma y$ = 80 MPa
$\tau xy$ = -110 MPa

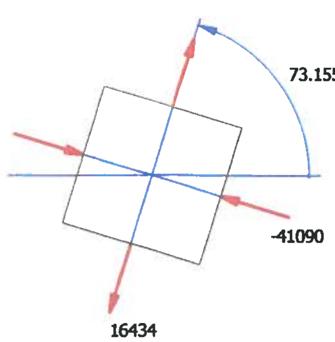
**Results:**

| | | | | |
|---|---|---|---|---|
| Maximum principal stress | $\sigma 1$ = | 113.305 | MPa | |
| Minimum principal stress | $\sigma 2$ = | -283.305 | MPa | |
| Maximum shear stress | $\tau max$ = | 198.305 | MPa | |
| Average normal stress | $\sigma avg$ = | -85.000 | MPa | |
| Principal planes | $\phi\sigma$ = | 73.155 | ° | ccw |
| Angle of maximum shear stress | $\phi\tau$ = | 28.155 | ° | ccw to $-\tau max$ |

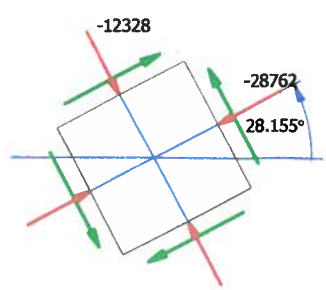



Original stress element




Principal stress element



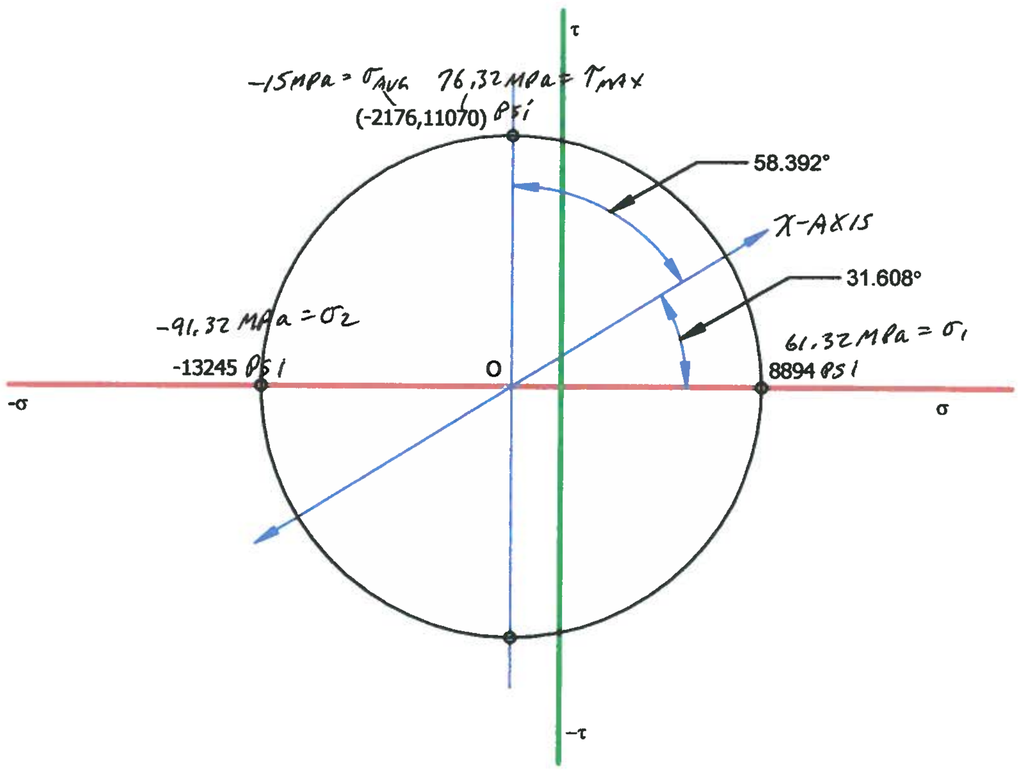

Maximum shear stress element

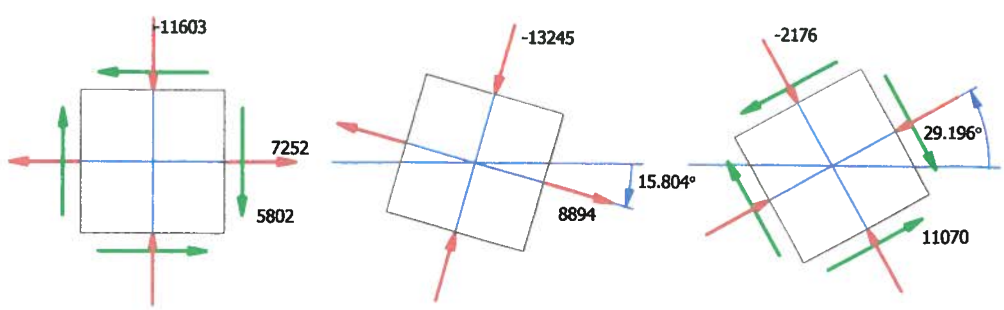
11

**Results:**

σx = 50 MPa
σy = -80 MPa
τxy = 40 MPa

| | | | | |
|---|---|---|---|---|
| Maximum principal stress | σ1 | = | 61.322 | MPa |
| Minimum principal stress | σ2 | = | -91.322 | MPa |
| Maximum shear stress | τmax | = | 76.322 | MPa |
| Average normal stress | σavg | = | -15.000 | MPa |
| Principal planes | ϕσ | = | 15.804 | ° CW |
| Angle of maximum shear stress | ϕτ | = | 29.196 | ° CCW |




Original stress element

Principal stress element

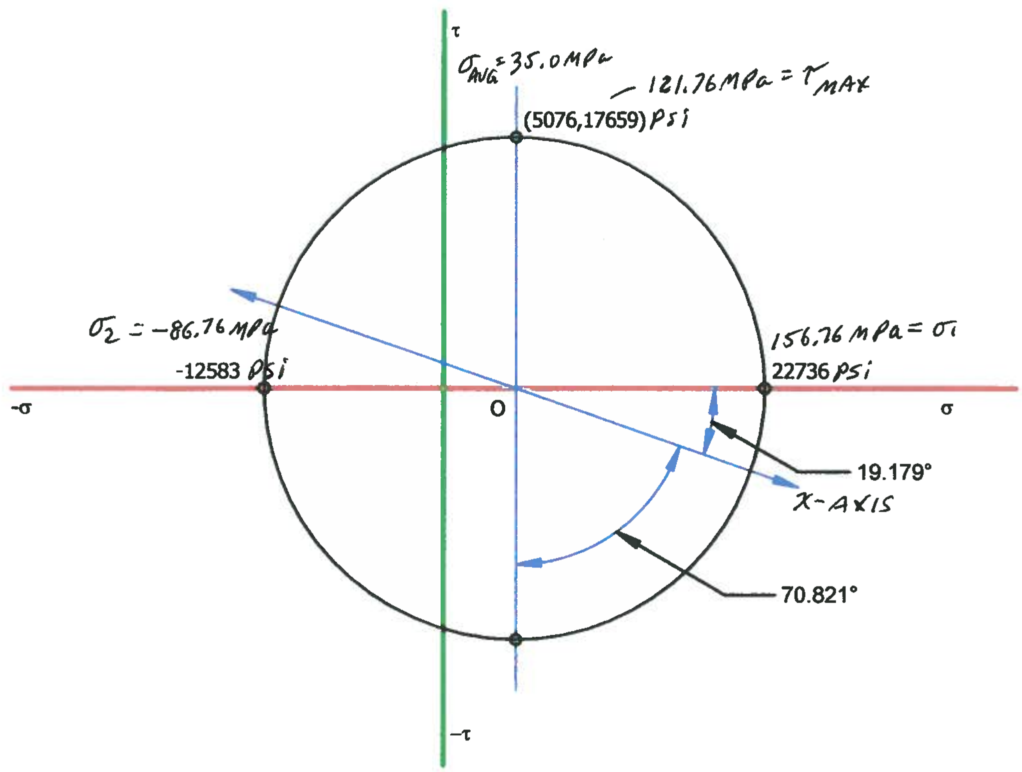
Maximum shear stress element

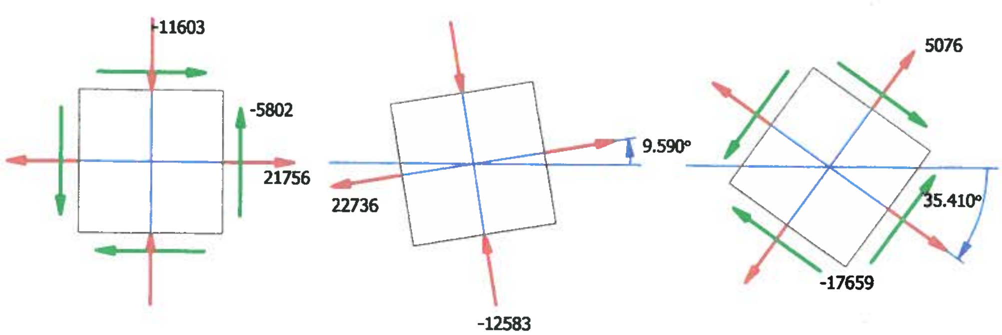
12

| | | |
|---|---|---|
| $\sigma x$ = 150 | MPa | |
| $\sigma y$ = -80 | MPa | |
| $\tau xy$ = -40 | MPa | |

**Results:**

| | | | | |
|---|---|---|---|---|
| Maximum principal stress | $\sigma 1$ = | 156.758 | MPa | |
| Minimum principal stress | $\sigma 2$ = | -86.758 | MPa | |
| Maximum shear stress | $\tau max$ = | 121.758 | MPa | |
| Average normal stress | $\sigma avg$ = | 35.000 | MPa | |
| Principal planes | $\phi\sigma$ = | 9.590 | ° | ccw |
| Angle of maximum shear stress | $\phi\tau$ = | 35.410 | ° | cw to $-\tau max$ |




Original stress element

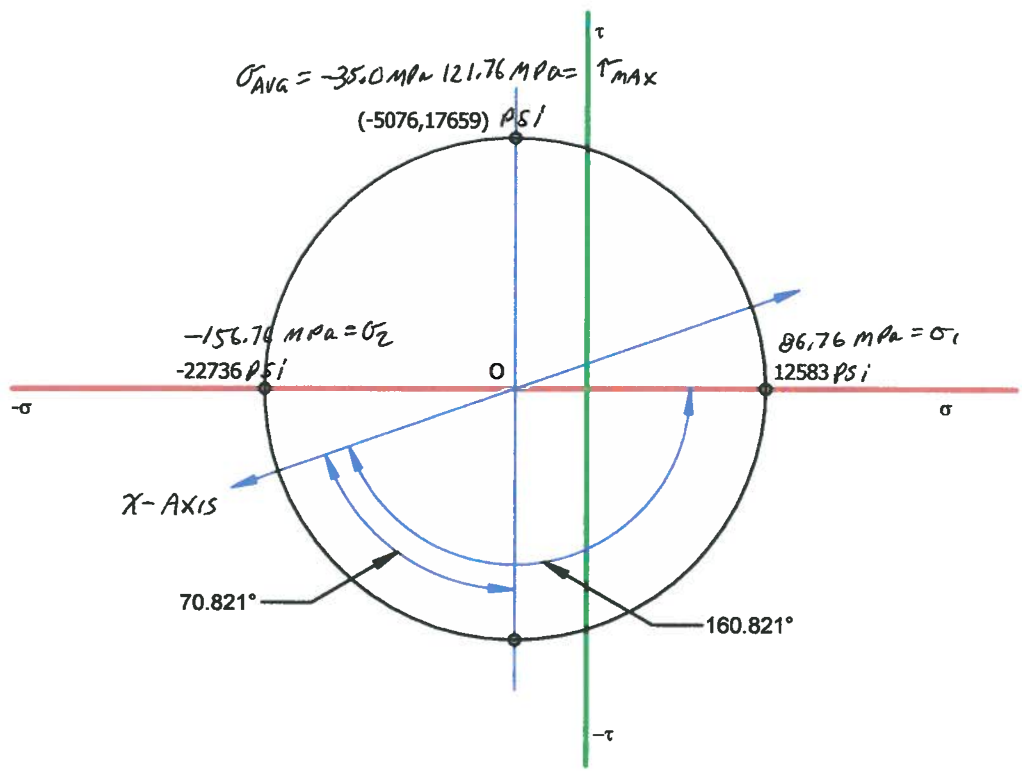
Principal stress element

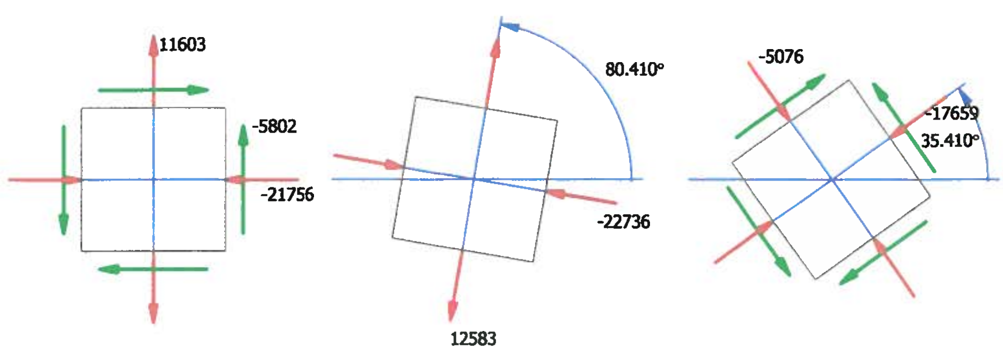
Maximum shear stress element

13

| | | |
|---|---|---|
| | $\sigma x$ = -150 | MPa |
| | $\sigma y$ = 80 | MPa |
| **Results:** | $\tau xy$ = -40 | MPa |

| | | | | |
|---|---|---|---|---|
| Maximum principal stress | $\sigma 1$ = | 86.758 | MPa | |
| Minimum principal stress | $\sigma 2$ = | -156.758 | MPa | |
| Maximum shear stress | $\tau max$ = | 121.758 | MPa | |
| Average normal stress | $\sigma avg$ = | -35.000 | MPa | |
| Principal planes | $\phi\sigma$ = | 80.410 | ° | ccw |
| Angle of maximum shear stress | $\phi\tau$ = | 35.410 | ° | ccw to $-\tau max$ |



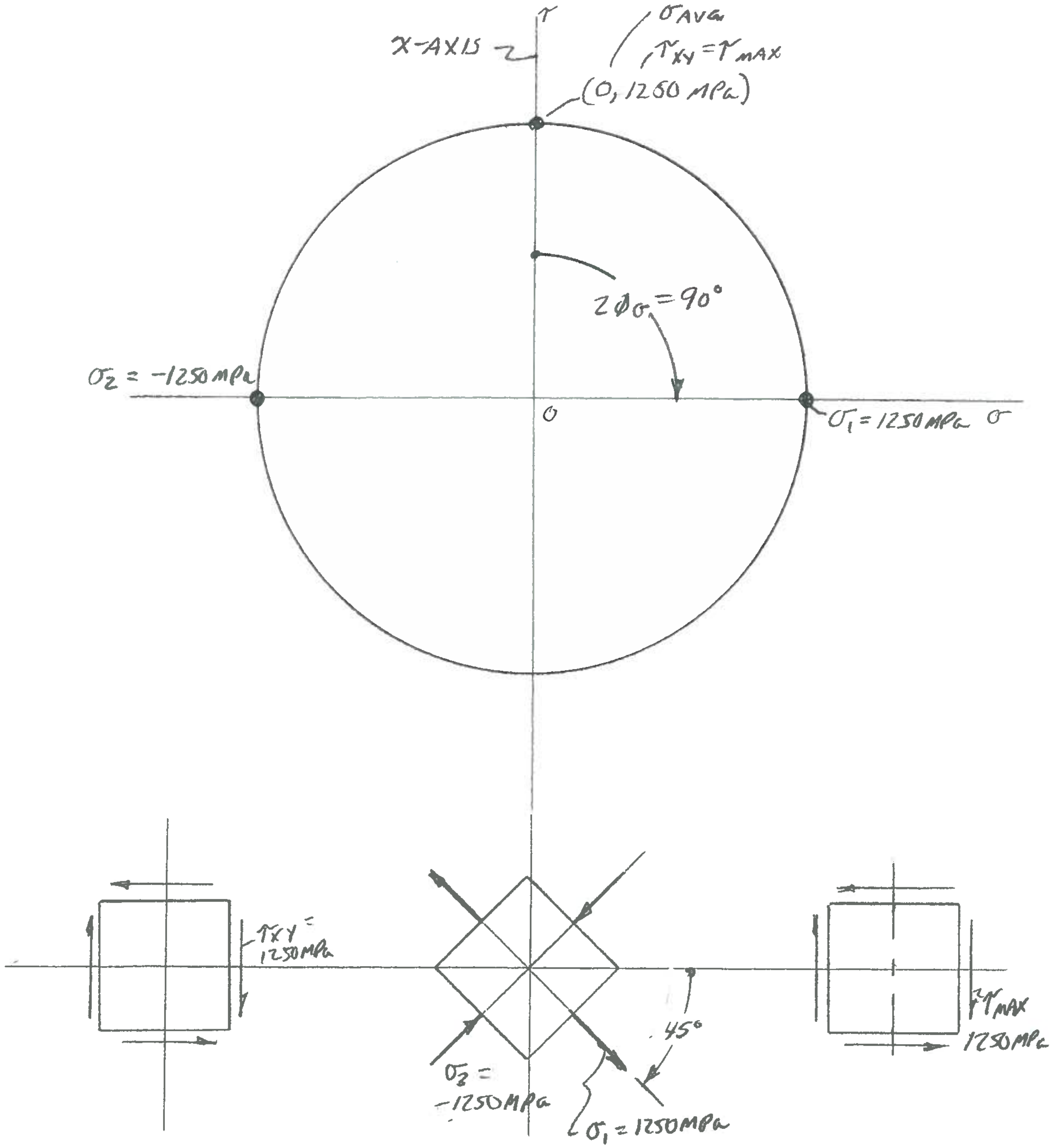




Original stress element

Principal stress element

Maximum shear stress element

$\boxed{14.}$ $\sigma_x = 0$, $\sigma_y = 0$, $\tau_{xy} = 1250\ MPa$




Original stress element

Principal stress element

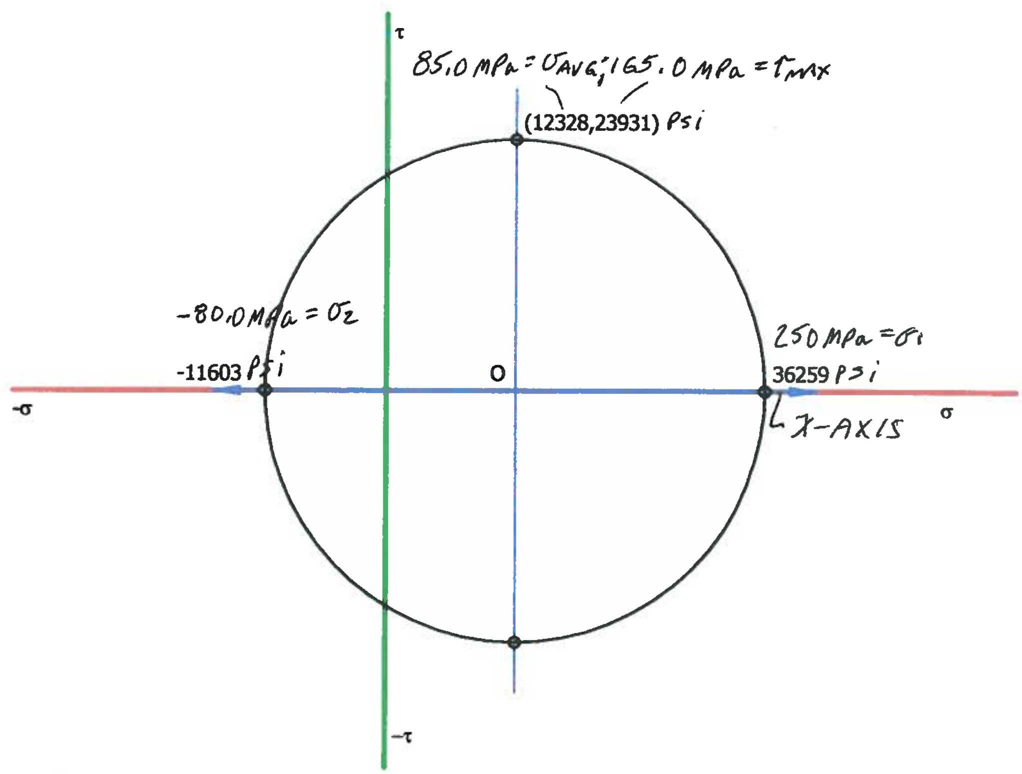
Maximum shear stress element

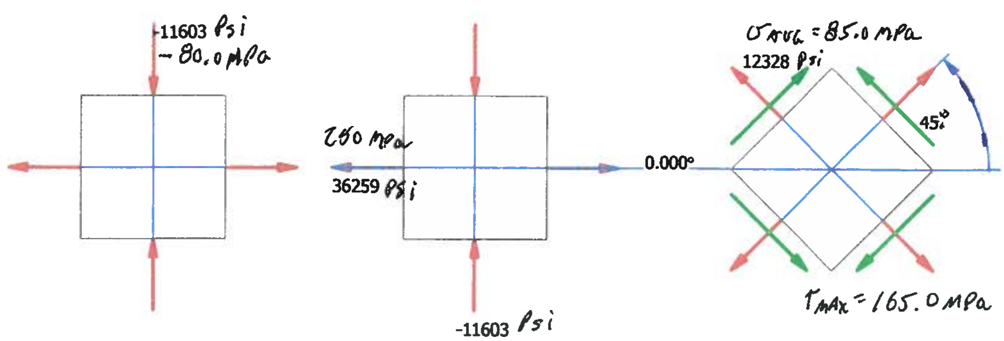
15

$\sigma x$ = 250 MPa
$\sigma y$ = -80 MPa
$\tau xy$ = 0 MPa

**Results:**

| | | | | |
|---|---|---|---|---|
| Maximum principal stress | $\sigma 1$ = | 250.000 | MPa | |
| Minimum principal stress | $\sigma 2$ = | -80.000 | MPa | |
| Maximum shear stress | $\tau max$ = | 165.000 | MPa | |
| Average normal stress | $\sigma avg$ = | 85.000 | MPa | |
| Principal planes | $\phi\sigma$ = | 0.000 | ° | ccw |
| Angle of maximum shear stress | $\phi\tau$ = | 45.000 | ° | ccw to -τmax |




Original stress element

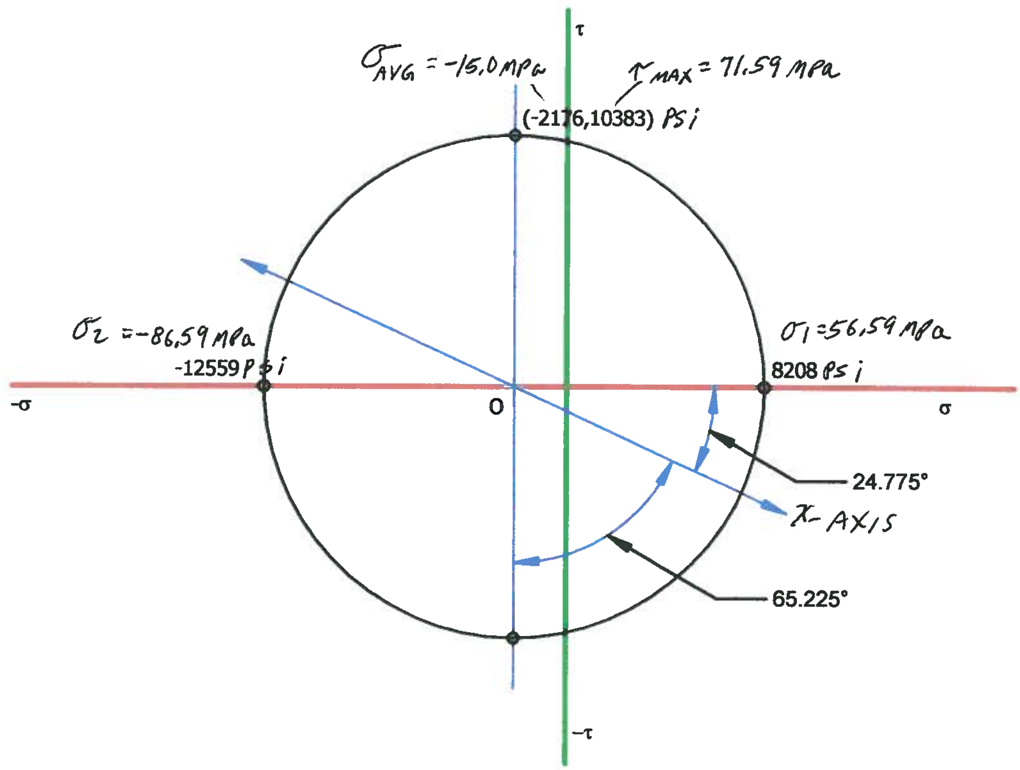
Principal stress element

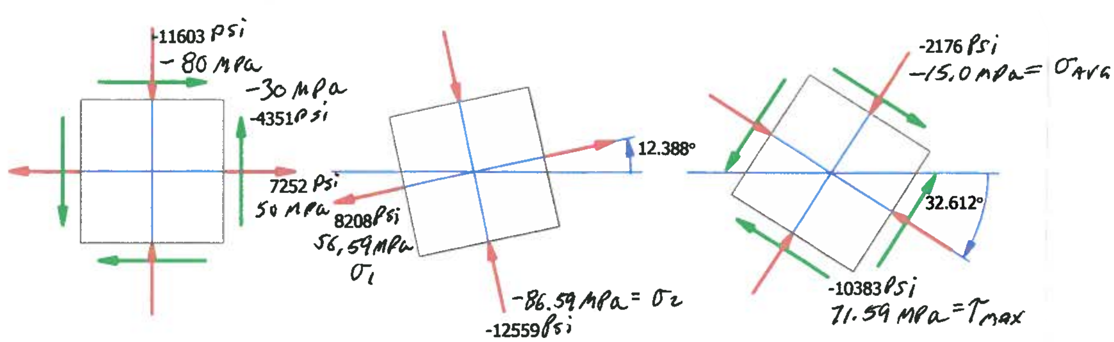
Maximum shear stress element

16

| | | |
|---|---|---|
| **Results:** | $\sigma x$ = 50 | MPa |
| | $\sigma y$ = -80 | MPa |
| | $\tau xy$ = -30 | MPa |

| | | | | | |
|---|---|---|---|---|---|
| Maximum principal stress | $\sigma 1$ | = | 56.589 | MPa | |
| Minimum principal stress | $\sigma 2$ | = | -86.589 | MPa | |
| Maximum shear stress | $\tau max$ | = | 71.589 | MPa | |
| Average normal stress | $\sigma avg$ | = | -15.000 | MPa | |
| Principal planes | $\phi\sigma$ | = | 12.388 | ° | ccw |
| Angle of maximum shear stress | $\phi\tau$ | = | 32.612 | ° | cw to $-\tau max$ |




Original stress element

Principal stress element

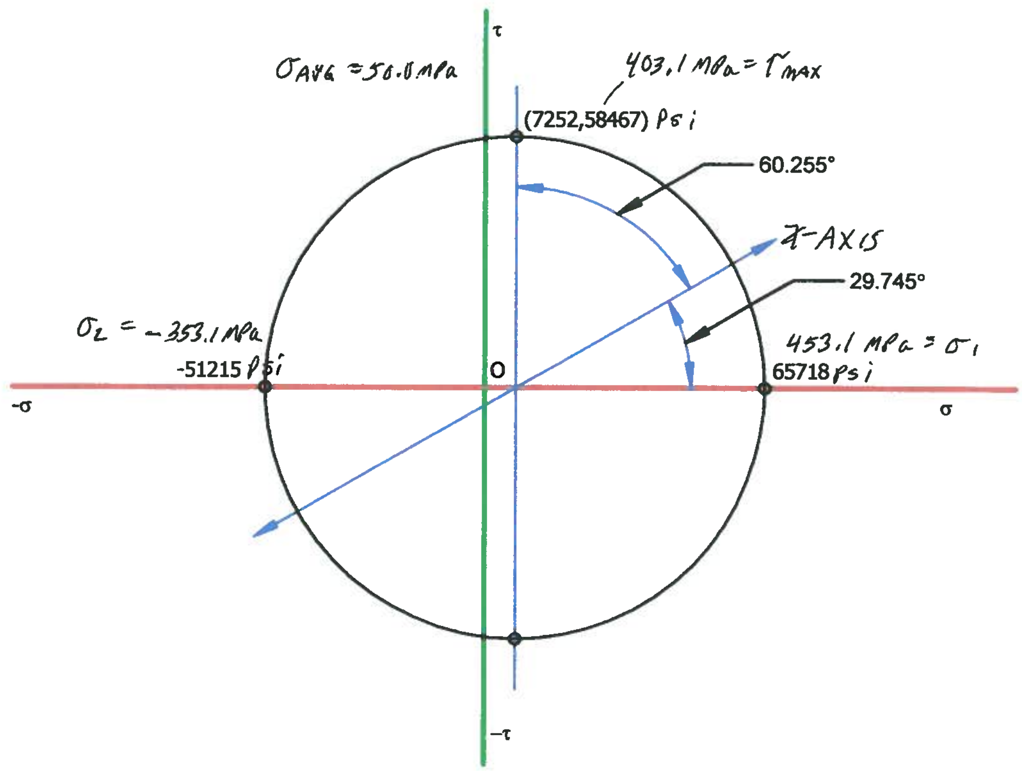
Maximum shear stress element

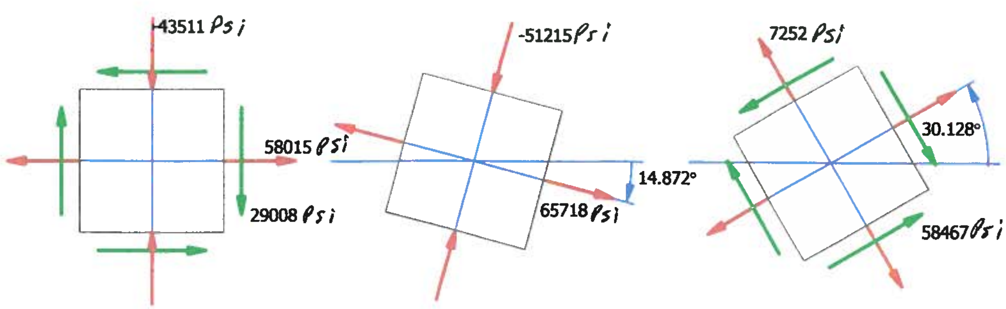
17

$\sigma x$ = 400 MPa
$\sigma y$ = -300 MPa
$\tau xy$ = 200 MPa

**Results:**

| | | | | |
|---|---|---|---|---|
| Maximum principal stress | $\sigma 1$ | = | 453.113 | MPa |
| Minimum principal stress | $\sigma 2$ | = | -353.113 | MPa |
| Maximum shear stress | $\tau max$ | = | 403.113 | MPa |
| Average normal stress | $\sigma avg$ | = | 50.000 | MPa |
| Principal planes | $\phi\sigma$ | = | 14.872 | ° CW |
| Angle of maximum shear stress | $\phi\tau$ | = | 30.128 | ° CCW |




Original stress element

Principal stress element

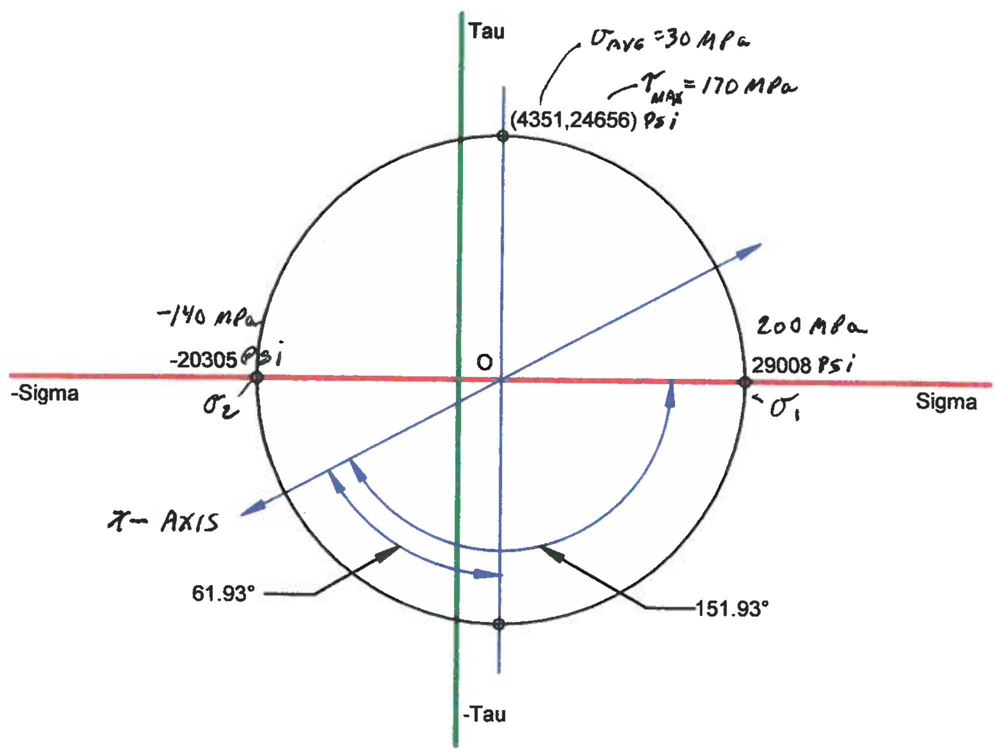
Maximum shear stress element

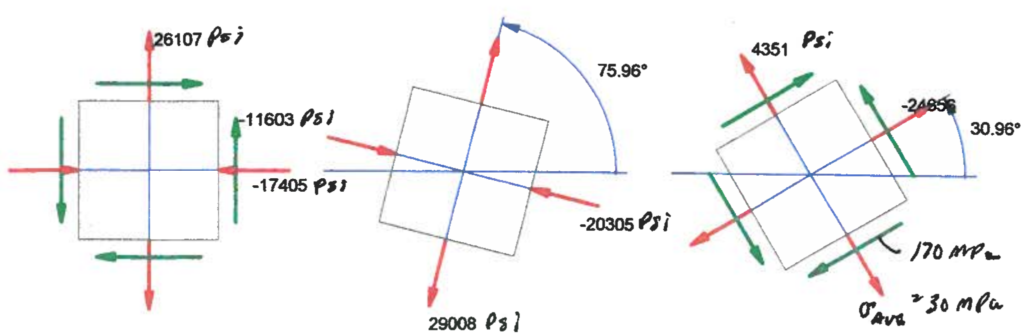
18.

$\sigma x$ = -120 MPa
$\sigma y$ = 180 MPa
$\tau xy$ = -80 MPa

**Results**

| | | | |
|---|---|---|---|
| Maximum principal stress | $\sigma 1$ | = | 200.000MPa |
| Minimum principal stress | $\sigma 2$ | = | -140.000MPa |
| Maximum shear stress | $\tau max$ | = | 170.000MPa |
| Average normal stress | $\sigma avg$ | = | 30.000MPa |
| Principal planes | $\phi\sigma$ | = | 75.964° ccw |
| Angle of maximum shear stress | $\phi\tau$ | = | 30.964° ccw to $-\tau max$ |




Original stress element

Principal stress element

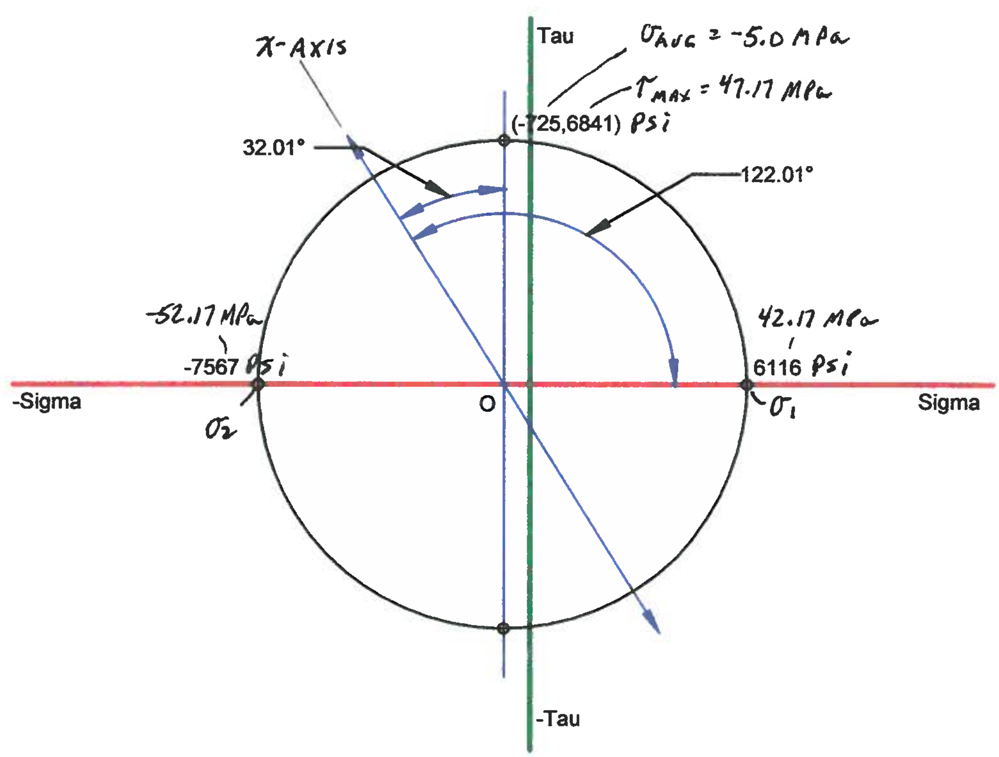
Maximum shear stress element

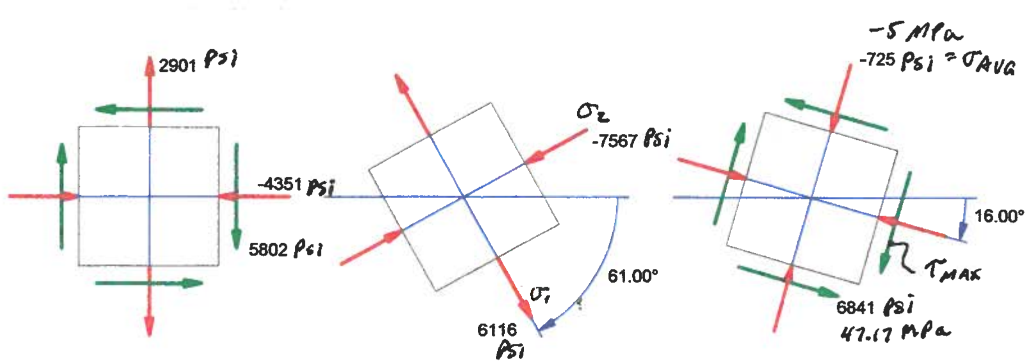
19,

$\sigma x$ = -30 MPa
$\sigma y$ = 20 MPa
$\tau xy$ = 40 MPa

**Results**

| | | | |
|---|---|---|---|
| Maximum principal stress | $\sigma 1$ | = | 42.170MPa |
| Minimum principal stress | $\sigma 2$ | = | -52.170MPa |
| Maximum shear stress | $\tau max$ | = | 47.170MPa |
| Average normal stress | $\sigma avg$ | = | -5.000MPa |
| Principal planes | $\phi\sigma$ | = | 61.003° CW |
| Angle of maximum shear stress | $\phi\tau$ | = | 16.003° CW |




Original stress element

Principal stress element

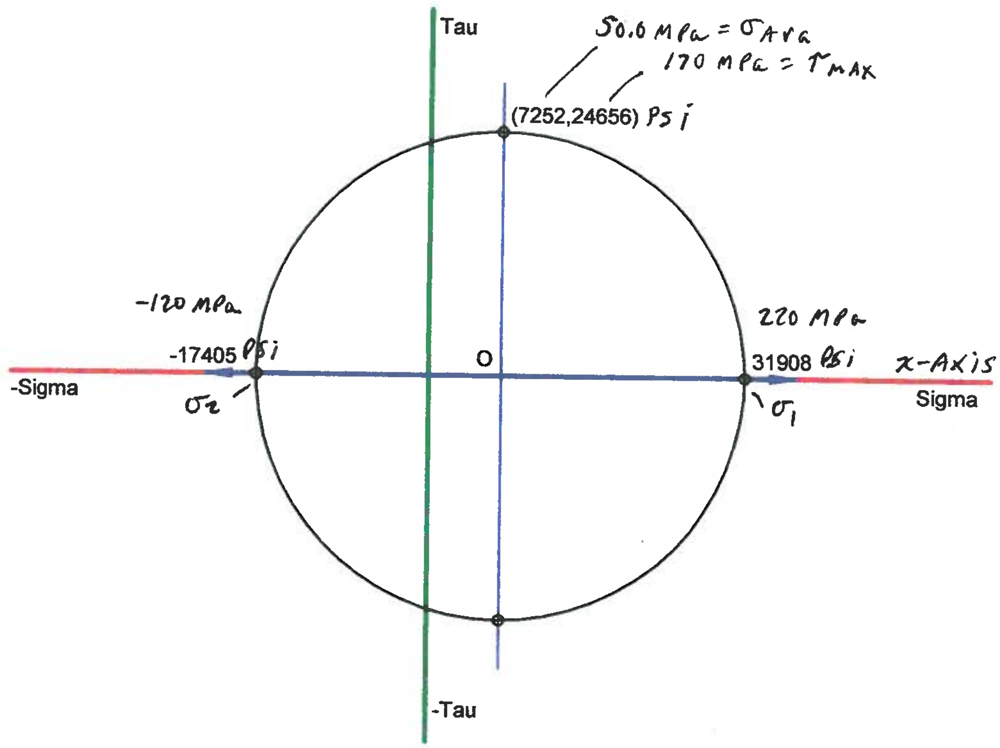
Maximum shear stress element

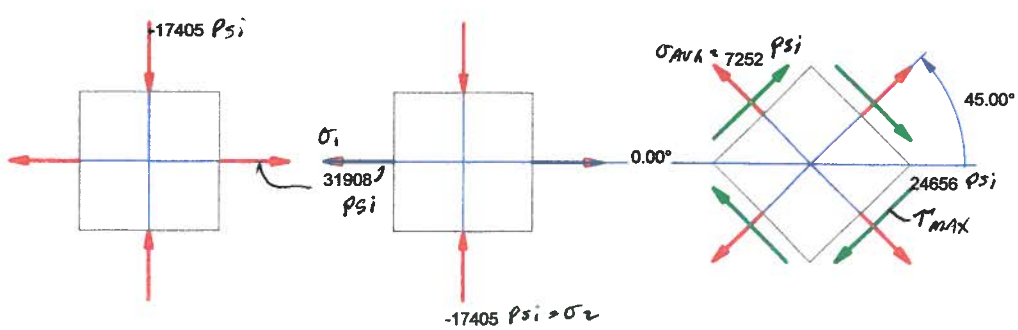
20.

σx = 220 MPa
σy = -120 MPa
τxy = 0 MPa

**Results**

| | | | |
|---|---|---|---|
| Maximum principal stress | σ1 | = | 220.000MPa |
| Minimum principal stress | σ2 | = | -120.000MPa |
| Maximum shear stress | τmax | = | 170.000MPa |
| Average normal stress | σavg | = | 50.000MPa |
| Principal planes | φσ | = | 0.000° ccw |
| Angle of maximum shear stress | φτ | = | 45.000° ccw to -τmax |




Original stress element

Principal stress element

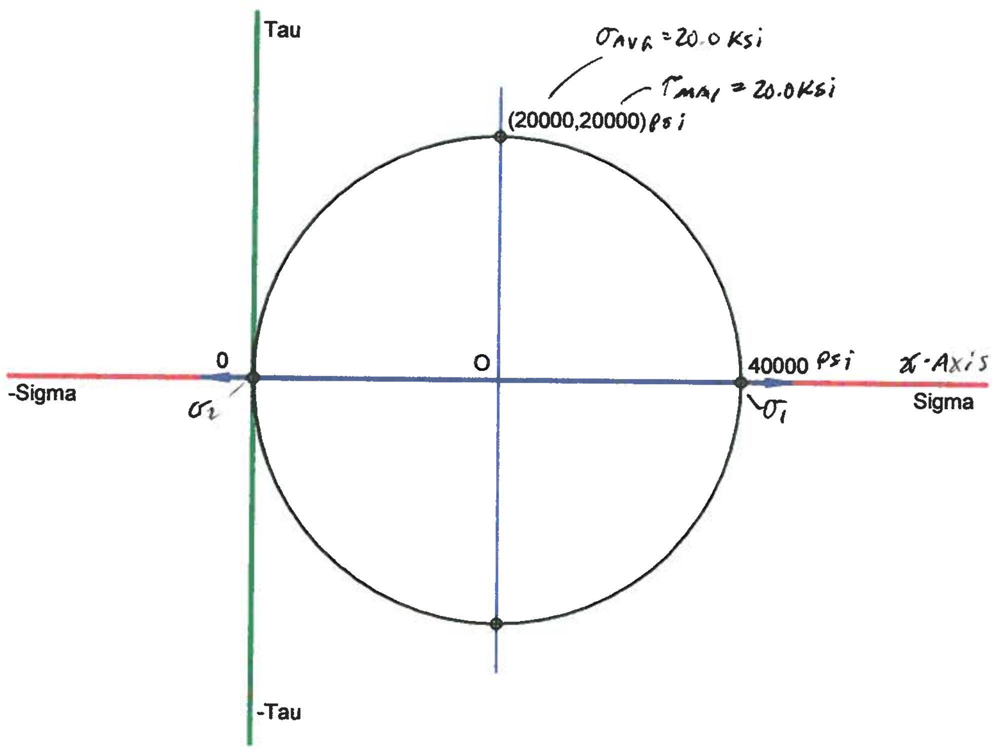
Maximum shear stress element

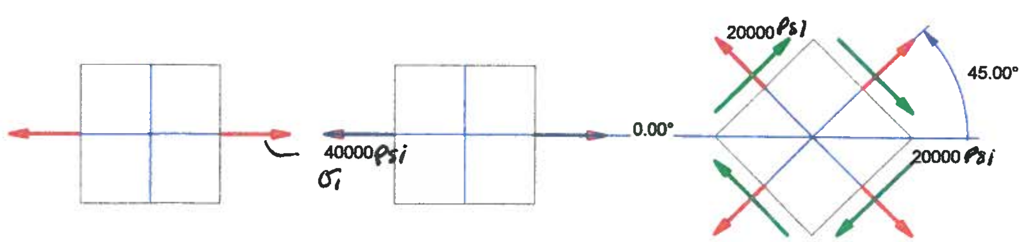
21,

$\sigma x$ = 40 Ksi
$\sigma y$ = 0 Ksi
$\tau xy$ = 0 Ksi

**Results**

| | | | |
|---|---|---|---|
| Maximum principal stress | $\sigma 1$ | = | 40.000Ksi |
| Minimum principal stress | $\sigma 2$ | = | 0.000Ksi |
| Maximum shear stress | $\tau max$ | = | 20.000Ksi |
| Average normal stress | $\sigma avg$ | = | 20.000Ksi |
| Principal planes | $\phi\sigma$ | = | 0.000° ccw |
| Angle of maximum shear stress | $\phi\tau$ | = | 45.000° ccw to $-\tau max$ |




Original stress element

Principal stress element

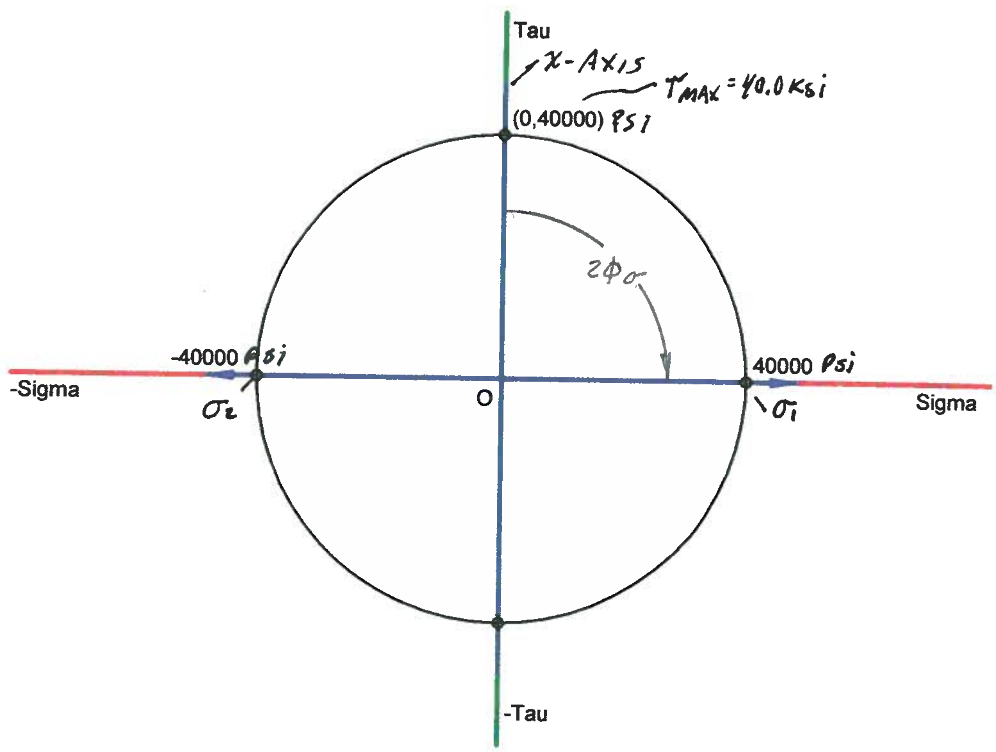
Maximum shear stress element

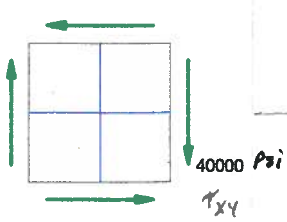
22.

| | | |
|---|---|---|
| $\sigma x$ = | 0 | Ksi |
| $\sigma y$ = | 0 | Ksi |
| $\tau xy$ = | 40 | Ksi |

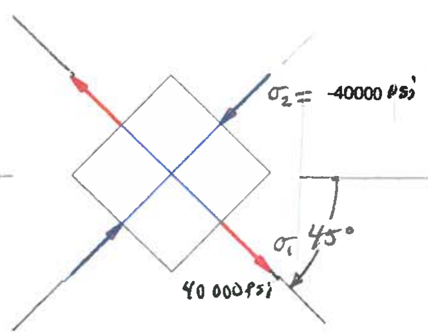
**Results**

| | | | |
|---|---|---|---|
| Maximum principal stress | $\sigma 1$ | = | 40.000Ksi |
| Minimum principal stress | $\sigma 2$ | = | -40.000Ksi |
| Maximum shear stress | $\tau max$ | = | 40.000Ksi |
| Average normal stress | $\sigma avg$ | = | 0.000psi |
| Principal planes | $\phi\sigma$ | = | 45.000° CW |
| Angle of maximum shear stress | $\phi\tau$ | = | 0.000° CCW |

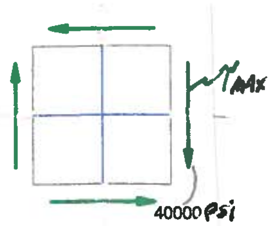



Original stress element




Principal stress element



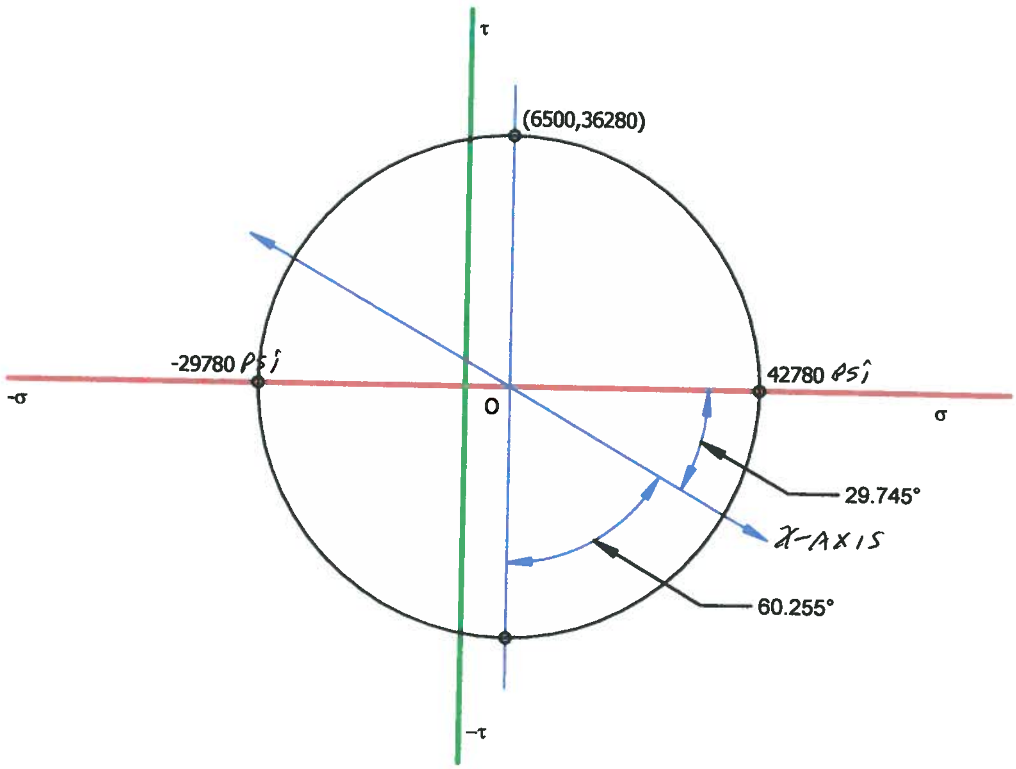

Maximum shear stress element

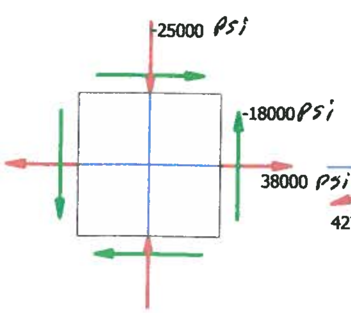
23

**Results:**

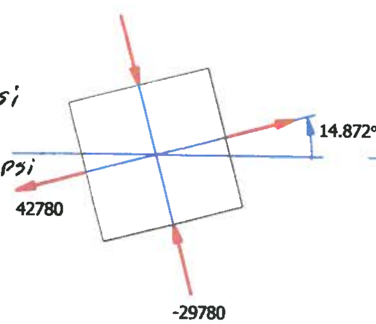
$\sigma x$ = 38 ksi
$\sigma y$ = -25 ksi
$\tau xy$ = -18 ksi

| | | | | |
|---|---|---|---|---|
| Maximum principal stress | $\sigma 1$ = | 42.780 | ksi | |
| Minimum principal stress | $\sigma 2$ = | -29.780 | ksi | |
| Maximum shear stress | $\tau max$ = | 36.280 | ksi | |
| Average normal stress | $\sigma avg$ = | 6.500 | ksi | |
| Principal planes | $\phi\sigma$ = | 14.872 | ° | ccw |
| Angle of maximum shear stress | $\phi\tau$ = | 30.128 | ° | cw to $-\tau max$ |

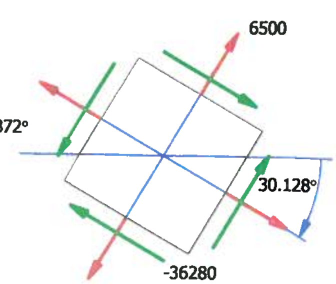



Original stress element




Principal stress element

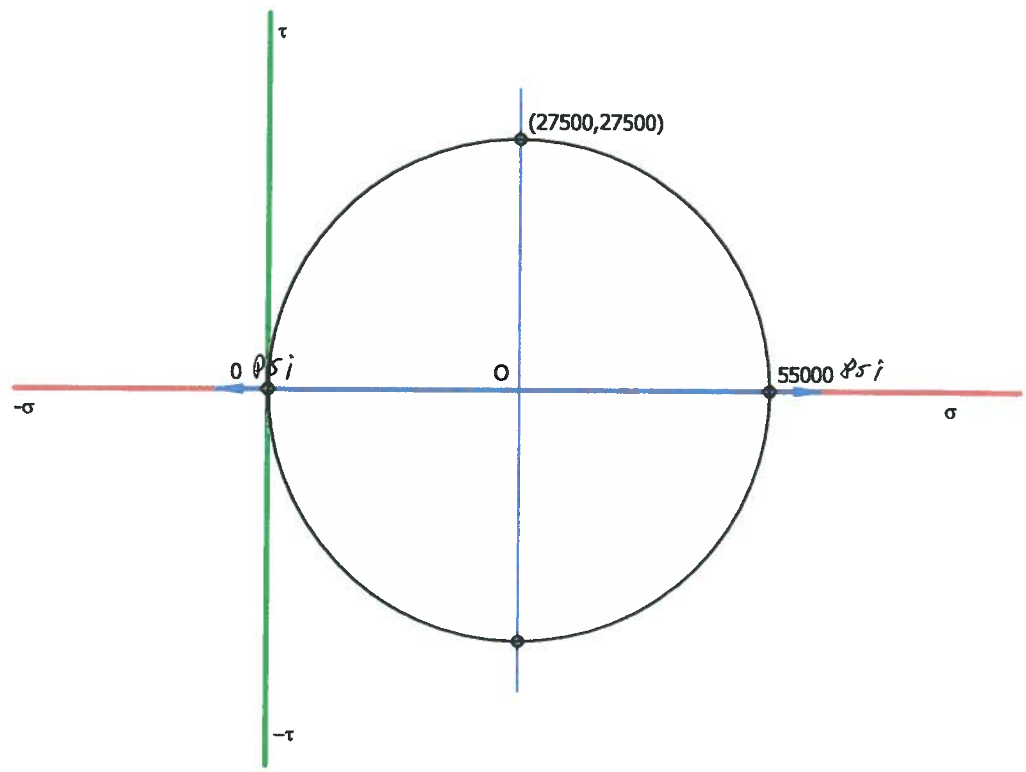


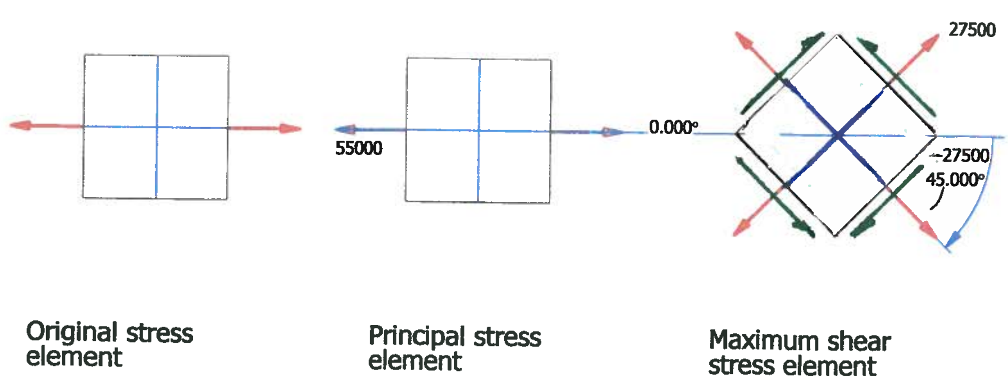

Maximum shear stress element

24

| | | |
|---|---|---|
| | $\sigma x$ = 55 | ksi |
| | $\sigma y$ = 0 | ksi |
| **Results:** | $\tau xy$ = 0 | ksi |

| | | | | | |
|---|---|---|---|---|---|
| Maximum principal stress | $\sigma 1$ | = | 55.000 | ksi | |
| Minimum principal stress | $\sigma 2$ | = | 0.000 | ksi | |
| Maximum shear stress | $\tau max$ | = | 27.500 | ksi | |
| Average normal stress | $\sigma avg$ | = | 27.500 | ksi | |
| Principal planes | $\phi\sigma$ | = | 0.000 | ° | ccw |
| Angle of maximum shear stress | $\phi\tau$ | = | 45.000 | ° | cw to $-\tau max$ |



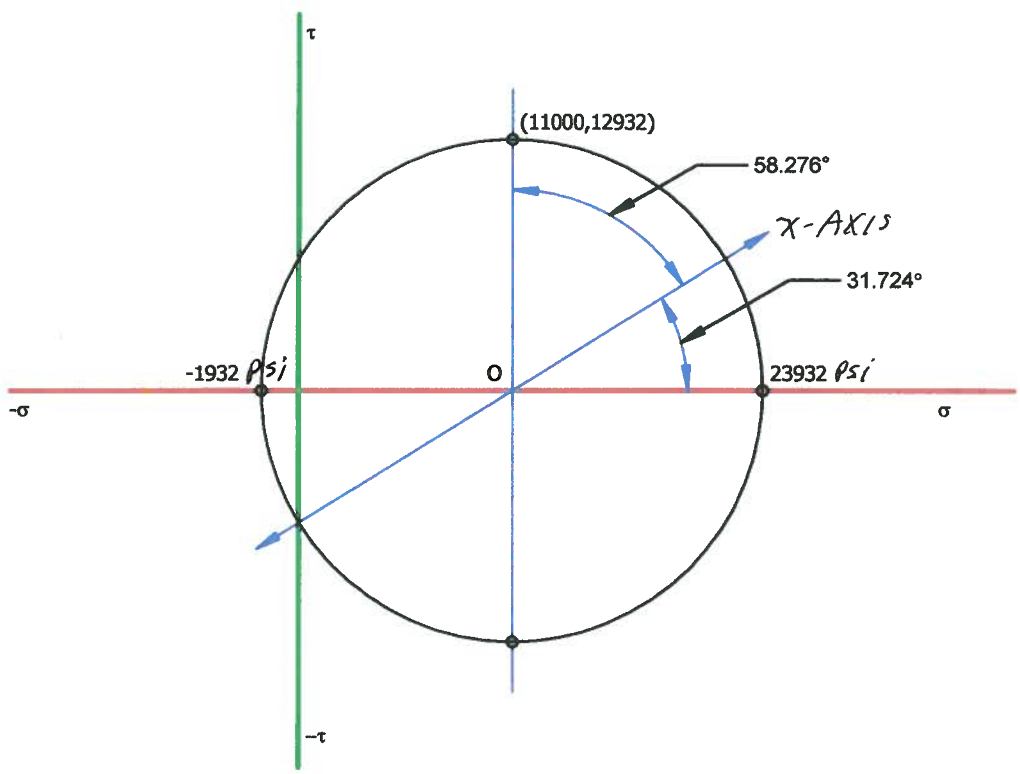



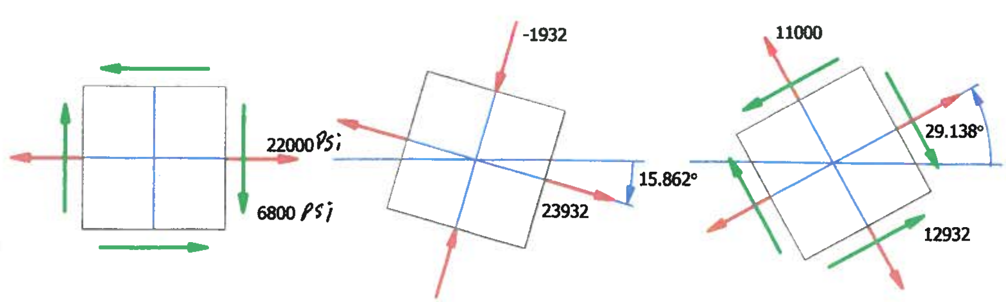
25

σx = 22 ksi
σy = 0 ksi
τxy = 6.8 ksi

**Results:**

| | | | | |
|---|---|---|---|---|
| Maximum principal stress | σ1 = | 23.932 | ksi | |
| Minimum principal stress | σ2 = | -1.932 | ksi | |
| Maximum shear stress | τmax = | 12.932 | ksi | |
| Average normal stress | σavg = | 11.000 | ksi | |
| Principal planes | φσ = | 15.862 | ° | CW |
| Angle of maximum shear stress | φτ = | 29.138 | ° | CCW |




Original stress element

Principal stress element

Maximum shear stress element

26

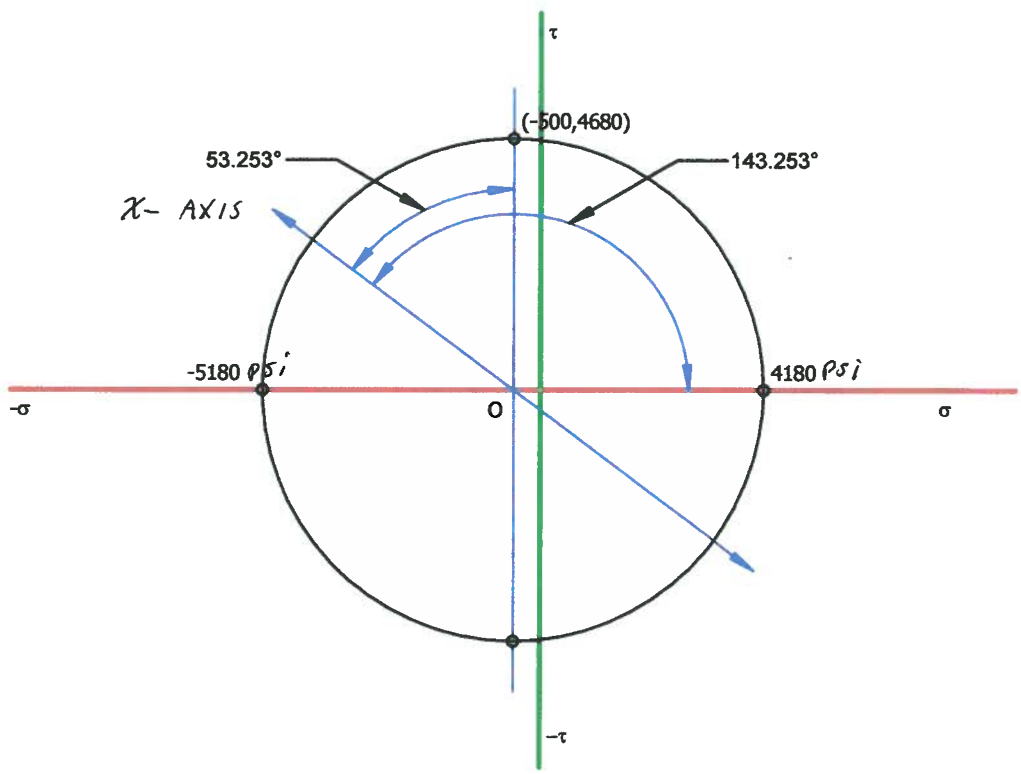
$\sigma x$ = -4250 psi

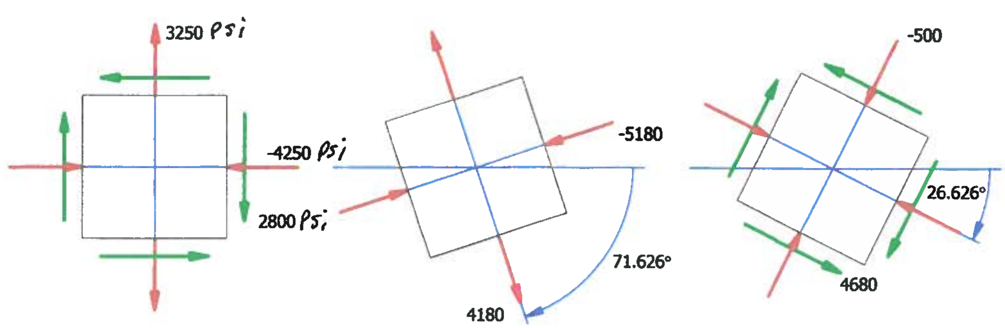
$\sigma y$ = 3250 psi

$\tau xy$ = 2800 psi

**Results:**

| | | | | |
|---|---|---|---|---|
| Maximum principal stress | $\sigma 1$ | = | 4180.011 | psi |
| Minimum principal stress | $\sigma 2$ | = | -5180.011 | psi |
| Maximum shear stress | $\tau max$ | = | 4680.011 | psi |
| Average normal stress | $\sigma avg$ | = | -500.000 | psi |
| Principal planes | $\phi\sigma$ | = | 71.626 | ° CW |
| Angle of maximum shear stress | $\phi\tau$ | = | 26.626 | ° CW |




Original stress element

Principal stress element

Maximum shear stress element

27 BOTH PRINCIPAL STRESSES ARE TENSILE – SAME SIGN

**Input data:**

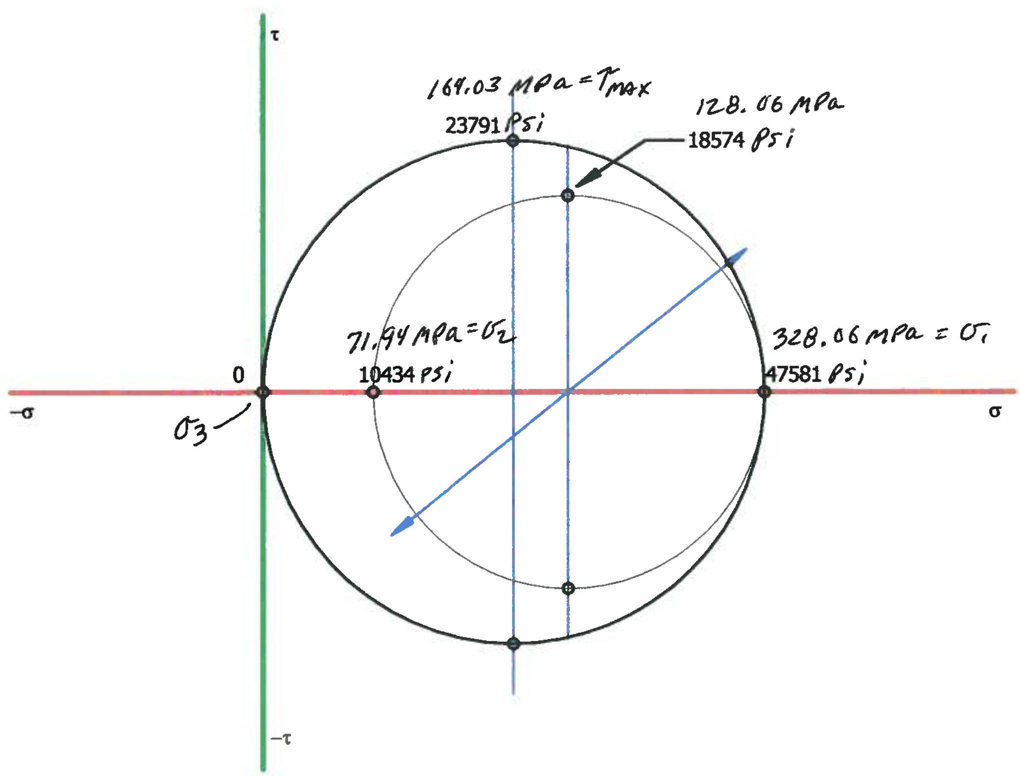
**Combined Stresses and Mohr's Circle**

| | | |
|---|---|---|
| Normal stress acting along x-axis | $\sigma x$ = 300 | MPa |
| Normal stress acting along y-axis | $\sigma y$ = 100 | MPa |
| Shear stress | $\tau xy$ = 80 | MPa |

**Results:**

| | | | | |
|---|---|---|---|---|
| Maximum principal stress | $\sigma 1$ | = | 328.062 | MPa |
| Minimum principal stress | $\sigma 2$ | = | 71.938 | MPa |
| Minimum principal stress | $\sigma 3$ | = | 0.000 | MPa |
| Maximum shear stress | $\tau max$ | = | 164.031 | MPa |
| Shear stress | $\tau$ | = | 128.062 | MPa |

Note: Both principle stresses have the same sign (both are tensile or both are compressive).
User must consider the resulting three-dimensional case.
Due to compound angles, elements as calculated are not applicable.



28 BOTH PRINCIPAL STRESSES ARE TENSILE - SAME SIGN.

**Input data:**

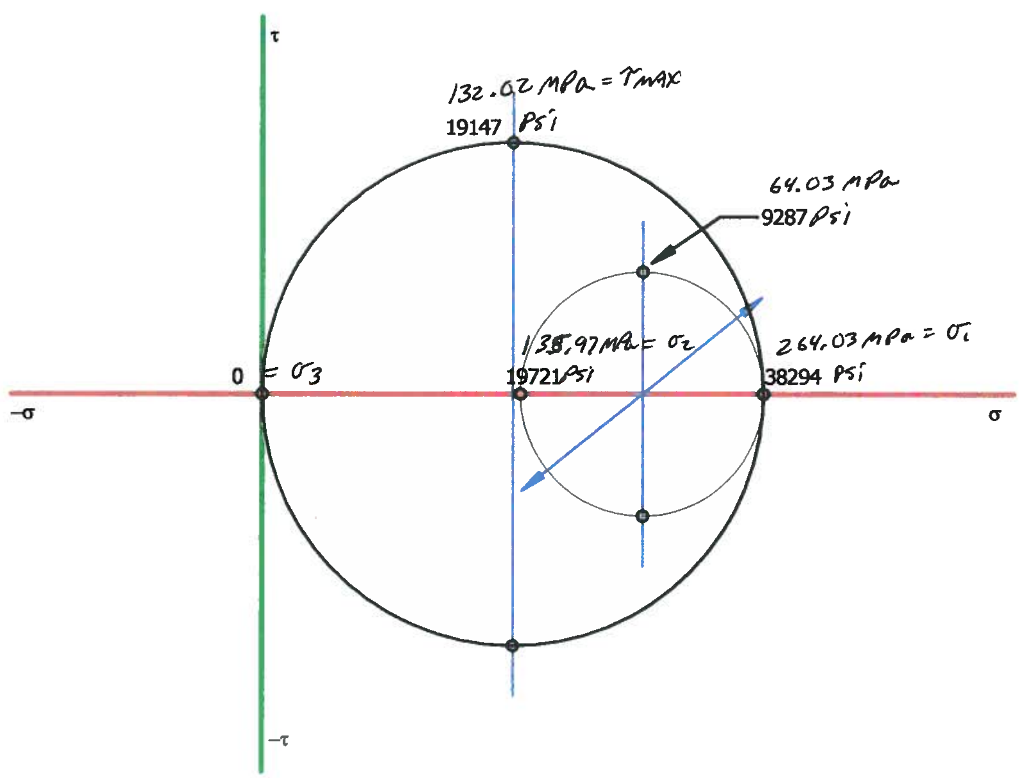
**Combined Stresses and Mohr's Circle**

| | | |
|---|---|---|
| Normal stress acting along x-axis | $\sigma x = 250$ | MPa |
| Normal stress acting along y-axis | $\sigma y = 150$ | MPa |
| Shear stress | $\tau xy = 40$ | MPa |

**Results:**

| | | | | |
|---|---|---|---|---|
| Maximum principal stress | $\sigma 1$ | = | 264.031 | MPa |
| Minimum principal stress | $\sigma 2$ | = | 135.969 | MPa |
| Minimum principal stress | $\sigma 3$ | = | 0.000 | MPa |
| Maximum shear stress | $\tau max$ | = | 132.016 | MPa |
| Shear stress | $\tau$ | = | 64.031 | MPa |

Note: Both principle stresses have the same sign (both are tensile or both are compressive).
User must consider the resulting three-dimensional case.
Due to compound angles, elements as calculated are not applicable.



29 BOTH PRINCIPAL STRESSES ARE COMPRESSIVE — SAME SIGN

**Input data:**

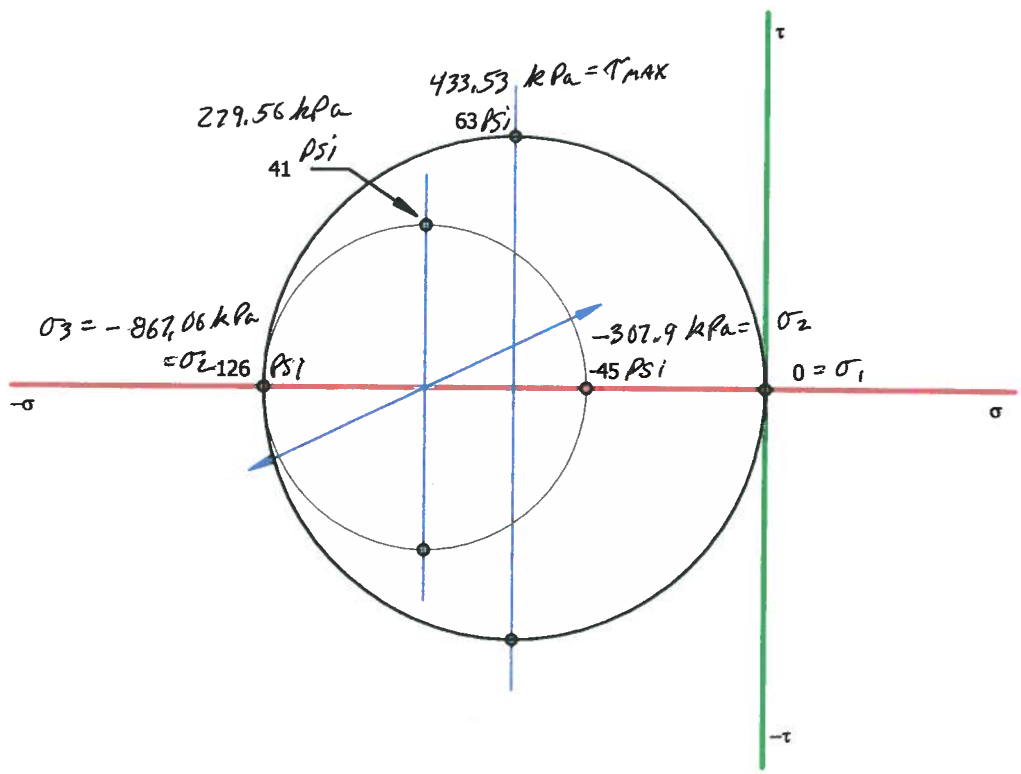
**Combined Stresses and Mohr's Circle**

| | | |
|---|---|---|
| Normal stress acting along x-axis | $\sigma x$ = -840 | kPa |
| Normal stress acting along y-axis | $\sigma y$ = -335 | kPa |
| Shear stress | $\tau xy$ = -120 | kPa |

**Results:**

| | | | | |
|---|---|---|---|---|
| Maximum principal stress | $\sigma 1$ | = | 0.000 | kPa |
| Minimum principal stress | $\sigma 2$ | = | -307.936 | kPa |
| Minimum principal stress | $\sigma 3$ | = | -867.064 | kPa |
| Maximum shear stress | $\tau max$ | = | 433.532 | kPa |
| Shear stress | $\tau$ | = | 279.564 | kPa |

Note: Both principle stresses have the same sign (both are tensile or both are compressive).
User must consider the resulting three-dimensional case.
Due to compound angles, elements as calculated are not applicable.



30 BOTH PRINCIPAL STRESSES ARE COMPRESSIVE – SAME SIGN

**Input data:**

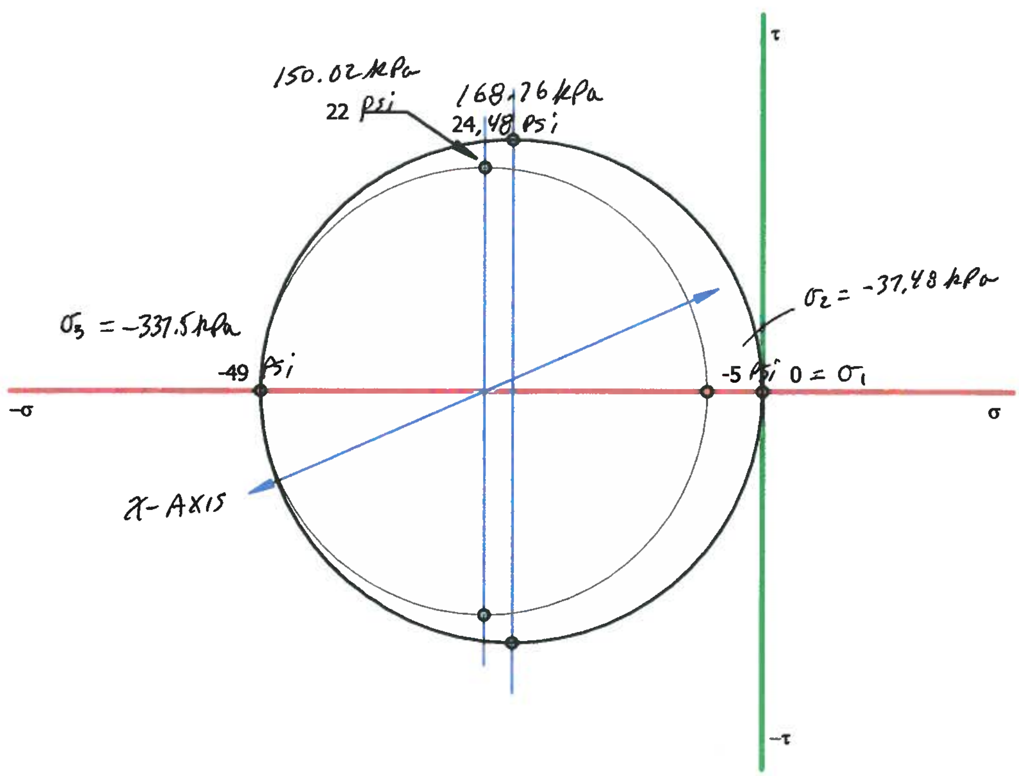
**Combined Stresses and Mohr's Circle**

| | |
|---|---|
| Normal stress acting along x-axis | $\sigma x$ = -325 kPa |
| Normal stress acting along y-axis | $\sigma y$ = -50 kPa |
| Shear stress | $\tau xy$ = -60 kPa |

**Results:**

| | | | | |
|---|---|---|---|---|
| Maximum principal stress | $\sigma 1$ | = | 0.000 | kPa |
| Minimum principal stress | $\sigma 2$ | = | -37.479 | kPa |
| Minimum principal stress | $\sigma 3$ | = | -337.521 | kPa |
| Maximum shear stress | $\tau max$ | = | 168.760 | kPa |
| Shear stress | $\tau$ | = | 150.021 | kPa |

Note: Both principle stresses have the same sign (both are tensile or both are compressive).
User must consider the resulting three-dimensional case.
Due to compound angles, elements as calculated are not applicable.



**31.** USE $\underline{D = 0.500\ IN}$ FOR SHAFT ABC. STRESS ELEMENT ON $\underline{BOTTOM}$.

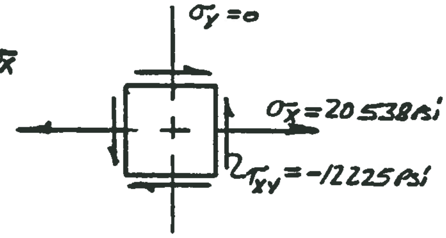
FROM FIG. 3-23: $M_B = 252\ LB\cdot IN.$ ; $T_B = 300\ LB\cdot IN.$

$Z = \pi D^3/32 = \pi(0.50\ IN)^3/32 = 0.01227\ IN^3$

$\sigma_B = \dfrac{M_B}{Z} = \dfrac{252\ LB\cdot IN.}{0.01227\ IN^3} = 20\,538\ psi = \sigma_x$

$Z_p = \pi D^3/16 = 2Z = 0.02454\ IN^3$

$\tau_B = \dfrac{T_B}{Z_p} = \dfrac{300\ LB\cdot IN.}{0.02454\ IN^3} = 12\,225\ psi$




$R = \tau_{MAX} = \sqrt{10\,269^2 + 12\,225^2} = 15\,966\ psi$

$\sigma_1 = 10\,269 + 15\,966 = 26\,235\ psi$

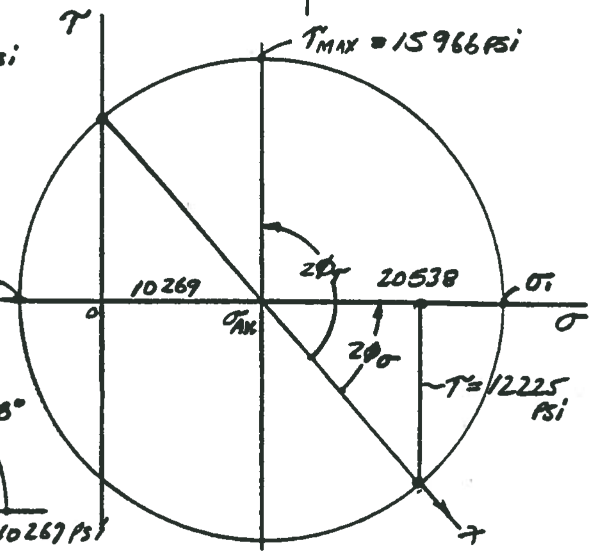
$\sigma_2 = 10\,269 - 15\,966 = -5\,697\ psi$

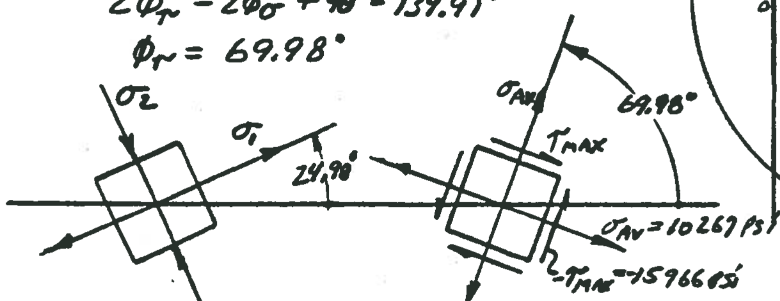
$2\phi_\sigma = TAN^{-1}\left(\dfrac{12225}{10269}\right) = 49.97°$

$\phi_\sigma = 24.98°\ CCW$

$2\phi_\tau = 2\phi_\sigma + 90° = 139.97°$

$\phi_\tau = 69.98°$




**32.** USE $\underline{D = 1.500\ IN.}$ FOR SHAFT ABC. STRESS ELEMENT ON $\underline{BOTTOM}$.

FROM SOLUTION FOR PROBLEM 3-48, $:M_B = 4572\ LB\cdot IN;\ T_B = 6400\ LB\cdot IN.$

$Z = \pi D^3/32 = 0.3313\ IN^3;\ \sigma_B = M_B/Z = 4572\ LB\cdot IN/0.3313\ IN^3 = 13\,800\ PSI = \sigma_x$

$Z_p = \pi D^3/16 = 2Z = 0.6627\ IN^3;\ \tau_B = T_B/Z_p = 6400/0.6627 = 9658\ PSI$

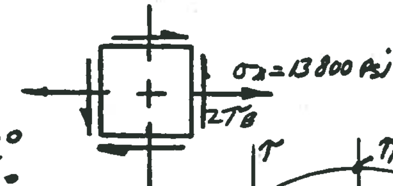
$R = \tau_{MAX} = \sqrt{6900^2 + 9658^2} = 11\,870\ PSI$

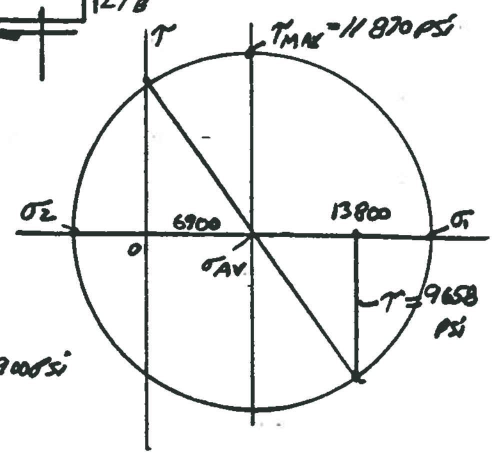
$\sigma_1 = 6900 + 11\,870 = 18\,770\ PSI$

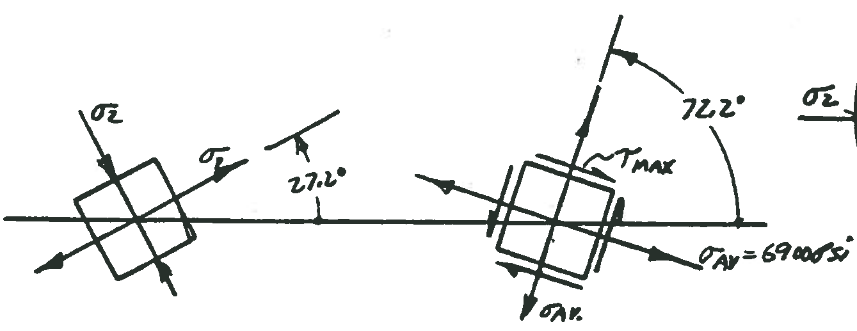
$\sigma_2 = 6900 - 11\,870 = -4970\ PSI$

$2\phi_\sigma = TAN^{-1}\left(\dfrac{9658}{6900}\right) = 54.46°;\ \phi_\sigma = 27.2°$

$2\phi_\tau = 2\phi_\sigma + 90° = 144.46°;\ \phi_\tau = 72.2°$



$\boxed{33.}$ USE $\underline{D = 2.25\ IN.}$ FOR SHAFT ABC.: STRESS ELEMENT ON $\underline{BOTTOM}$.

FROM SOLUTION FOR PROBLEM 3-49, $\therefore M_B = 8640\ LB{\cdot}IN.;\ T_B = 400\ LB{\cdot}IN$

$Z = \pi D^3/32 = 1.118\ IN^3;\ \underline{\sigma_B} = M/Z = 8640\ LB{\cdot}IN/1.118\ IN^3 = \underline{7726\ PSI}$ TENSION

$Z_P = \pi D^3/16 = 2Z = 2.237\ IN^3;\ \underline{\tau_B} = T_B/Z_P = 400\ LB{\cdot}IN/2.237\ IN^3 = \underline{179\ PSI}$

$R = \tau_{MAX} = \sqrt{179^2 + 3863^2} = 3867\ psi$

$\sigma_1 = 3863 + 3867 = 7730\ psi$

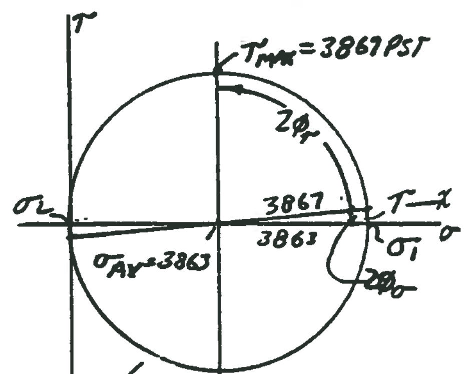
$\sigma_2 = 3863 - 3867 = -4\ psi$

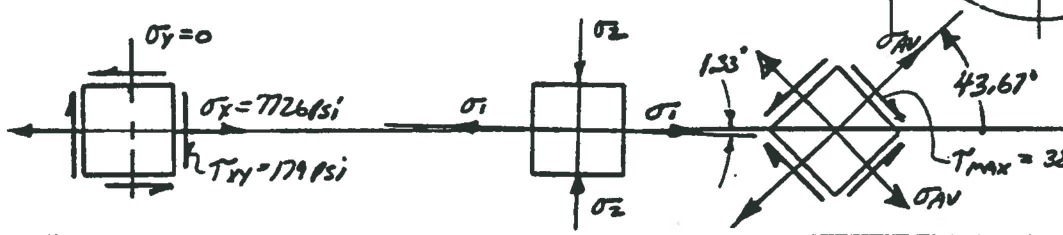
$2\phi_\sigma = TAN^{-1}(179/3863) = 2.65°$

$\phi_\sigma = 1.33°$ CW

$2\phi_\tau = 90° - 2\phi_\sigma = 87.35°$

$\phi_\tau = 43.67°$ CCW




$\boxed{34.}$ USE $\underline{D = 50.0\ mm}$ FOR SHAFT AB: STRESS ELEMENT ON $\underline{BOTTOM}$.

FROM SOLUTION FOR PROBLEM 3-50, $\therefore M_A = -0.650\ kN{\cdot}m;\ T_A = 0.375\ kN{\cdot}m$

$Z = \pi D^3/32 = 12\,272\ mm^3;\ \underline{\sigma_A} = M/Z = \dfrac{650\ N{\cdot}m}{12272\ mm^3} \cdot \dfrac{10^3\ mm}{m} = \underline{-52.97\ MPa}$ COMPRESSION.

$Z_P = \pi D^3/16 = 2Z = 24\,544\ mm^3;\ \underline{\tau_A} = \dfrac{T}{Z_P} = \dfrac{375\ N{\cdot}m}{24544\ mm^3} \cdot \dfrac{10^3\ mm}{m} = \underline{15.28\ MPa}$

$R = \tau_{MAX} = \sqrt{15.28^2 + 26.48^2} = 30.57\ MPa$

$\sigma_1 = -26.48 + 30.57 = 4.09\ MPa$

$\sigma_2 = -26.48 - 30.57 = -57.05\ MPa$

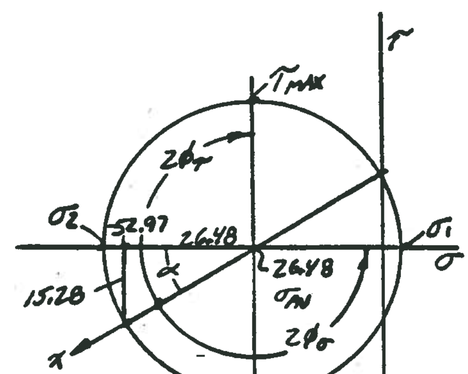
$\alpha = Tan^{-1}(15.28/26.48) = 30.0°$

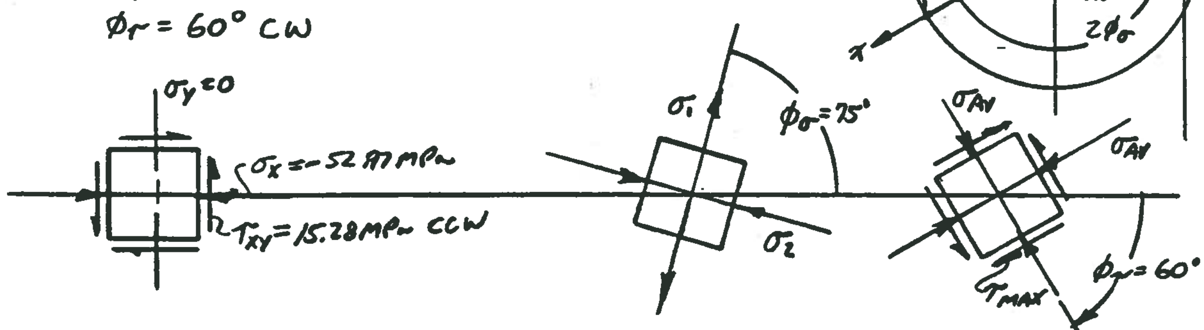
$2\phi_\sigma = 180° - \alpha = 150°$

$\phi_\sigma = 75°$ CCW

$2\phi_\tau = 90° + \alpha = 120°$

$\phi_\tau = 60°$ CW



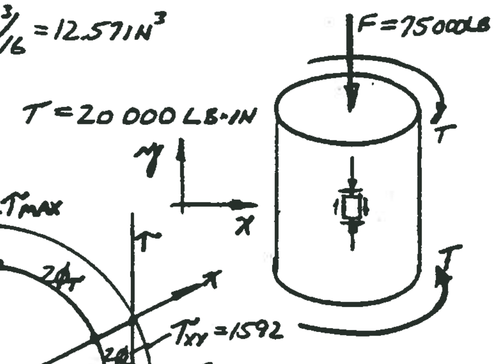



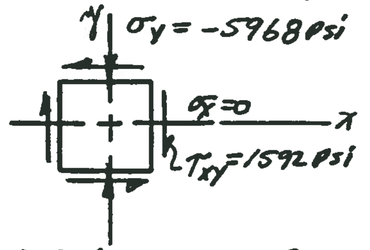
**35.**

$D = 4.00\,IN$ ; $A = \pi D^2/4 = 12.57\,IN^2$; $Z_p = \pi D^3/16 = 12.57\,IN^3$

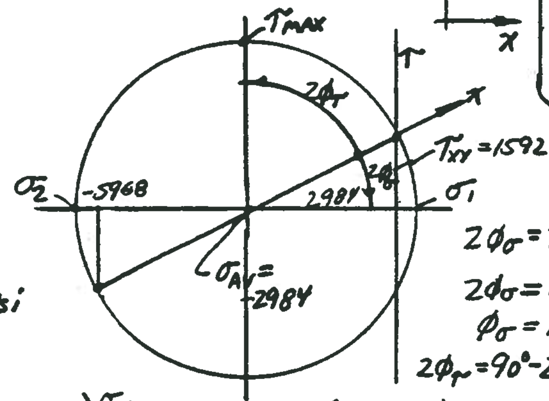
$\sigma = \dfrac{-F}{A} = \dfrac{-75\,000\,LB}{12.57\,IN^2} = -5968\,psi$

$\tau = \dfrac{T}{Z_p} = \dfrac{20\,000\,LB\cdot IN}{12.57\,IN^3} = 1592\,psi$




SURFACE ELEMENT




$R = \tau_{MAX} = \sqrt{1592^2 + 2984^2} = 3382\,psi$

$\sigma_1 = -2984 + 3382 = 398\,psi$

$\sigma_2 = -2984 - 3382 = -6366\,psi$

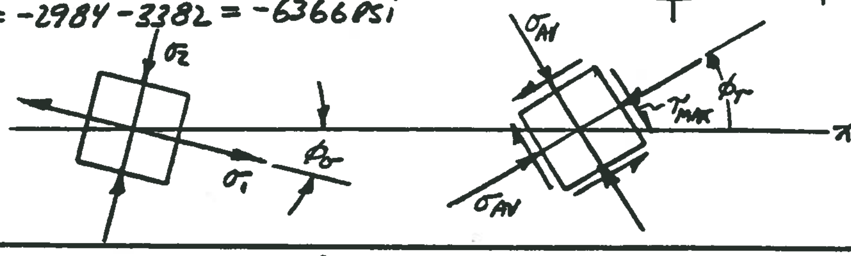
$2\phi_\sigma = Tan^{-1}\left(\dfrac{1592}{2984}\right)$

$2\phi_\sigma = 28.08°$

$\phi_\sigma = 14.04°\ CW$

$2\phi_\tau = 90° - 2\phi_\sigma = 61.92°$

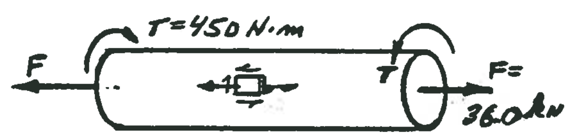
$\phi_\tau = 30.96°\ CCW$




**36.**

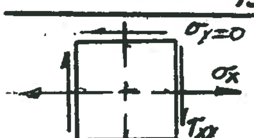
$D = 20\,mm$ ; $A = \dfrac{\pi D^2}{4} = 314\,mm^2$

$\sigma_x = \dfrac{F}{A} = \dfrac{36000\,N}{314\,mm^2} = 114.6\,MPa$ TENSION




$Z_P = \dfrac{\pi D^3}{16} = 1571\,mm^3$

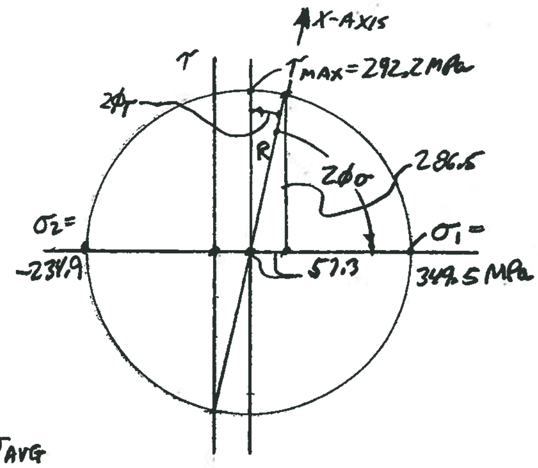
$\tau_{x\text{-}y} = T/Z_P = \dfrac{450\,N\cdot m}{1571\,mm^3} \times \dfrac{10^3\,mm}{m} = 286.48\,MPa$




$\sigma_{AVG} = \dfrac{114.6 + 0}{2} = 57.3\,MPa$

$R = \tau_{MAX} = \sqrt{57.3^2 + 286.5^2}$

$\tau_{MAX} = 292.2\,MPa$

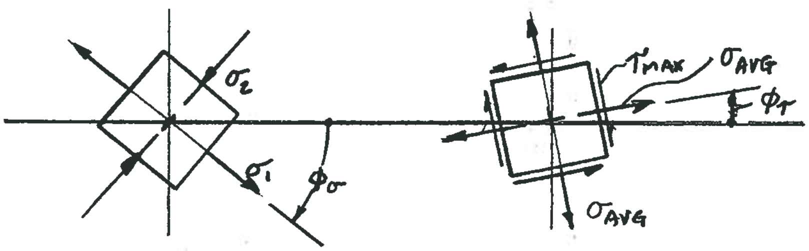



$\sigma_1 = 57.3 + 292.2 = 349.5\,MPa$

$\sigma_2 = 57.3 - 292.2 = -234.9\,MPa$

$2\phi_\sigma = Tan^{-1}(286.5/57.3) = 78.7°$; $\phi_\sigma = 39.35°$

$2\phi_\tau = 90 - 2\phi_\sigma = 11.3°$; $\phi_\tau = 5.65°$



# CHAPTER 5
# DESIGN FOR DIFFERENT TYPES OF LOADING

**Stress Ratio**

$\boxed{1.}$

$\sigma = F/A\ ;\ A = \frac{\pi (10\,mm)^2}{4} = 78.54\,mm^2$

$\sigma_{MAX} = 3500\,N / 78.54\,mm^2 = \underline{44.6\,MPa}$

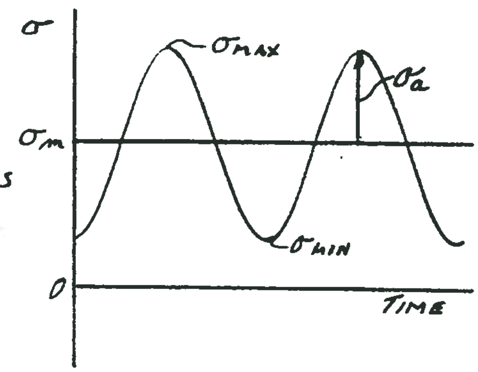
$\sigma_{MIN} = 500\,N/A = \underline{6.37\,MPa}$

$F_m = (3500 + 500)/2 = 2000\,N$

$\sigma_m = 2000\,N/A = \underline{25.5\,MPa}$

$\sigma_a = \sigma_{MAX} - \sigma_m = 44.6 - 25.5 = \underline{19.10\,MPa}$

$R = \sigma_{MIN}/\sigma_{MAX} = 6.37/44.6 = \underline{0.143}$




$\boxed{2.}$

$\sigma = F/A\ ;\ A = (10\,mm)(30\,mm) = 300\,mm^2$

$\sigma_{MAX} = 20 \times 10^3\,N / 300\,mm^2 = \underline{66.7\,MPa}$

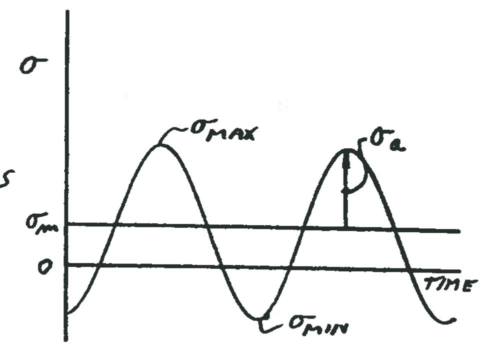
$\sigma_{MIN} = -8.0 \times 10^3\,N/A = \underline{-26.7\,MPa}$

$F_m = (20 - 8)/2 = 6\,kN$

$\sigma_m = 6 \times 10^3\,N/A = \underline{20.0\,MPa}$

$\sigma_a = \sigma_{MAX} - \sigma_m = 66.7 - 20.0 = \underline{46.7\,MPa}$

$R = \sigma_{MIN}/\sigma_{MAX} = -26.7/66.7 = \underline{-0.40}$




$\boxed{3.}$

$\sigma = F/A\ ;\ A = (0.40\,IN)^2 = 0.16\,IN^2$

$\sigma_{MAX} = 860\,LB / 0.16\,IN^2 = \underline{5375\,psi}$

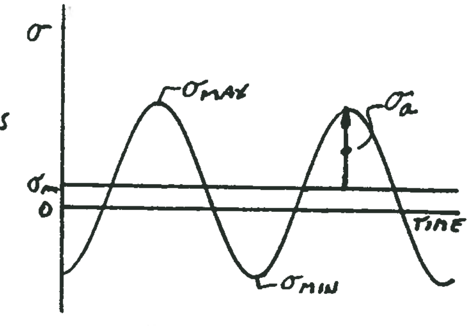
$\sigma_{MIN} = -120\,LB / 0.16\,IN^2 = \underline{-750\,psi}$

$F_m = (860 - 120)/2 = 370\,LB$

$\sigma_m = 370\,LB/A = \underline{2313\,psi}$

$\sigma_a = \sigma_{MAX} - \sigma_m = 5375 - 2313 = \underline{3062\,psi}$

$R = \sigma_{MIN}/\sigma_{MAX} = -750/5375 = \underline{-0.140}$




$\boxed{4.}$

$\sigma = F/A\ ;\ A = \pi D^2/4 = \pi (0.375)^2/4 = 0.1104\,IN^2$

$\sigma_{MAX} = 1800\,LB / 0.1104\,IN^2 = \underline{16297\,psi}$

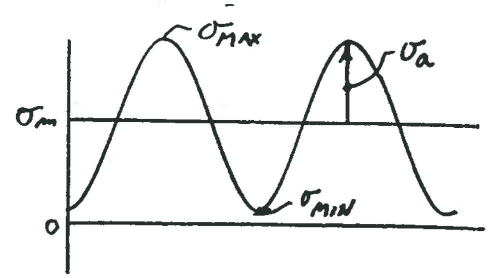
$\sigma_{MIN} = 150\,LB / 0.1104\,IN^2 = \underline{1358\,psi}$

$F_m = (1800 + 150)/2 = 975\,LB$

$\sigma_m = 975\,LB/A = \underline{8828\,psi}$

$\sigma_a = \sigma_{MAX} - \sigma_m = 16297 - 8828 = \underline{7470\,psi}$

$R = \sigma_{MIN}/\sigma_{MAX} = 1358/16297 = \underline{0.083}$



**5.** $\sigma = F/A$ ; $A = \frac{\pi D^2}{4} = \frac{\pi (3.0\,mm)^2}{4} = 7.069\,mm^2$

$\sigma_{MAX} = 780N / 7.07\,mm^2 = \underline{110.3\,MPa}$

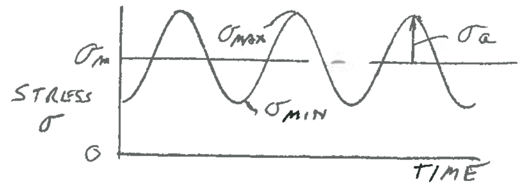
$\sigma_{MIN} = 360N/A = \underline{50.9\,MPa}$

$F_m = (780+360)/2 = 570\,N$

$\sigma_m = 570N/A = \underline{80.6\,MPa}$

$\sigma_a = 110.3 - 80.6 = \underline{29.7\,MPa}$

$R = 50.9/110.3 = \underline{0.462}$




**6.** $\sigma = M/S$ ; $S_y = 1.48\,IN^3$ FOR 4x2x1/4 TUBE — APP 15-14

AT B

$\sigma_{MAX} = \frac{14400}{1.48} = 9730\,psi$

$\sigma_{MIN} = \frac{10560}{1.48} = 7135\,psi$

$\sigma_m = (9730+7135)/2 = 8432\,psi$

$\sigma_a = \sigma_{MAX} - \sigma_m = 9730 - 7135$

$\sigma_a = 2595\,psi$

$R = \sigma_{MIN}/\sigma_{MAX} = \frac{7135}{9730}$

$R = 0.233$

AT C

$\sigma_{MAX} = \frac{14400}{1.48} = 9730\,psi$

$\sigma_{MIN} = \frac{3840}{1.48} = 2595\,psi$

$\sigma_m = (9730+2595)/2 = 6162\,psi$

$\sigma_a = \sigma_{MAX} - \sigma_m = 9730 - 6162$

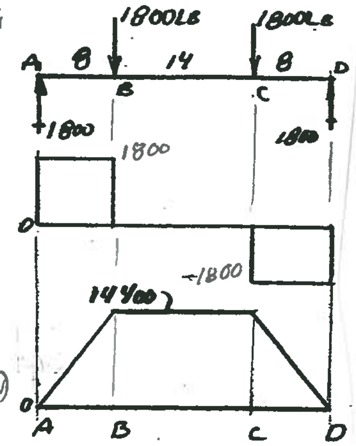
$\sigma_a = 3568\,psi$

$R = \sigma_{MIN}/\sigma_{MAX} = \frac{2595}{9730}$

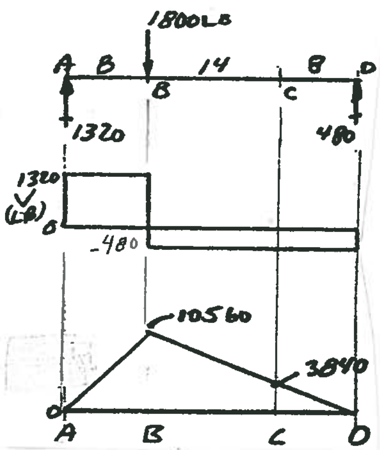
$R = 0.267$

STRESS VS. TIME DIAGRAM–SAME AS FOR PROB. 5.

LOADING CASE I




LOADING CASE II




**7.** BEAM: $\sigma = \frac{M}{S}$ : $S = 3.04\,IN^3$ FOR S4×7.7

$M_{MAX} = (500\,LB)(60\,IN) = 30000\,LB\cdot IN = M_1$

$M_{MIN} = (500\,LB)(10\,IN) = 5000\,LB\cdot IN = M_2$

$M_m = (30000 + 5000)/2 = 17500\,LB\cdot IN$

$M_a = 30000 - 17500 = 12500\,LB\cdot IN$

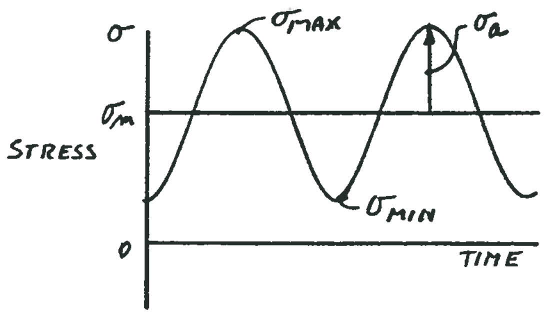
$\sigma_{MAX} = \frac{30000\,LB\cdot IN}{3.04\,IN^3} = \underline{9868\,psi}$

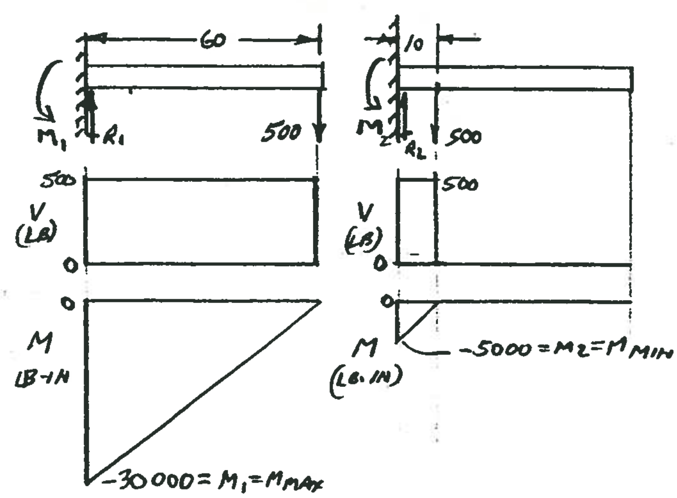
$\sigma_{MIN} = 5000/3.04 = \underline{1645\,psi}$

$\sigma_m = 17500/3.04 = \underline{5757\,psi}$

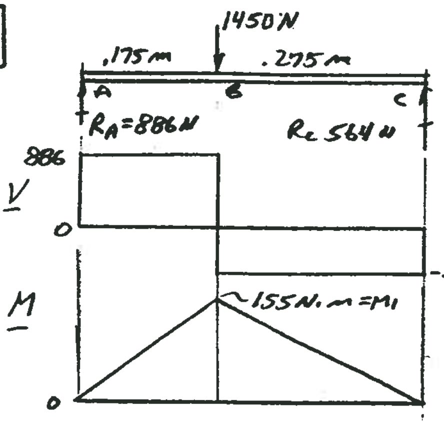
$\sigma_a = 12500/3.04 = \underline{4112\,psi}$

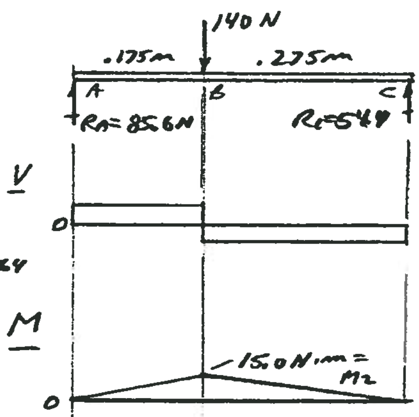
$R = \frac{\sigma_{MIN}}{\sigma_{MAX}} = 1645/9868 = \underline{0.167}$



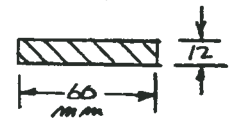



**8.**




BEAM CROSS SECTION




$S = \frac{BH^2}{6} = \frac{(60)12^2}{6}$

$S = 1440\ mm^3$

$\sigma = \frac{M}{S}$ BENDING

$\sigma_{MAX} = \frac{M_1}{S} = \frac{155\ N\cdot m}{1440\ mm^3} \cdot \frac{10^3\ mm}{m} = \underline{107.7\ MPa}$

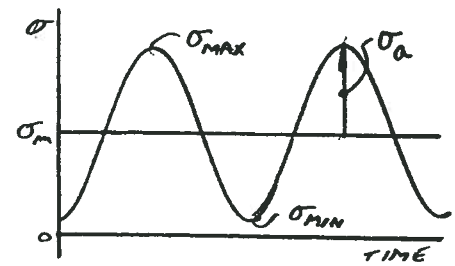
$\sigma_{MIN} = \frac{M_2}{S} = \frac{15.0 \times 10^3\ N\cdot mm}{1440\ mm^3} = \underline{10.4\ MPa}$

$\sigma_m = (107.7 + 10.4)/2 = \underline{59.1\ MPa}$

$\sigma_a = \sigma_{MAX} - \sigma_m = 107.7 - 59.1 = \underline{48.6\ MPa}$

$R = \sigma_{MIN}/\sigma_{MAX} = 10.4/107.7 = \underline{0.097}$

STRESS




**9.**

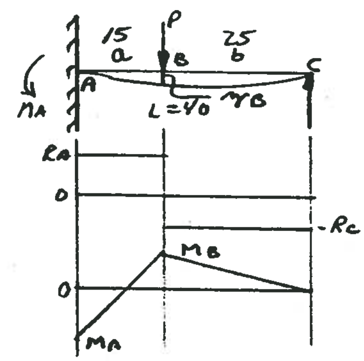
SPRING IS A SUPPORTED CANTILEVER - CASE (b) APP A14-3.
DEFLECTION PROPORTIONAL TO FORCE. BENDING MOMENT PROPORTIONAL TO FORCE.
DEFLECTION AT LOAD B:

$y_B = \frac{-P a^3 b^2}{12 E I L^3}(3L + b)$


$I = \frac{(5)(.6)^3}{12} = 0.090\ mm^4$

$S = \frac{5(.6)^2}{6} = 0.30\ mm^3$

$E = 207 \times 10^3\ N/mm^2$




SOLVE FOR P:

$P = \frac{12 E I L^3 y_B}{a^3 b^2 (3L + b)} = \frac{(12)(207 \times 10^3)(0.090)(40)^3 y_B}{(15)^3 (25)^2 [3(40) + 25]}\ N$

$P = 46.78\ y_B$

FOR $y_1 = 0.25\ mm$; $P_1 = 46.78(0.25) = \underline{11.7\ N}$; FOR $y_2 = 0.40\ mm$, $\underline{P_2 = 18.7\ N}$

MOMENTS:

$M_A = \frac{-Pab}{2L^2}(b + L) = \frac{-P(15)(25)}{2(40)^2}(25 + 40) = \underline{-7.617\ P \quad MAXIMUM}$

$M_B = \frac{Pa^2 b}{2L^3}(b + 2L) = \frac{P(15)^2(25)}{2(40)^3}[25 + 2(40)] = 4.614 P$

SUMMARY

$P_2 = 18.7\ N$; $M_{A2} = 7.617(18.7) = 142.5\ N\cdot mm$; $\sigma_{A2} = \frac{M}{S} = \frac{142.5\ N\cdot mm}{0.30\ mm^3} = \underline{475\ MPa = \sigma_{MAX}}$

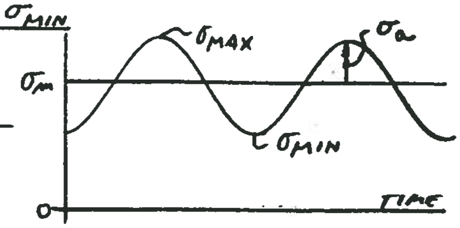
$P_1 = 11.7\ N$: $M_{A1} = 89.1\ N\cdot mm$; $\sigma_{A1} = \underline{297\ MPa = \sigma_{MIN}}$

$\sigma_m = (475 + 297)/2 = \underline{386\ MPa}$

$\sigma_a = 475 - 386 = \underline{89.0\ MPa}$

$R = \sigma_{MIN}/\sigma_{MAX} = 297/475 = \underline{0.625}$

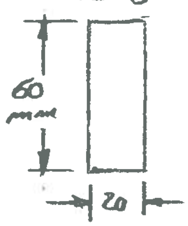
STRESS



**10.** FIND $S_n'$; SAE 1040 CD; $S_u = 80$ ksi; $S_n = 31$ ksi FOR CD CURVE FIG. 5-8.
0.75 IN DIA. $C_s = 0.90$ FIG. 5-9
$C_m = 1.0$ WROUGHT STEEL; $C_{ST} = 1.0$ REV. BENDING; $C_R = 0.81$ (99% R)
$S_n' = S_n C_s C_m C_{ST} C_R = 31\text{ ksi}(0.90)(1.0)(1.0)(0.81) = \underline{22.6\text{ ksi}}$

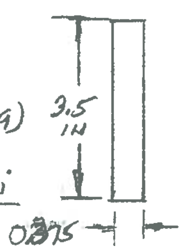
**11.** FIND $S_n'$: SAE 5160 OQT 1300; $S_u = 793$ MPa; $S_n = 300$ MPa FIG. 5-8. MACHINED.
20.0 mm DIA. $C_s = 0.90$ FIG. 5-9; $C_m = 1.0$ WROUGHT STEEL;
$C_{ST} = 1.0$ REV. BENDING; $C_R = 0.81\ (R=0.99)$
$S_n' = S_n C_s C_m C_{ST} C_R = 300\text{ MPa}(0.90)(1.0)(1.0)(0.81) = \underline{219\text{ MPa}}$

**12.** FIND $S_n'$: SAE 4130 WQT 1300; $S_u = 676$ MPa; $S_n = 260$ MPa FIG. 5-8
EQ. (5-8): $D_e = 0.808\sqrt{hb} = 0.808\sqrt{(60)(20)} = 28.0$ mm
$C_s = 0.87$ FIG. 5-9; $C_m = 1.0$ WROUGHT STEEL
$C_{ST} = 1.0$ REV. BENDING; $C_R = 0.81\ (R=0.99)$
$S_n' = S_n C_s C_m C_{ST} C_R = 260\text{ MPa}(0.87)(1.0)(1.0)(0.81) = \underline{183\text{ MPa}}$




**13.** FIND $S_n'$: SAE 301 ST. STL. 1/2 HARD; $S_u = 150$ ksi; $S_n = 52$ ksi FIG 5-8.
$C_s = 1.0$ FOR AXIAL TENSILE STRESS. $C_m = 1.0$ WROUGHT STEEL
$C_{ST} = 0.80$ AXIAL TENSILE STRESS. $C_R = 0.75$ FOR $R = 0.999$.
$S_n' = S_n C_s C_m C_{ST} C_R = 52\text{ ksi}(1.0)(1.0)(0.80)(0.75) = \underline{31.2\text{ ksi}}$

**14.** FIND $S_n'$: ASTM A242; $S_u = 70$ ksi; $S_n = 27.0$ ksi, FIG. 5-8
EQ (5-8): $D_e = 0.808\sqrt{hb} = 0.808\sqrt{(3.5)(0.375)} = 0.926$; $C_s = 0.883$
$C_m = 1.0$ WROUGHT STEEL; $C_{ST} = 1.0$ REV. BENDING; $C_R = 0.81\ (R=0.99)$
$S_n' = S_n C_s C_m C_{ST} C_R = (27.0\text{ ksi})(0.883)(1.0)(1.0)(0.81) = \underline{19.3\text{ ksi}}$




**Design and Analysis**

**Problems 15 - 18 are open-ended design problems for which there is no unique answer. The *General Design Procedure* from Section 5-9 should be used. The loading and support conditions should be compared with the cases described in Section 5-8 to determine the appropriate design stress. A design factor should be specified using the guidelines in Section 5-7 When needed, the endurance strength should be computed from Equation 5-6 in Section 5-4.**

**15. The link is subjected to a fluctuating normal stress. Use Case 5 from Section 5-8 See also the solution for Problem 1.**

**16. The rod is subjected to a fluctuating normal stress. Use Case 5 from Section 5-8 See also the solution for Problem 4.**

**17. The strut is subjected to a fluctuating normal stress. Use Case 5 from Section 5-8 See also the solution for Problem 2.**

**18. The latch part is subjected to a fluctuating normal stress. Use Case 5 from Section 5-8 See also the solution for Problem 5.**

**19.** FIG. P5-8. SEE ALSO PROBLEM 8 SOLUTION. FLUCTUATING LOAD

$\frac{1}{N} = \frac{\sigma_m}{S_u} + \frac{K_t \sigma_a}{S_n'}$ CASE 5 $\sigma_m = 59.1\ MPa$ ; $\sigma_a = 48.6\ MPa$

FIND N. SAE 1020 HR; $S_y = 207\ MPa$ ; $S_u = 379\ MPa$ ; $S_n = 140\ MPa$

$C_s$: RECTANGLE: $D_e = .808\sqrt{12(60)} = 21.7\ mm$ ; $C_s = 0.89$ FIG. 5-8 HOT ROL.

$C_m = 1.0$ (WROUGHT STEEL), $C_{ST} = 1.0$ (REV. BENDING); LET $R = 0.99 - C_R = 0.81$

$S_n' = (0.89)(1.0)(1.0)(0.81)(140) = 101\ MPa$

$\frac{1}{N} = \frac{59.1}{379} + \frac{(1.0)(48.6)}{101} = 0.631$ ; $\underline{N = 1.57}$ LOW

YIELD: $N = \frac{S_y}{K_t(\sigma_m + \sigma_a)}$ $N_y = \frac{207}{1(59.1 + 48.6)} = 1.92$

**20.** DESIGN PROBLEM – NO UNIQUE SOLUTION.

SUGGESTIONS: KEEP 60 mm WIDTH FOR SEAT APPLICATION.

CONSIDER HIGHER STRENGTH MATERIAL; THINNER STOCK FORMED INTO CHANNEL SHAPE . CONSIDER TAPERING CROSS SECTION DEPTH – DEEPER AT LOAD – LESS DEEP NEAR SUPPORTS WHERE MOMENT IS SMALLER. CONSIDER: REF. k – APP. 15-7 OR REF. g – APP 15-8. ALUMINUM.

**21.** DATA ARE SAME AS IN PROB. 7 WHERE AN S4X7.7 STEEL BEAM WAS PROPOSED. FLUCTUATING STRESS. USE CASE 5. $A_T = 2.26\ in^2$

$\frac{1}{N} = \frac{\sigma_m}{S_u} + \frac{K_t \sigma_a}{S_n'}$ • SPECIFY ASTM A36 STEEL $S_y = 36\ ksi$ , $S_u = 58\ ksi$

• $C_s$: $A_{95} = 0.05 A_T = 0.05(2.26\ in^2) = 0.113\ in^2 = 0.0766\ D_e^2$

$S_n = 20\ ksi$ (FIG. 5-8) • $D_e = \sqrt{0.113/0.0766} = 1.215 \Rightarrow C_s = 0.86$

$C_m = 1.0$ ; $C_{st} = 1.0$ ; LET $R = 0.99 - C_R = 0.81$.

$S_n' = (.86)(0.81)(20) = 13.9\ ksi$ : $\sigma_m = 5757\ psi$ , $\sigma_a = 4112\ psi$ – PROB. 7.

$\frac{1}{N} = \frac{5757\ psi}{58\,000\ psi} + \frac{1.0(4112)\ psi}{13\,900\ psi} = 0.395$ ; $\underline{N = 2.53}$

CHECK YIELD:

$N = \frac{S_y}{K_t(\sigma_a + \sigma_m)} = \frac{36000}{1(4112 + 5757)} = 3.65$

N = 2.53 IS SATISFACTORY IF NO UNUSUAL CONDITIONS OR UNCERTAINTY OF DATA EXIST.

**22.** DATA SAME AS PROBLEM 9. FLUCTUATING NORMAL STRESS – CASE 5.

$\sigma_m = 386\ MPa$ ; $\sigma_a = 89\ MPa$. FROM APP. 3 AISI 4140 OQT 400 HAS THE HIGHEST $S_u$ WITH > 10% ELONGATION FOR GOOD DUCTILITY.

$S_u = 2000\ MPa$ ; $S_y = 1730\ MPa$ ; $S_n = 450\ MPa$ (FIG 5-8); LET $R = 99\% - C_R = 0.81$

$S_n' = (0.81)(450\ MPa) = 364\ MPa$ | FOR RECT.: $D_e = 0.808\sqrt{hb} = 0.808\sqrt{0.6(5)} = 1.40\ mm$ ; $C_s = 1.0$

$\frac{1}{N} = \frac{\sigma_m}{S_u} + \frac{K_t \sigma_a}{S_n'} = \frac{386}{2000} + \frac{(1.0)(89)}{364} = 0.438$ ; $\underline{N = 2.29}$

SUGGEST TRYING TO FIND AN EVEN STRONGER MATERIAL FROM OTHER REFERENCES. MAY ADJUST WIDTH OR THICKNESS OF SPRING STOCK. USE ANALYSIS FROM PROB. 9 TO COMPUTE FORCE VS. DEFLECTION FOR SPRING. CONSIDER MOVING LATCH PIN FARTHER FROM FIXED END OF SPRING. THIS IS A GOOD SPREADSHEET PROBLEM.

**23.** DATA SAME AS PROB. 6. FLUCTUATING NORMAL STRESS, CASE 5.

AT B: $\sigma_m = 8432$ psi, $\sigma_a = 2595$ psi. | FOR ASTM A500 GRADE B: $S_u = 58$ ksi, $S_y = 46$ ksi

AT C: $\sigma_m = 6162$ psi, $\sigma_a = 3568$ psi. | $S_n = 20$ ksi; $R = 0.99 \rightarrow C_R = 0.81$; $C_s$ FOR 4×4×1/4 TUBE

$A_T = 3.59$ in$^2$; $A_{95} = 0.05(3.59) = 0.180$ in$^2$ $= 0.0766\,D_e^2$; $D_e = 1.53$ in $\rightarrow C_s = 0.84$; $S_n' = (0.84)(.81)(20) = 13.6$ ksi

AT C: $\frac{1}{N} = \frac{6162}{58\,000} + \frac{(1.0)(3568)}{13\,600} = 0.369$; $\boxed{N = 2.71}$ CHECK YIELD: $N = \frac{46000}{1(3429+5922)} = 4.92$ OK

AT B: $N = 2.97 > 2.71$ OK. 4×4×1/4 TUBING WEIGHS 8.78 LB/FT. APP 15-14. A LIGHTER BEAM CAN BE DESIGNED BY PLACING 4 IN SIDE VERTICAL AND USING A THINNER WALL. IN APP 15-15, THE SOURCE - JORGENSEN OFFERS 4×2×0.134; 5.223 LB/FT. WITH 4.0 IN SIDE VERTICAL, $S_x = 1.58$ in$^3$. ORIGINAL $S_x = 1.48$ SO BEAM IS SAFE. WT. IS REDUCED BY ≈ 40%.

**24.** PISTON ROD. FIG P5-24. DIA = 0.60 IN. $A = \pi D^2/4 = 0.283$ in$^2$

FLUCTUATING LOAD. $F_{max} = 1500$ LB TENS.; $F_{min} = -400$ LB COMP. CASE 5.

$F_m = (1500-400)/2 = 550$ LB; $F_a = 1500 - 550 = 950$ LB

$\sigma_m = F_m/A = 550\text{ LB}/0.283\text{ in}^2 = 1943$ psi; $\sigma_a = F_a/A = 950\text{ LB}/0.283\text{ in}^2 = 3357$ psi

SAE 4130 WQT 1300: $S_u = 98$ ksi; $S_y = 89$ ksi, $S_n = 37$ ksi (FIG. 5-8)

$C_s = 0.93$; $C_m = 1.0$; $C_{st} = 0.80$ (AXIAL); $R = 99\% - C_R = 0.81$

$S_n' = (0.93)(1.0)(0.80)(0.81)\,37$ ksi $= 22.3$ ksi

$\frac{1}{N} = \frac{\sigma_m}{S_u} + \frac{(K_t)(\sigma_a)}{S_n'} = \frac{1943\text{ psi}}{98000\text{ psi}} + \frac{(1.0)(3357\text{ psi})}{22\,300\text{ psi}} = 0.170$; $\underline{N = 5.87}$

SAFE BUT HIGH. SHOULD ALSO CHECK FOR $K_t$ IN FINAL DESIGN. IF ROD DIA. IS REDUCED TO 0.50 IN. $A = 0.196$ in$^2$

$\sigma_m = 2801$ psi; $\sigma_a = 4838$ psi; $\underline{N = 4.15}$ BETTER. USE D = 0.50 IN.

**25.** BRITTLE MATERIAL - STATIC LOAD - CASE 1: $N = S_{uc}/\sigma_{max}$

$\sigma_{max} = \frac{K_t F}{A} = \frac{(1.99)(75000\text{ lb})}{\pi(4.00\text{ in.})^2/4} = 11877$ psi COMPRESSION

$r/d = 0.25\text{ in}/4.00\text{ in} = 0.0625$; $D/d = 5.00\text{ in}/4.00\text{ in} = 1.25$: THEN $K_t = 1.99$

$N = \frac{S_{uc}}{\sigma_{max}} = \frac{140\,000\text{ psi}}{11877\text{ psi}} = \underline{11.8}$

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**26.** BRITTLE MATERIAL - STATIC LOAD - CASE 1: $N = S_{ut}/\sigma_{max}$

$\sigma_{max} = \frac{K_t F}{A} = \frac{(1.99)(12\,000\text{ lb})}{\pi(4.00\text{ in})^2/4} = 1900$ psi; [$K_t$ SAME AS PROB. 25]

$N = \frac{S_{ut}}{\sigma_{max}} = \frac{40\,000\text{ psi}}{1900\text{ psi}} = \underline{21.0}$

**27.** BRITTLE MATERIAL - BIAXIAL STRESS - SECTION 5-11.1

USE MODIFIED MOHR METHOD | $\sigma_1, \sigma_2$ FOUND FROM MOHR CIRCLE STRESS ELEMENT IN FILLET AREA

<u>AXIAL COMPRESSIVE</u> STRESS FROM PROB.: $\sigma_{max} = -11\,877$ psi $= \sigma_y$

CONTINUED - NEXT PAGE.

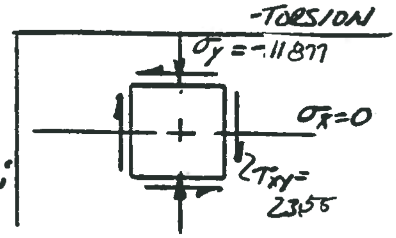
## 27. CONTINUED

TORSION: $\tau = \frac{K_t T}{Z_p}$ : FOR $r/d = 0.0625$ ; $D/d = 1.25$ , $K_t = 1.48$

SEE PROB. 25.

$Z_p = \frac{\pi D^3}{16} = \frac{\pi (4.00 IN)^3}{16} = 12.57 IN^3$

$\tau = (1.48)(20\,000\ lb \cdot IN) / 12.57\ IN^3 = 2355\ psi$




FROM MOHR CIRCLE: $R = \sqrt{2355^2 + 5939^2} = 6389\ psi$

$\sigma_1 = \sigma_{AV} + R = -5939 + 6389 = 450\ psi$ TENSION

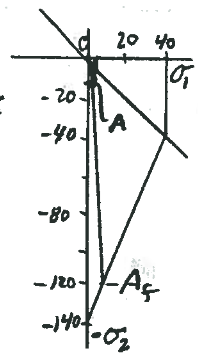
$\sigma_2 = \sigma_{AV} - R = -5939 - 6389 = -12328\ psi$ COMPR.

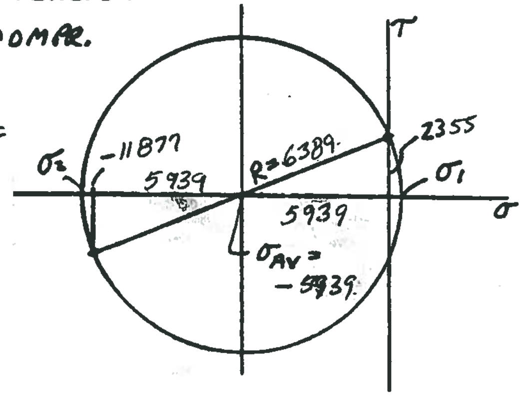
GRAPHICAL SOLUTION
4TH QUADRANT

PT. A AT $\sigma_1 = 450\ psi$, $\sigma_2 = -12328\ psi$
LINE $OA = 12336\ psi$
PT $A_f$ IS FAILURE POINT
LINE $OA_f' = 120300\ psi$ (SCALED)

$N = OA_f/OA = 9.75$

VERY SAFE




## 28. DUCTILE MATERIAL - STATIC LOAD - CASE 2:

SAE 1137 CD ; $s_y = 565\ MPa$

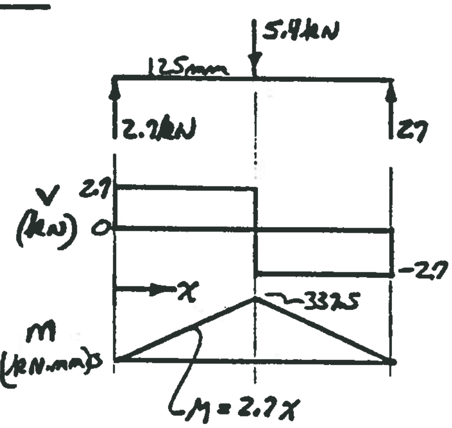
$\sigma_d = s_y/N = 565\ MPa/3 = 188\ MPa$

IN MIDDLE OF SHAFT, $M = 337.5\ kN \cdot mm$

REQ'D $S = \frac{M}{\sigma_d} = \frac{337.5 \times 10^3\ N \cdot mm}{188\ N/mm^2} = 1795\ mm^3$

$S = \pi D^3/32$ : $D = \sqrt[3]{32S/\pi} = 26.3\ mm$

USE PREFERRED VALUE $D = 28\ mm$
(TABLE A2-1)



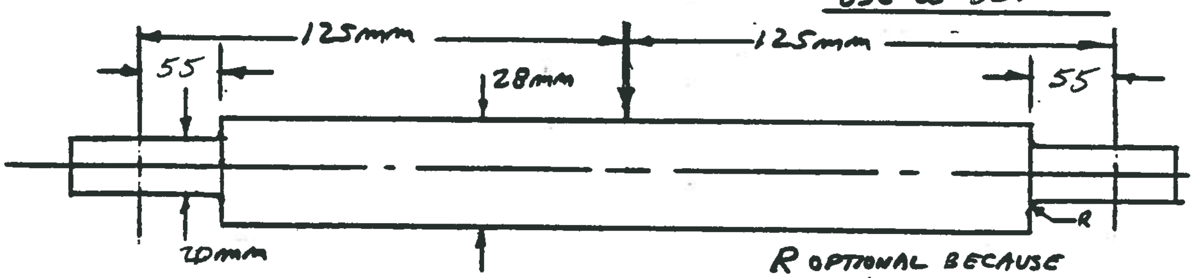

FIND $x$ WHICH WOULD BE SAFE FOR $d = 20\ mm$

$S = \pi d^3/32 = \pi (20)^3/32 = 785.4\ mm^3$

$M = \sigma_d S = (188\ N/mm^2)(785.4\ mm^3) = 147{,}650\ N \cdot mm = 147.7\ kN \cdot mm$

BUT $M = 2.7x$ : $x = \frac{M}{2.7} = \frac{147.7\ kN \cdot mm}{2.7\ kN} = 54.7\ mm$ (MAX)

USE $a = 55\ mm$




R OPTIONAL BECAUSE OF STATIC STRESS

**29.** FIGURE P5-28. SEE ALSO PROB. 28.

CASE 4; REPEATED REVERSED NORMAL STRESS; $\sigma_d = s_n'/N$

SAE 1137 CD $s_u = 676$ MPa; $s_n = 250$ MPa (FIG 5-8); FOR d = 20 mm,

$C_s = 0.90$, $C_m = 1.0$, $C_{st} = 1.0$, $R = 99\% - C_R = 0.81$; $s_n' = (.90)(.81)(250) = 182$ MPa

$\sigma_d = s_n'/N = 182/3 = 60.7$ MPa; $M_{MAX} = 337.5\times10^3$ N·mm AT LOAD (PROB 28)

REQ'D $S = \dfrac{M}{\sigma_d} = \dfrac{337.5\times10^3\ N\cdot mm}{60.7\ N/mm^2} = 5563\ mm^3$; BUT $S = \pi D^3/32$

REQ'D $D = \sqrt[3]{32\,S/\pi} = 38.4$ mm; USE $D = 40.0$ mm | NEW $C_s = 0.83$ RECOMPUTE $D_{MIN} = 38.4$ mm $D = 40.0$ mm OK

ALLOWABLE DISTANCE "a". LET $R = 2.0$ mm, $R/d = 2.0/20 = 0.100$; $K_t = 1.80$

$\sigma = \dfrac{K_t M}{S}$; $M_{MAX} = \dfrac{\sigma_d S}{K_t}$; $S = 785.4\ mm^3$ (PROB 28); $D/d = 40/20 = 2.00$

$M_{MAX} = \dfrac{60.7\ N}{mm^2}\cdot\dfrac{785.4\ mm^3}{1.80} = 26485$ N·mm = 26.5 kN·mm = 2.7x (PROB 28)

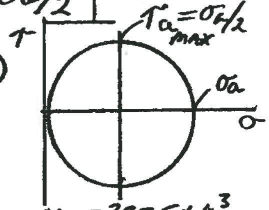
$x_{MAX} = M_{MAX}/2.7 = \dfrac{26.5\ kN\cdot mm}{2.7\ kN} = 9.81$ mm; USE a = 9.0 mm (SMALL)

**30.** SEE FIG. P5-28 AND PROB. 28 AND 29. $K_t = 2.0$ FOR KEYSEAT

EQN. 5-22: $\dfrac{1}{N} = \dfrac{K_t(\tau_a)_{MAX}}{s_{sn}'} + \dfrac{(\tau_m)_{MAX}}{s_{su}}$ | $(\tau_m)_{MAX} = T/Z_p$ | $(\tau_a)_{MAX} = \sigma_a/2$

MOHR CIRCLE FOR ALTERNATING STRESS

$\sigma_a = \dfrac{M_a}{S} = \dfrac{M_a}{Z_p/2} = \dfrac{2M_a}{Z_p}$; THEN $(\tau_a)_{MAX} = \dfrac{M_a}{Z_p}$




$\dfrac{1}{N} = \dfrac{K_t M_a}{Z_p s_{sn}'} + \dfrac{T}{Z_p s_{su}}$; REQ'D $Z_p = N\left[\dfrac{K_t M_a}{s_{sn}'} + \dfrac{T}{s_{su}}\right]$ (I)

SAE 1137 CD: $s_u = 676$ MPa, $s_n = 250$ MPa

ASSUME D = 50 mm, $C_s = 0.81$, $C_R = 0.81$

$s_{sn}' = 0.577 s_n' = (0.50)(0.81)(0.81)(250) = 82.0$ MPa

$s_{su} = 0.75 s_u = 0.75(676) = 507$ MPa

$M_a = 337.5\times10^3$ N·mm PROB. 3-28

REQ'D $Z_p = 3\left[\dfrac{(2.0)(337.5\times10^3)}{82.0} + \dfrac{150\times10^3}{507}\right] = 25579\ mm^3 = \pi D^3/16$

$D_{MIN} = \sqrt[3]{16 Z_p/\pi} = \sqrt[3]{\dfrac{16(25579)}{\pi}} = 50.69$ mm; SPECIFY D = 52.0 mm

LOCATION OF STEP FROM 20 mm TO 50 mm:

FOR $K_t$: BENDING D = 52 mm, d = 20 mm, r = 2.0 mm, $K_t = 1.82$

$Z_p = \pi D^3/16 = \pi(20)^3/16 = 1571\ mm^3$; SOLVE EQ (I) FOR $M_a$

$M_a = \dfrac{s_{sn}'}{K_t}\left[\dfrac{Z_p}{N} - \dfrac{T}{s_{su}}\right]$

FOR d = 20 mm; $C_s = 0.90$; $s_{sn}' = (0.90)(0.81)(250)(0.577) = 105$ MPa

$M_a = \dfrac{105\ N/mm^2}{1.82}\left[\dfrac{1571\ mm^3}{3} - \dfrac{150\times10^3\ N\cdot mm}{507\ N/mm^2}\right] = 13143$ N·mm

FROM PROB. 3-28, $M = (2700\ N)x$

$x_{MAX} = M_a/2700 = \dfrac{13143\ N\cdot mm}{2700\ N} = 4.86$ mm VERY SMALL

X IS FROM MIDDLE OF BEARING TO STEP.

REDESIGN IS REQUIRED, CONSIDER LARGER d OR MATERIAL WITH HIGHER STRENGTH.

**31.** $S_y = 30\,ksi$ ; $\tau_d = 0.5S_y/N = 0.5(30)/3 = 5\,ksi = 5000\,psi$

CASE 3 : COMBINED STRESS – STATIC LOAD, MAX SHEAR STRESS METHOD. ASSUME AXIAL COMPRESSION IS SMALL. THEN MAX $\tau$ OCCURS AT FRONT AND REAR – COMBINED BENDING AND TORSION. USE EQUIVALENT TORQUE METHOD – CH. 4, EQ. 4-16 AND 4-17

$T = (200\,LB)(18\,IN) = 3600\,LB\text{-}IN$

$M = (400\,LB)(18\,IN) = 7200\,LB\text{-}IN$

NOTE: BENDING DUE TO 200 LB LOAD IS ONLY 4000 LB-IN AND ACTS AT A DIFFERENT POINT.

$$T_e = \sqrt{M^2+T^2} = \sqrt{7200^2+3600^2} = 8050\,LB\text{-}IN$$

$$\tau_{MAX} = T_e/Z_p = \tau_d \text{ : THEN } Z_p = \frac{T_e}{\tau_d} = \frac{8050\,LB\text{-}IN}{5000\,LB/IN^2} = 1.61\,IN^3$$

BUT $S = Z_p/2 = 0.805\,IN^3 \rightarrow$ FROM APP. A-15: USE $2\frac{1}{2}$ IN SCH. 40 PIPE

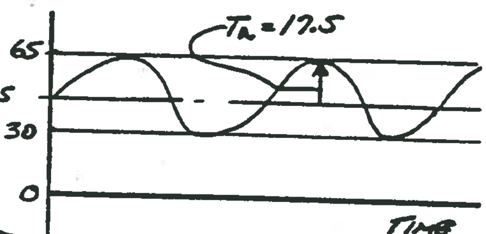
$S = 1.064\,IN^3$; $Z_p = 2.128\,IN^3$; $A = 1.704\,IN^2$

CHECK $$\tau_{MAX} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \sqrt{(-7028/2)^2 + (1693)^2} = 3903\,psi \text{ OK } < \tau_d$$

$$\sigma = \sigma_b - \sigma_c = \frac{-M}{S} - \frac{400\,LB}{A} = \frac{-7200}{1.064} - \frac{400}{1.704} = -7002\,psi$$

$$\tau = \frac{T}{Z_p} = \frac{3600}{2.128} = 1692\,psi$$

**32.** FLUCTUATING SHEAR STRESS:




EQN 5-22: $$\frac{1}{N} = \frac{\tau_m}{S_{su}} + \frac{K_t\tau_a}{S_{sn}'}$$

$$\tau_m = \frac{T_m}{Z_p}; \quad \tau_a = \frac{T_a}{Z_p}$$

$$\frac{1}{N} = \frac{T_m}{Z_pS_{su}} + \frac{K_tT_a}{Z_pS_{sn}'} = \frac{1}{Z_p}\left[\frac{T_m}{S_{su}} + \frac{K_tT_a}{S_{sn}'}\right]$$

$$Z_p = \frac{\pi D^3}{16} = N\left[\frac{T_m}{S_{su}} + \frac{K_tT_a}{S_{sn}'}\right] = 2\left[\frac{47.5\times10^3\,N\cdot mm}{870\,N/mm^2} + \frac{2.5(17.5\times10^3)\,N\cdot mm}{146\,MPa}\right]$$

$S_{su} = 0.75\,S_u = 0.75(1160\,MPa) = 870\,MPa$

USE $S_{sn}' = 0.577\,S_n'$ SAE 4140 OQT 1000; $S_u = 1160\,MPa$, $S_y = 1050\,MPa$

FOR $S_u = 1160\,MPa$ ; $S_n = 400\,MPa$ (FIG 5-8): LET $C_s = 0.90$, $C_R = 0.81$

$S_{sn}' = (0.577)(0.9)(0.81)(400\,MPa) = 146\,MPa$

REDO $Z_p = 708\,mm^3 = \pi D^3/16$

$$D_{MIN} = \sqrt[3]{16Z_p/\pi} = \sqrt[3]{\frac{16(708)\,mm^3}{\pi}} = 15.3\,mm$$

SPECIFY $D = 16.0\,mm$

ACTUAL $C_s = 0.92$ OK

$Z_p = \pi(16)^3/16 = 804\,mm^3$

CHECK YIELDING: EQN 5-23

$$\tau_a = \frac{T_a}{Z_P} = \frac{17500\,N\cdot mm}{804\,mm^3} = 21.77\,MPa; \quad \tau_m = \frac{T_m}{Z_p} = \frac{47500}{804} = 59.06\,MPa$$

$$N_y = \frac{S_y/2}{K_t(\tau_a+\tau_m)} = \frac{1050/2}{2.5(21.77+59.06)} = 2.60 > 2.0 \text{ OK}$$

33.

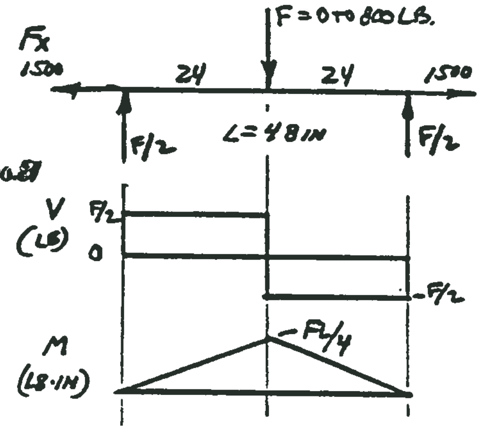
FLUCTUATING NORMAL STRESS:

CASE 5: $\frac{1}{N} = \frac{\sigma_m}{S_u} + \frac{K_t \sigma_a}{S_n'}$ FIG. 5-8

$S_y = 58\text{KSI}$; $S_u = 75\text{KSI}$; $N = 3$; $S_n = 28\text{KSI}$

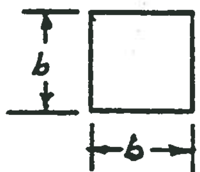
ASSUME MACHINED SURFACE AND $C_s = 0.9$, $C_R = 0.81$

$S_n' = (0.9)(0.81)(28\text{ksi}) = 20.4\text{ksi} = 20\,400\text{ psi}$

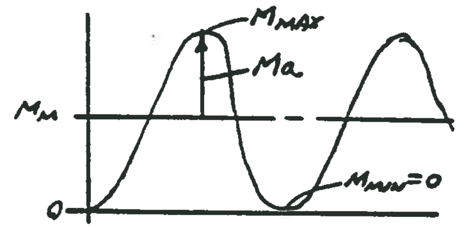



$M_{MAX} = \frac{FL}{4} = \frac{800(48)}{4} = 9600 \text{ LB·IN.}$

$M_m = M_a = 4800 \text{ LB·IN}$




$S = b^3/6$; $A = b^2$




$\sigma_m = \frac{F_x}{A} + \frac{M_m}{S}$

$\sigma_m = \frac{1500}{b^2} + \frac{4800}{b^3/6} = \frac{1500}{b^2} + \frac{28800}{b^3}$

$\sigma_a = \frac{M_a}{S} = \frac{4800}{b^3/6} = \frac{28800}{b^3}$

(I) $\frac{1}{N} = \frac{\frac{1500}{b^2} + \frac{28800}{b^3}}{75000} + \frac{\frac{28800}{b^3}}{20{,}400} = \frac{0.020}{b^2} + \frac{0.384}{b^3} + \frac{1.412}{b^3} = \frac{0.020}{b^2} + \frac{1.796}{b^3}$

TERM INVOLVING $b^2$ IS SMALL: $b \approx \sqrt[3]{N(1.796)} = 1.75 \text{ IN} \rightarrow$ USE $\boxed{b = 1.80 \text{ IN}}$, PREFERRED SIZE

RECHECK: $C_s$ FOR $b = 1.80$ IN SQUARE. $D_e = 0.808\sqrt{bh} = 0.808\sqrt{b^2} = 0.808\,b$

$D_e = 0.808(1.80) = 1.454$ IN. THEN $C_s = \left(\frac{1.454}{0.3}\right)^{-0.11} = 0.84$

$S_n' = (0.84)(0.81)(28\text{KSI}) = 19.05\text{KSI} = 19050 \text{ PSI}$

EQUATION (I) BECOMES:

$\frac{1}{N} = \frac{\frac{1500}{b^2} + \frac{28800}{b^3}}{75000} + \frac{\frac{28800}{b^3}}{19050} = \frac{0.020}{b^2} + \frac{0.384}{b^3} + \frac{1.512}{b^3} = \frac{0.020}{b^2} + \frac{1.896}{b^3}$

$\frac{1}{N} = \frac{0.020}{(1.80)^2} + \frac{1.896}{(1.80)^3} = 0.331$; $\underline{N = 3.02 \quad \text{OK}}$ $\underline{\text{SPECIFY } b = 1.80 \text{ IN}}$

**34** FLUCTUATING COMBINED STRESS:

CASE 5: $\frac{1}{N} = \frac{(\tau_m)_{max}}{S_{su}} + \frac{K_t(\tau_a)_{max}}{S_{sn}'}$

$\tau$ DUE TO TORQUE: (FIG. 3-10)

$\tau = \frac{T}{Q} = \frac{T}{0.208\,b^3} = \frac{1200}{.208\,b^3} = \frac{5769}{b^3}$

FROM PROB. 33, MEAN STRESS

$\sigma_m = \underbrace{\frac{1500}{b^2}}_{\text{(SMALL)}} + \frac{28800}{b^3} \approx \frac{28800}{b^3}$

$R = (\tau_m)_{max} = \sqrt{\left(\frac{5769}{b^3}\right)^2 + \left(\frac{14400}{b^3}\right)^2} = \frac{15513}{b^3}$

ALT. STRESS: $\tau_a = \sigma_a/2$ FOR BENDING ONLY

$(\tau_a)_{max} = \frac{1}{2}\,\frac{28800}{b^3} = \frac{14400}{b^3}$

THEN $\frac{1}{N} = \frac{\frac{15513}{b^3}}{56250} + \frac{\frac{14400}{b^3}}{10860} = \frac{1.602}{b^3}$

FOR $N = 3$, $b = 1.69$ IN $\rightarrow$ USE $b = 1.80$

CHECK:

$\sigma_m = \frac{1500}{(1.80)^2} + \frac{28800}{(1.80)^3} = 5401$ psi

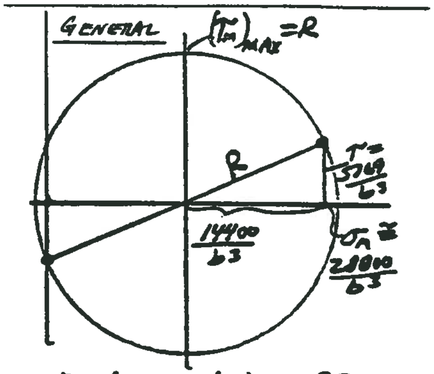
$\tau_m = \frac{5769}{(1.80)^3} = 989$ psi

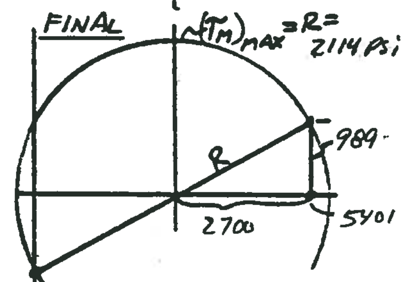
FROM MOHR CIRCLE: $(\tau_m)_{max} = 2876$ psi

$\tau_a = 14400/(1.80)^3 = 2469$ psi

$\frac{1}{N} = \frac{2876}{56250} + \frac{2469}{10860} = 0.279 \rightarrow N = 3.59$ OK    USE $b = 1.80$ IN

MOHR'S CIRCLE-MEAN STRESS




$S_{su} = 0.75\,S_{u} = 0.75(75\,ksi) = 56.25\,ksi$

$S_{sn}' = 0.577\,S_n' = (0.577)(0.83)(0.81)(28\,ksi) = 10\,860$ psi

PROBLEMS 35, 36 AND 37 ALL DUCTILE MATERIALS - STEADY LOAD: CASE 1.

**35.** $\sigma = \frac{F}{A} = \frac{4500\,N}{\pi(18^2 - 12^2)/4\ mm^2} = 31.8$ MPa; $N = S_y/\sigma = 290/31.8 = 9.11$

SAE 1040 HR: $S_y = 290$ MPa

**36.** $\sigma = F/A = 5000\,N/(12)^2\,mm^2 = 34.7$ MPa ; $N = S_y/\sigma$   CASE 1.

a) 1020 HR: $N = 207/34.7 = 5.96$
b) 8650-OQT 1000; $N = 1070/34.7 = 30.8$
c) DUCTILE IRON, 60-40-18: $N = 276/34.7 = 7.95$
d) ALUM. 6061-T6: $N = 276/34.7 = 7.95$
e) TI-6Al-4V: $N = 827/34.7 = 23.8$

(a–e) ALL HIGH - REDUCE

f) PVC: N BASED ON TENSILE STRENGTH; $N = 41/34.7 = 1.18$
g) PHENOLIC " " " " " ; $N = 45/34.7 = 1.30$

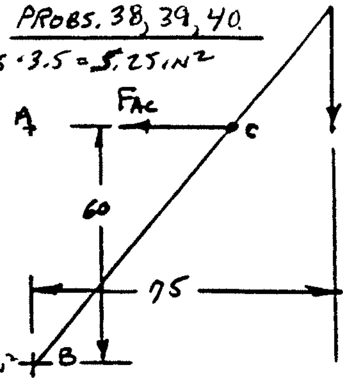
(f–g) LOW - INCREASE

**37.** $\sigma = \frac{F}{A} = \frac{12\,600\ LB}{[(2.25)^2 - (2.00)^2]\ IN^2} = 11859$ psi; $N = S_y/\sigma = 40\,000/11859 = 3.37$

**38** DUCTILE MATERIAL - STEADY LOAD - CASE 2 : PROBS. 38, 39, 40

SAE 1144 CD $S_y = 90$ KSI $= 90\,000$ PSI

$A = 1.5 \cdot 3.5 = 5.25\ \text{IN}^2$




$\Sigma M_B = 0 = 75F - 60C$ ; $C = 75(2500)/60 = 3125\ \text{LB} = F_{AC}$

$\sigma = F_{AC}/A = \dfrac{3125\ \text{LB}}{(1.5)(3.5)} = 595\ \text{PSI}$

$N = S_y/\sigma = 90\,000/595 = \underline{151}$ VERY HIGH

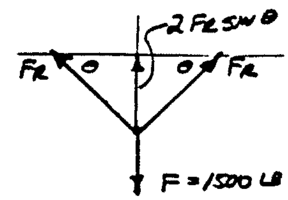
CONSIDER A SMALL ROD OF SAE 1020 HR STEEL

$\sigma_d = S_y/N = 30\,000/3 = 10\,000$ PSI ; $A = \dfrac{F_{AC}}{\sigma_d} = 0.3125\ \text{IN}^2$

A SQUARE BAR 3/4 IN ON A SIDE WOULD DO. $A = 0.44\ \text{IN}^2$

OR FROM APP. 15-15: RECT. TUBE - $1.00 \times 2.00 \times 0.065$; $A = 0.373\ \text{IN}^2$

**39** $\Sigma F_V = 0 = 1500\ \text{lb} - 2 F_R \sin 45°$ CASE 2




$F_R = 1500/2(\sin 45°) = 1061\ \text{LB}$

$\sigma_d = S_y/N = 42000/3 = 14\,000\ \text{PSI} = F_R/A$

$A = \dfrac{F_R}{\sigma_d} = \dfrac{1061\ \text{LB}}{14000\ \text{LB/IN}^2} = 0.0758\ \text{IN}^2 = \pi D^2/4$ ; $D = 0.311$ IN.

USE 5/16 IN.

**40** $F_R = 1500/2(\sin 15°) = 2898\ \text{LB}$ : $A = \dfrac{F_R}{\sigma_d} = \dfrac{2898}{14000} = 0.207\ \text{IN}^2$ ; $D = 0.513$ IN.

USE 9/16 IN.

**41** REPEATED REVERSED AXIAL LOAD: $C_s = 1.00$ ; $C_{ST} = 0.8$ AXIAL LOAD

$C_R = 0.81$

CASE 4 : $N = S_n'/\sigma_{max}$ : $\sigma_{max} = K_t F/A = \dfrac{1.83(7500\text{N})}{(6)(9)\text{mm}^2} = 254$ MPa

$\left.\begin{aligned} r/t &= 1.5/9 = 0.167 \\ t/w &= 9/12 = 0.75 \end{aligned}\right\}$ $K_t = 1.83$ USING FIG. 3-26(a)

SIZE SAE 4140 OQT 1000

$S_u = 1160$ MPa $\rightarrow S_n = 400$ MPa (FIG. 5-8) ; $S_n' = (1.00)(0.8)(400)(0.81) = 259$ MPa

AXIAL

$N = 259/254 = 1.02$ LOW $\rightarrow$ USE LARGER BAR AND/OR STRONGER MATERIAL.

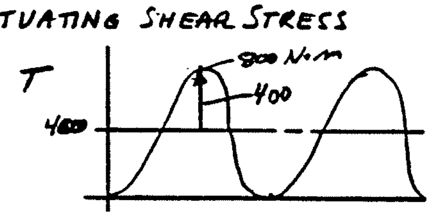
**42** REPEATED REVERSED SHEAR STRESS: CASE 4 : $C_s = 0.81$ , $C_R = 0.81$

$\tau_{MAX} = \dfrac{T}{Z_P} = \dfrac{800 \times 10^3\ \text{N}\cdot\text{mm}}{\pi(50\ \text{mm})^3/16} = 32.6$ MPa : SAE 1040 WQT 1000 : $S_u = 780$ MPa ; $S_n = 280$ MPa (FIG. A4-1) (FIG. 5-8)

$S_{sn}' = (0.5)(0.81)(0.81)(280) = 91.9$ MPa $S_{sn} = 0.5 S_n'$

$N = S_{sn}'/\tau_{MAX} = 91.9/32.6 = \underline{2.82\ \text{OK}}$

**43** REPEATED - ONE DIRECTION SHEAR STRESS: FLUCTUATING SHEAR STRESS




CASE 5 : EQ. 5-22 : $\dfrac{1}{N} = \dfrac{\tau_m}{S_{su}} + \dfrac{K_t \tau_a}{S_{sn}'}$

$Z_P = \pi D^3/16 = 24544\ \text{mm}^3$

$\tau_m = \tau_a = 400 \times 10^3\ \text{N}\cdot\text{mm}/24544\ \text{mm}^3 = 16.30$ MPa

$S_{su} = 0.75 S_u = 0.75(780\ \text{MPa}) = 585$ MPa

$\dfrac{1}{N} = \dfrac{16.30}{585} + \dfrac{(1.0)(16.30)}{91.9} = 0.205$ ; $\underline{N = 4.87}$ ($S_{sn}'$ FROM PROB. 42)

**44** DUCTILE MATERIAL – STATIC LOAD – CASE 2 : $N = 0.5\,S_y/\tau_{MAX}$

LET $N = 3.0$ DESIGN DECISION

$$\tau_{MAX} = \frac{T}{Z_p} = \frac{88.0\ \text{LB·IN}}{\pi(.401\text{N})^3/16} = 7003\ \text{PSI}$$

REQ'D $S_y = N\,\tau_{MAX}/0.5 = 3(7003)/0.5 = 42017\ \text{PSI}$

ALUMINUM 2024-T4 HAS $S_y = 47000$ PSI

**45** FLUCTUATING SHEAR STRESS: CASE 5

ASSUME $C_s = 0.82$, $C_R = 0.81$, $N = 3$

$$T_{MAX} = \frac{63000\,(hp)}{n} = \frac{63000(110)}{560} = 12375\ \text{LB-IN.}$$

$T_m = T_a = T_{MAX}/2 = 6188$ LB-IN ; $\tau_m = \tau_a = \frac{T}{Z_p}$

$S_{su} = 0.75 S_u = 0.75(208) = 156$ ksi ; $S_u = 208$ ksi $\rightarrow S_n = 64$ ksi (FIG. 5-8)

$S_{sn}' = (0.50)(0.82)(0.81)(64) = 21.25$ ksi $= 21250$ psi

$$\frac{1}{N} = \frac{T_m}{Z_p S_{su}} + \frac{T_a}{Z_p S_{sn}'} = \frac{1}{Z_p}\left[\frac{T_m}{S_{su}} + \frac{T_a}{S_{sn}'}\right] \therefore Z_p = N\left[\frac{T_m}{S_{su}} + \frac{T_a}{S_{sn}'}\right]$$

$$Z_p = 3\left[\frac{6188}{156000} + \frac{6188}{21250}\right] = 0.993\ \text{IN}^3;\ D = \sqrt[3]{16 Z_p/\pi} = 1.72\ \text{IN}\quad \text{USE } D = 1.75\ \text{IN.}$$

**46** STEADY LOAD – CASE 3 : $N = 0.5\,S_y/\tau_{MAX}$.

$$T = \frac{P}{n} = \frac{28\times10^3\ \text{N·m/s}}{45\ \text{RAD/S}} = 622\ \text{N·m} = 622\times10^3\ \text{N·mm}$$

$$Z_p = \frac{\pi(D^4 - d^4)}{16D} = \frac{\pi(40^4 - 30^4)}{16(40)}\ \text{mm}^3 = 8590\ \text{mm}^3$$

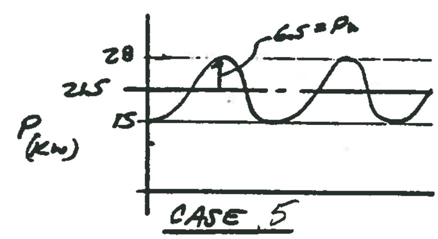
$$\tau_{MAX} = T/Z_p = 622\times10^3\ \text{N·mm}/8590\ \text{mm}^3 = 72.4\ \text{MPa}$$

$$N = 0.5S_y/\tau_{MAX} \therefore S_y = N\tau_{MAX}/0.5 = 3(72.4)/0.5 = 434\ \text{MPa}$$

AISI 1040 COLD DRAWN HAS $S_y = 490$ MPa.

**47** FLUCTUATING SHEAR STRESS – CASE 5 : EQ. 5-22

(SEE PROBLEM 46.)




$T_m = P_m/n = 21500/45 = 478$ N·m

$T_a = P_a/n = 6500/45 = 144$ N·m

$\tau_m = T_m/Z_p = 478000/8590 = 55.6$ MPa

$\tau_a = T_a/Z_p = 144000/8590 = 16.8$ MPa

$$\frac{1}{N} = \frac{\tau_m}{S_{su}} + \frac{K_t\tau_a}{S_{sn}'} = \frac{55.6}{S_{su}} + \frac{(1.0)(16.8)}{S_{sn}'}$$

$C_s = 0.83$
$C_R = 0.81$

AFTER TRIALS:

AISI 1144 C.D. ; $S_y = 621$ MPa ; $S_u = 690$ MPa $\rightarrow S_n = 253$ MPa (FIG 5-8)

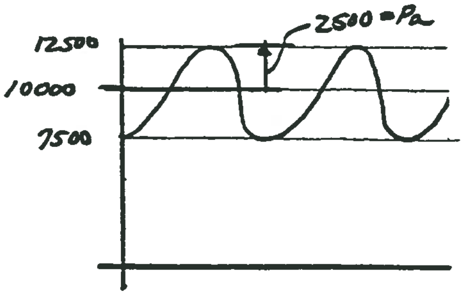
$S_{su} = 0.75S_u = 0.75(690) = 518$ MPa ; $S_{sn}' = 0.50\,S_n' = (0.50)(0.83)(0.81)(253) = 85.0$ MPa

$$\frac{1}{N} = \frac{55.6}{518} + \frac{16.8}{85.0} = 0.305\ ;\ N = 1/0.305 = 3.28\quad \text{OK}$$

FLUCTUATING NORMAL STRESS : CASE 5 : EQ. 5-20
PROBLEMS 48, 49, 50, 51.

**48**

CASE 5: $\frac{1}{N} = \frac{\sigma_m}{S_u} + \frac{K_t \sigma_a}{S_n'}$




$A = (w-d)t = (1.50-0.50)(0.50) = 0.50\ in^2$

$\sigma_m = \frac{10000}{.5} = 20000\ psi$

$\sigma_a = \frac{2500}{0.5} = 5000\ psi$

$d/w = .5/1.50 = 0.333 \rightarrow K_t = 2.31$

$S_y = 107\ ksi;\ S_u = 118\ ksi \rightarrow S_n = 43\ ksi;\ S_n' = (0.8)(0.81)(43) = 27.9\ ksi$ ← AXIAL

$\frac{1}{N} = \frac{20000}{118000} + \frac{2.31(5000)}{27900} = 0.583 \rightarrow N = 1.71$ (LOW)

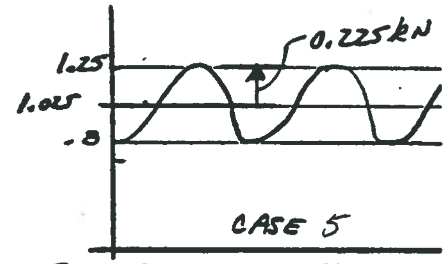
**49**

$A = \pi(6)^2/4 = 28.27\ mm^2$

$\sigma_m = \frac{F_m}{A} = \frac{1025\ N}{28.27\ mm^2} = 36.25\ MPa$

$\sigma_a = \frac{F_a}{A} = \frac{225\ N}{28.27\ mm^2} = 7.96\ MPa$

$\left.\begin{array}{l} r/d = 0.5/6 = 0.083 \\ D/d = 9/6 = 1.50 \end{array}\right\} K_t = 2.03$




$S_y = 779\ MPa;\ S_u = 841\ MPa$

$S_n = 300\ MPa$

$S_n' = (0.8)(0.81)(300) = 194.4\ MPa$ (AXIAL, CR)

$\frac{1}{N} = \frac{36.25}{841} + \frac{(2.03)(7.96)}{194.4} = 0.126;\ N = 7.92$

COULD USE WEAKER MATERIAL

FOR YIELD $N = 8.68$ OK

**50**

FROM PROB. 62, CHAPTER 3, MAX STRESS OCCURS AT BOTTOM

$K_t = 1.86;\ S_y = 86000\ psi;\ S_u = 121000\ psi;\ S_n = 43000\ psi$

$S_n' = (0.8)(.81)(43000) = 27864\ psi$ (AXIAL) $;\ \sigma_m = \frac{F_m}{A} = \frac{600\ LB}{\pi(0.5)^2/4} = 3056\ psi$

$\sigma_a = \sigma_m$

CASE 5:

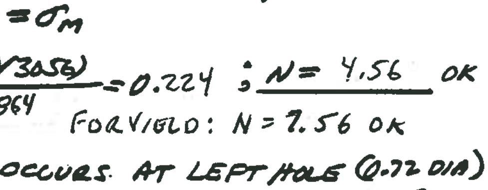
$\frac{1}{N} = \frac{\sigma_m}{S_u} + \frac{K_t\sigma_a}{S_n'} = \frac{3056}{121000} + \frac{(1.86)(3056)}{27864} = 0.224;\ N = 4.56$ OK

FOR YIELD: $N = 7.56$ OK

**51**

FROM PROB. 63, CHAPTER 3, MAX STRESS OCCURS AT LEFT HOLE (0.72 DIA)

$K_t = 2.15:\ A = (1.40-0.72)(0.50) = 0.34\ in^2$




$\sigma_m = \frac{F_m}{A} = \frac{5100}{.34} = 15000\ psi$

CASE 5

$\sigma_a = \frac{F_a}{A} = \frac{1100}{.34} = 3235\ psi$

$\frac{1}{N} = \frac{\sigma_m}{S_u} + \frac{K_t\sigma_a}{S_n'} = \frac{15000}{145000} + \frac{2.15(3235)}{33050} = 0.314$

$S_y = 125\ ksi$

$S_u = 145\ ksi$

$S_n = 51\ ksi$

$S_n' = (0.8)(0.81)(51) = 33.05\ ksi$ (CST, CR)

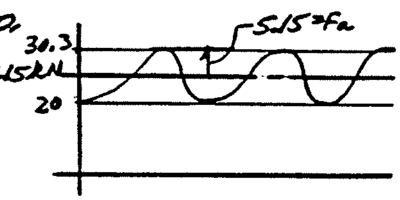
$N = 3.19$

FOR YIELD: $N = \frac{S_y}{K_t(\sigma_a + \sigma_m)} = \frac{125000}{2.15(3235+15000)}$

$N = 3.19$ O.K.

[52] FROM PROB 3-64, $\sigma_{MAX} = 16650$ PSI INCLUDING $K_t$: CASE 1 : $N = S_{ut}/\sigma$
REQ'D. $S_{ut} = N\sigma = 3(18281) = 54843$ PSI → USE GRADE 60 A CAST IRON

[53] CASE 5: $\frac{1}{N} = \frac{\sigma_m}{s_u} + \frac{K_t \sigma_a}{s_n'}$ : NOTE THAT A DIRECT SOLUTION IS NOT POSSIBLE BECAUSE BOTH $s_u$ AND $s_n'$ ARE UNKNOWN. ALSO DATA FOR ENDURANCE FOR TITANIUM ARE NOT DIRECTLY AVAILABLE HERE. AS AN ESTIMATE WE WILL USE FIG 5-8 AND THE DISCUSSION FOR STEEL TO OBTAIN $s_n'$. ALSO NOTE FROM PREVIOUS PROBLEMS, $s_n' \approx s_u/4$. THIS PERMITS SOLUTION FOR $s_u$. AFTER MATERIAL SELECTION, FINAL "N" CAN BE COMPUTED.




FROM PROBLEM 3-65, $K_t = 2.30$

$A = \pi(30)^2/4 = 707\ mm^2$

$\sigma_m = \frac{F_m}{A} = \frac{25.15 \times 10^3 N}{707 mm^2} = 35.57 MPa$

$\sigma_a = \frac{F_a}{A} = \frac{5.15 \times 10^3 N}{707 mm^2} = 7.28 MPa$

$\frac{1}{N} = \frac{\sigma_m}{s_u} + \frac{K_t \sigma_a}{s_n'} = \frac{35.57}{s_u} + \frac{(2.30)(7.28)}{s_u/4} = \frac{102.5}{s_u}$

THEN $s_u \approx N(102.5) \approx 3(102.5) = 308$ MPa : TRY Ti 50A; $s_y = 276$ MPa; $s_u = 345$ MPa

FROM FIG. 5-8: $s_n \approx 120$ MPa (ESTIMATE)

$s_n' = (0.8)(0.81)(130) = 84.2$ MPa

$\frac{1}{N} = \frac{35.57}{345} + \frac{(2.30)(7.28)}{84.2} = 0.302 \rightarrow N = 3.31$ OK Ti-50A

[54] FROM PROB. 3-66, $K_t = 1.43$ : $T_m = 1100$ LB·IN $= T_a$

$Z_p = \pi(1.25)^3/16 = 0.383\ IN^3$

$\tau_m = T_m/Z_p = 1100/0.383 = 2868$ PSI $= \tau_a$ → CASE 5

$\frac{1}{N} = \frac{\tau_m}{s_{su}} + \frac{K_t \tau_a}{s_{sn}'}$ ASSUME $s_{sn}' \approx s_{su}/4$ [SEE PROB 53]

$\frac{1}{N} = \frac{2868}{s_{su}} + \frac{1.43(2868)}{s_{su}/4} \approx \frac{19273}{s_{su}}$ ∴ $s_{su} \approx 3(19273) \approx 57819$ psi

BUT $s_{su} = 0.75 s_u$; $s_u = s_{su}/0.75 = 57819/0.75 = 77092$ PSI

TRY AISI 1137 OQT 1300; $s_y = 60$ KSI; $s_u = 87$ KSI; $s_n = 33$ KSI

$s_{sn}' = 0.50\, s_n'$ ∴ $s_{sn}' = (0.50)(.81)(.85)(33000) = 11360$ psi

$s_{su} = 0.75 s_u = 0.75(87000\ psi) = 65250$ PST

$\frac{1}{N} = \frac{2868}{65250} + \frac{1.43(2868)}{11360} = 0.405$; $N = 2.47$ LOW

TRY SAE 1040 WQT 1000; $s_u = 113$ KSI, $s_y = 88$ KSI, $s_n = 42$ KSI

$s_{su} = 0.75(113) = 84.75$ KSI; $s_{sn}' = (0.50)(0.81)(0.85)(42) = 14.458$ KSI

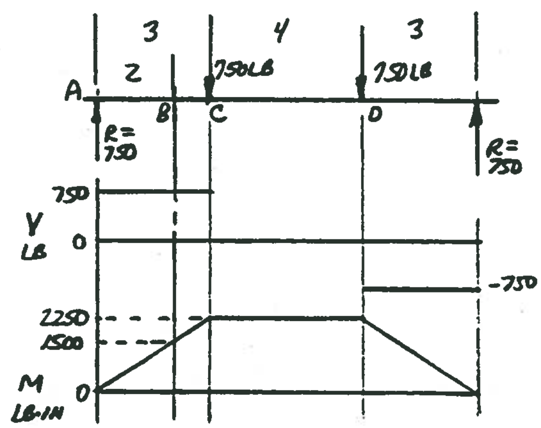
$\frac{1}{N} = \frac{2868}{84750} + \frac{1.43(2868)}{14458} = 0.3175$ $N = 3.15$ OK

**55.** USE CASE 1 BECAUSE HIGHER STRENGTH DUCTILE IRONS ARE FAIRLY BRITTLE.
FROM PROB. 3-67, $\sigma = 32\,564$ psi INCLUDING $K_t$. : $N = S_{ut}/\sigma$
REQ'D. $S_{ut} = N\sigma = 3(32\,564) = 97\,692$ psi → USE GRADE 100-70-03.

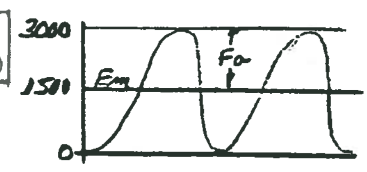
**56.** FROM PROB. 3-68, $\sigma = 49\,323$ psi INCLUDING $K_t$ : CASE 4 : $N = S_n'/\sigma$
REQ'D $S_n' = N\sigma = 3(49323) = 147\,969$ psi
REFERRING TO FIG. 5-8, THIS IS VERY HIGH. NO PRACTICAL MATERIAL
REDESIGN THE MEMBER.

**57.** LOAD IS REPEATED – ONE DIRECTION – TORSIONAL SHEAR STRESS;
FLUCTUATING SHEAR STRESS – CASE 5 : $T_m = T_a = \dfrac{100\text{ LB·IN}}{2} = 50$ LB·IN.
AT FILLET: $D/d = 0.50/0.30 = 1.67$; $r/d = 0.025/0.30 = 0.083$ : $K_t = 1.43$
$Z_p = \pi d^3/16 = \pi(0.30)^3/16 = 0.00530\text{ IN}^3$
$$\tau_m = \tau_a = \frac{T}{Z_p} = \frac{50\text{ LB·IN}}{0.0053\text{ IN}^3} = 9431\text{ psi}$$
SAE 8740 OQT 1000: $S_y = 167$ ksi ; $S_u = 175$ ksi ; THEN $S_n \approx 60$ ksi (FIG. 5-8)
$S_{sn}' = (0.50)S_n' = (0.50)(0.81)(1.0)\,60\text{ ksi} = 24.3$ ksi ; $S_{su} = 0.75S_u = 0.75(175) = 131.3$ ksi
$$\frac{1}{N} = \frac{\tau_m}{S_{su}} + \frac{K_t\tau_a}{S_{sn}'} = \frac{9431}{131300} + \frac{1.43(9431)}{24300} = 0.627 \text{ ; } N = 1.60 \text{ LOW}$$

**58.** STEADY LOAD – BRITTLE MAT'L.: CASE 1
AT MIDDLE – BETWEEN C AND D:
$Z = bH^2/6 = (0.75)(2.25)^2/6 = 0.633\text{ IN}^3$
$\sigma = M/Z = 2250\text{ LB·IN}/0.633\text{ IN}^3 = 3556$ psi
AT STEP – POINT B:
$Z = bh^2/6 = (0.75)(1.25)^2/6 = 0.1953\text{ IN}^3$
$r/h = 0.20/1.25 = 0.16$ ; $H/h = 2.25/1.25 = 1.80$ } $K_t = 1.63$
$$\sigma_{MAX} = \frac{K_t M}{Z} = \frac{1.63(1500)}{0.1953} = 12\,519$$
$N = S_{ut}/\sigma_{MAX} = 40000/12\,519 = 3.19$



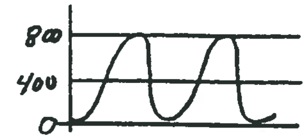

**59.** REPEATED ONE DIRECTION : CASE 5 : EQ. 5-20 : $\boxed{\text{SEEK } 2.5 < N < 3.0}$
$D = 0.50$ IN. : $A = \pi D^2/4 = 0.196\text{ IN}^2$ ; ASSUME $K_t = 1.0$
$\sigma_m = \sigma_a = F/A = 1500\text{ LB}/0.196\text{ IN}^2 = 7639$ psi




1ST TRIAL: SAE 1040 CD : $S_y = 71$ ksi ; $S_u = 80$ ksi
$S_n \approx 30$ ksi (FIG 5-8) ; $S_n' = (C_R)(C_{ST})S_n = (0.81)(0.8)(30) = 19.4$ ksi
$$\frac{1}{N} = \frac{\sigma_m}{S_u} + \frac{K_t\sigma_a}{S_n'} = \frac{7639}{80000} + \frac{1.0(7639)}{19\,400} = 0.489 \text{ ; } N = 2.04 \text{ LOW}$$
2ND: SAE 1040 WQT 1000 : $S_u = 112$ ksi ; $S_y = 87$ ksi ; 23% EL. ; $S_n = 42$ ksi ; $S_n' = 27.2$ ksi
$$\frac{1}{N} = \frac{7639}{112\,000} + \frac{(1)(7639)}{27\,200} = 0.349 \text{ ; } N = 2.86 \text{ OK}$$
CHECK YIELD: EQ. 5-21
$$N = \frac{S_y}{K_t(\sigma_a + \sigma_m)} = \frac{87000}{1(7639+7639)} = 5.69 \text{ OK}$$

**60** REPEATED – ONE DIRECTION: CASE 5: SPECIFY A STEEL

$F_{MAX} = 800\,LB$; $F_{MIN} = 0$; $\delta_{MAX} = 0.010\,IN$; $L = 25.0\,IN.$




$F_m = F_a = 400\,LB.$

CONSIDER DEFLECTION FIRST:

$$REQ'D.\ A = PL/E\delta = \frac{(800)(25.0)}{(30\times10^6)(0.010)} = 0.0667\,IN^2 = S^2$$

$$S = \sqrt{A} = \sqrt{0.0667\,IN^2} = 0.258\,IN. : \underline{TRY\ S = 0.300\,IN.}$$


CROSS SECTION

STRESS ANALYSIS: ASSUME $K_t = 1.0$

$$\sigma_m = \sigma_a = \frac{F}{A} = \frac{400\,LB}{(0.30\,IN)^2} = 4444\,psi$$

TRY SAE 1040 CD: $S_y = 71\,KSI$; $S_u = 80\,KSI$; 12% ELONGATION

FROM FIG 5-8: $S_n = 30\,KSI$: LET $C_s = 1.0$; $C_{ST} = 0.80$ (AXIAL); $C_R = 0.81$

$$S_n' = (1.0)(0.80)(0.81)(30) = 19{,}400\,psi$$

$$EQ.\ 5\text{-}20:\ \frac{1}{N} = \frac{\sigma_m}{S_u} + \frac{K_t\sigma_a}{S_n'} = \frac{4444}{80000} + \frac{1.0(4444)}{19400} = 0.285;\ \underline{N = 3.51\ OK.}$$

**61** REPEATED – ONE DIRECTION: CASE 5: $F_{MAX} = 1200\,LB$; $F_{MEAN} = F_a = 600\,LB.$

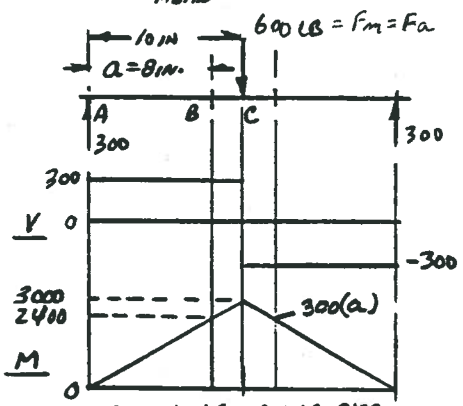
a) $$\frac{1}{N} = \frac{\sigma_m}{S_y} + \frac{K_t\sigma_a}{S_n'}$$

FOR ILLUSTRATION USE SAME MATERIAL AS IN PROB. 60: SAE 1040 CD

$S_u = 80\,KSI$; $S_n = 30\,KSI$

$S_n' = (0.90)(1.0)(0.81)(30) = 21.9\,KSi$

└SIZE └BENDING




AT C: $M = 3000\,LB\cdot IN$; $K_t = 1.0$

$$S = \pi D^3/32 = \pi(2.0)^3/32 = 0.785\,IN^3$$

$$EQ.\ 5\text{-}20:\ \frac{1}{N} = \frac{\sigma_m}{S_u} + \frac{K_t\sigma_a}{S_n'}$$

$$\sigma_m = \sigma_a = M/S = 3000\,LB\cdot IN/0.785\,IN^3 = 3820\,psi$$

$C_s$ – NONROTATING CIRC.

$D_e = 0.370\,D = 0.370(2.0) = 0.74\,IN$

$C_s = (.74/0.3)^{-0.11} = 0.90$

$$\frac{1}{N} = \frac{3820}{80\,000} + \frac{1.0(3820)}{21900} = 0.222;\ \underline{N = 4.50}\ HIGHER\ THAN\ (b).$$

b) AT SECTION B: $M = 2400\,LB\cdot IN$, $r/d = 0.20/2.0 = 0.10$; $D/d = 3.0/2.0 = 1.50$

$$K_t = 1.74:\ \sigma_m = \sigma_a = \frac{M}{S} = \frac{2400}{0.785} = 3056\,psi$$

$$\frac{1}{N} = \frac{3056}{80\,000} + \frac{(1.74)(3056)}{21\,900} = 0.281;\ N = 3.56 - LOWER\ THAN\ (a).$$

**62** REDESIGN BEAM IN PROB. 61. NOTE THAT N IS INVERSELY PROPORTIONAL TO MOMENT M AND DISTANCE $\underline{a}$. THEN $\underline{a}$ MUST BE REDUCED BY: $a' = a \times 3.54/4.50 = 0.787(a) = 0.787(8.0) = 6.30\,IN.$ (SAY 6.25 IN)

THEN $M = 300(a) = 1875\,LB\cdot IN.$; $\sigma_m = \sigma_a = M/S = 1875/0.785 = 2389\,psi$

$$\frac{1}{N} = \frac{2389}{80\,000} + \frac{1.74(2389)}{21\,900} = 0.220;\ \underline{N = 4.55\ OK}\ \text{HIGHER THAN (a).}$$

SPECIFY $\underline{a = 6.25\ IN.}$

**63** REFER TO PROBS. 61, 62. : NEW $r = 0.40$; $r/d = 0.40/2.0 = 0.20$

$D/d = 3.0/2.0 = 1.50$ ; $K_t = 1.47$

IF $a = 8.00$ IN AS GIVEN ; $M = 2400$ LB·IN AT B; $\sigma_m = \sigma_a = 3056$ PSI

$$\frac{1}{N} = \frac{\sigma_m}{s_u} + \frac{K_t\,\sigma_a}{s_n'} = \frac{3056}{80\,000} + \frac{1.47\,(3056)}{21\,900} = 0.305\ ;\ N = 3.27$$

DIMENSION $a$ MUST BE REDUCED TO GET $N \geq 4.50$ AS AT C.

$$a' = a \times 4.21/4.50 = 8.0\,(0.936) = 7.49\ \text{IN}\ ;\ \text{LET}\ \underline{a = 7.25\ \text{IN.}}$$

THEN $M = (7.25)(300) = 2175$ LB·IN ; $\sigma = M/S = 2771$ PSI

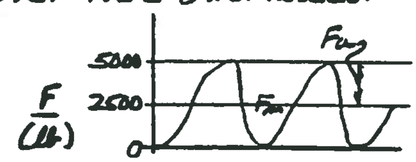
$$\frac{1}{N} = \frac{2771}{80\,000} + \frac{(1.47)(2771)}{21\,900} = 0.2206\ ;\ \underline{N = 4.53}\ \text{OK}$$

**64.** REPEATED - ONE DIRECTION : FLUCTUATING STRESS : CASE 5, EQ 5-20

SAE 1040 HR : $s_y = 42$ KSI ; $s_u = 72$ KSI ; $s_n = 23$ KSI FIG 5-8 HOT ROLLED.

$C_s = 1.0$ DIRECT TENSION; $C_R = 0.81$; $C_{st} = 0.80$ AXIAL LOAD

$s_n' = (0.8)(0.81)(23\,\text{KSI}) = 14.9$ KSI




$F_m = F_a = 2500$ lb.

a) AT PIN HOLE : $d = 0.25$ IN. DIA., $w = 1.00$ IN.

$d/w = 0.25$ ; $K_t = 4.40$

$$\sigma_{NOM} = \frac{F}{(w-d)t} = \frac{2500\,\text{LB}}{(1.0-0.25)(0.25)\,\text{IN}^2} = 13333\ \text{PSI}$$

$$\frac{1}{N} = \frac{\sigma_m}{s_u} + \frac{K_t\,\sigma_a}{s_n'} = \frac{13333}{72000} + \frac{(4.40)(13333)}{14900} = 4.12\ ;\ N = 0.243$$

INDICATES FAILURE.

b) AT FILLETS : $r/h = 0.02/1.00 = 0.02$ ; $H/h = 2.0/1.0 = 2.0$

ON FATIGUE : MIN. $r/h = 0.04$ THEN $K_t = 3.68$

$$\sigma_{NOM} = \frac{F}{A} = \frac{2500\,\text{LB}}{(1.00)(0.25)\,\text{IN}^2} = 10000\ \text{PSI}$$

$$\frac{1}{N} = \frac{10000}{72000} + \frac{(3.68)(10000)}{14900} = 2.61\ ;\ N = 0.383\ \underline{\text{FAILURE}}.$$

**65** IMPROVEMENTS: 1.) INCREASE THICKNESS, 2.) INCREASE FILLET RADIUS, 3.) USE STRONGER MATERIAL, 4.) INCREASE PIN HOLE SIZE -OR- CHANGE MANNER OF APPLYING FORCE TO THE PART TO ELIMINATE HOLE -OR- MAKE PART THICKER AT THE HOLES THAN IN MIDDLE OF THE PART. MATERIAL MAY BE REMOVED IN 2.00 IN. SECTION NEAR MIDDLE OF PART TO OFFSET ADDED MATERIAL ELSEWHERE. COULD TRY TITANIUM WITH LOWER DENSITY THAN STEEL. THIS PROBLEM MAY BE TOO RESTRICTIVE TO PERMIT A PRACTICAL SOLUTION WITH DATA IN THIS BOOK. MAY HAVE TO ACCEPT LOWER $N < 3.0$ OR SOME INCREASE IN WEIGHT OF THE COMPONENT.

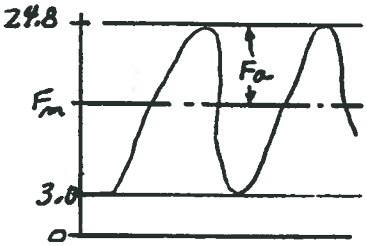
**66** FLUCTUATING NORMAL STRESS – CASE 5 – EQ. 5-20.

SAE 1040 CD: $S_y = 71$ ksi; $S_u = 80$ksi; $S_n = 30$ksi FIG. 5-8: $C_R = 0.81$

$C_s = 1.0$; $C_{ST} = 0.80$ AXIAL: $S_n' = 0.8(0.81)(30) = 19\,400$ psi

$F_m = (24.8+3.0)/2 = 13.9$ kN $(1.0\text{ LB}/4.448\text{ N}) = 3125$ LB

$F_a = 24.8 - 13.9 = 10.9$ kN $(1.0\text{ LB}/4.448\text{ N}) = 2450$ LB




$t = 0.375$ IN.; $w = 1.50$ IN; $d = 0.625$ IN; $d/w = 0.417$

AT PIN HOLES: $K_t = 2.84$

$$\sigma_m = \frac{F_m}{(w-d)t} = \frac{3125}{(1.50-0.625)(0.375)} = 9524 \text{ psi}$$

$$\sigma_a = \frac{F_a}{(w-d)t} = \frac{2450}{(1.50-0.625)(0.375)} = 7467 \text{ psi}$$

$$\frac{1}{N} = \frac{\sigma_m}{S_u} + \frac{K_t\sigma_a}{S_n'} = \frac{9524}{80\,000} + \frac{(2.84)(7467)}{19400} = 1.212;\ N = 0.825$$

FAILURE.

AT MIDDLE HOLE: $K_t = 2.22$; SAME NOMINAL STRESSES.

$$\frac{1}{N} = \frac{9524}{80\,000} + \frac{(2.22)(7467)}{19\,400} = 0.854;\ N = 1.17 \text{ LOW.}$$

PART MUST BE REDESIGNED.

**67** REPEATED REVERSED LOAD – BENDING – CASE 4: $N = S_n'/\sigma_{MAX}$

SAE 1340 OQT 1300: $S_y = 517$ MPa; $S_u = 690$ MPa; $S_n = 260$ MPa; $C_R = 0.81$

$C_s = 0.98$, $C_{ST} = 1.0$, $C_m = 1.0$: $S_n' = (0.98)(0.81)(260) = 206$ MPa

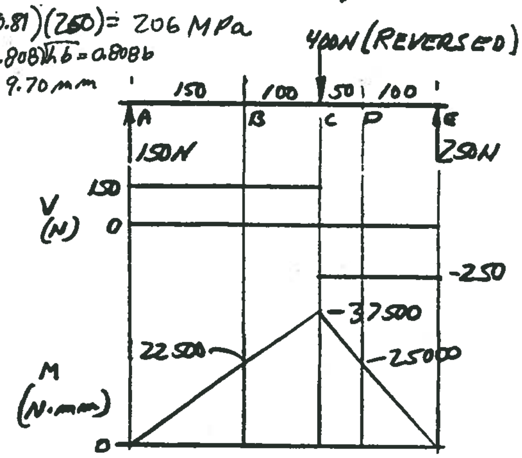
$R_E = 400\text{ N}(250/400) = 250$ N

$R_A = 400\text{ N}(150/400) = 150$ N

$C_S$ AT B: $D_e = 0.808\sqrt{tb} = 0.808b$

$D_e = 0.808(12) = 9.70$ mm

$C_S = 0.98$




AT B: $S = S^3/6 = 12^3/6 = 288$ mm$^3$

$r/h = 2.0/12.0 = 0.167$; $H/h = 20/12 = 1.67$

$K_t = 1.60$

$$\sigma_{MAX} = \frac{K_t M}{S} = \frac{(1.60)(22500)}{288} = 125 \text{ MPa}$$

$$N = S_n'/\sigma_{MAX} = 206/125 = 1.648$$

AT C: $S = bh^2/6 = 12(20)^2/6 = 800$ mm$^3$

$$\sigma_{MAX} = \frac{37500(1.0)}{800} = 46.9 \text{ MPa } (< \text{AT B.})$$

AT D: FROM e FATIGUE: $d/w = 14/20 = 0.70 - K_t = 1.40$

$$\sigma_{NOM} = \frac{6Mw}{(w^3-d^3)(t)} = \frac{6(25000)(20)}{(20^3-14^3)(12)} = 47.6 \text{ MPa};\ \sigma_{MAX} = K_t\sigma_{NOM}$$

$\sigma_{MAX} = K_t\,\sigma_{NOM} = (1.40)(47.6) = 66.6$ MPa ($<$ AT B.)

MINIMUM $N = 1.648$ AT B (LOW)

**68** SEE PROB. 67: FOR $N = 2.5$; $S_n' = N\,\sigma_{MAX} = 2.5(125) = 312.5$ MPa $= (0.98)(0.81)\,S_n$

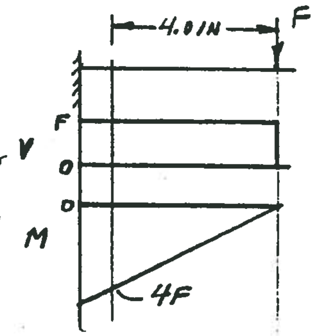
THEN $S_u$ REQ'D $= \approx 1150$ MPa (FIG 5-8) $S_n = 394$ MPa

FROM APP A4-3: SAE 1340 OQT 900 HAS $S_u = 1150$ MPa

**69** FLUCTUATING NORMAL STRESS – CASE 5: $F_{MIN} = 300\,LB$; $F_{MAX} = 700\,LB$

$F_m = (300+700)/2 = 500\,LB$; $F_a = 700 - 500 = 200\,LB$

AT FILLET: $r/d = 0.06/1.25 = 0.048$; $D/d = 2.0/1.25 = 1.60$




$K_t = 2.35$

$S = \pi d^3/32 = \pi(1.25)^3/32 = 0.192\ IN^3$

$\sigma_m = \frac{M_m}{S} = \frac{4F_m}{S} = \frac{4(500)\,LB\cdot IN}{0.192\,IN^3} = 10\,430\ PSI$

$\sigma_a = \frac{M_a}{S} = \frac{4F_a}{S} = \frac{4(200)\,LB\cdot IN}{0.192\,IN^3} = 4172\ PSI$

$C_s$ – NON-ROTATING CIRC. SHAFT $D_e = .370 = 0.463\,IN$

$C_s = 0.95$

SAE 1050 HR: $S_y = 49\,KSI$; $S_u = 90\,KSI$

$S_n = 26\,KSI$ (FIG 5-8 HR CURVE); $S_n' = (0.95)(0.81)(26) = 20.0\,KSI$

$\frac{1}{N} = \frac{\sigma_m}{S} + \frac{K_t\sigma_a}{S_n'} = \frac{10\,430}{90\,000} + \frac{(2.35)(4172)}{20\,000} = 0.606$; $N = 1.65$ LOW

**70** SEE PROB. 69: INCREASE $N$ TO 3.0 OR HIGHER BY USING LARGER $r$.
BEST POSSIBLE IMPROVEMENT WOULD BE $K_t = 1.0$ WITH LARGE $r$.

$\frac{1}{N} = \frac{\sigma_m}{S_y} + \frac{K_t\sigma_a}{S_n'} = \frac{10430}{90\,000} + \frac{(1.0)(4172)}{20\,000} = 0.324$; $N = 3.08$ OK

CONSIDER GRADUAL TAPER.

**71.** SEE PROB. 69: INCREASE MATERIAL STRENGTH TO GET $N \geq 3.0$.
TRY SAE 1340 OQT 900: $S_y = 158\,KSI$; $S_u = 169\,KSI$ (APP. 4-3) 17% ELONG.
USE MACHINED SURFACE: $S_n = 58\,KSI$; $S_n' = (0.95)(0.81)(58) = 44.63\,KSI$

$\frac{1}{N} = \frac{\sigma_m}{S} + \frac{K_t\sigma_a}{S_n'} = \frac{10\,430}{169\,000} + \frac{(2.35)(4172)}{44\,630} = 0.2814$; $N = 3.55$ OK

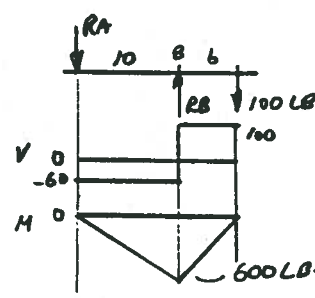
**72** REPEATED REVERSED STRESS: CASE 4: DESIGN FOR $N \geq 3.0$.
SPECIFY MATERIAL: $\sigma_d = S_n'/N$.

$\sigma_{MAX}$ AT B: $M_B = 600\,LB\cdot IN$

$S = \pi d^3/32 = \pi(1.00)^3/32 = 0.0982\ IN^3$

AT FILLET: $r/d = 0.06/1.0 = 0.06$; $D/d = 1.38/1.0 = 1.38$

$K_t = 1.99$




LET $\sigma_d = \sigma_{MAX} = \frac{K_t M}{S} = \frac{1.99(600)\,LB\cdot IN}{0.0982\,IN^3} = 12\,159\ PSI$

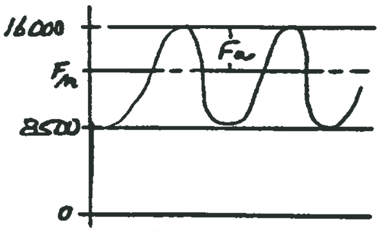
REQ'D $S_n' = N\sigma = (3.0)(12\,159) = 36\,477\ PSI$

$C_R = 0.81$, $C_s = 0.88$; $S_n = S_n'/(0.88)(0.81) = 51\,174\ PSI$

FROM FIG. 5-8, REQ'D $S_u = 145\,000\ PSI$ (MACHINED SURFACE)

ONE POSSIBLE SOLUTION: SAE 3140 OQT 1000, $S_u = 152\,000\ PSI$, 17% ELONG.

73 FLUCTUATING NORMAL STRESS – CASE 5:




SAE 1340 OQT 700: $S_y = 197$ KSI ; $S_u = 221$ KSI

$S_n' = (0.80)(0.81)(65\text{KSI}) = 42.1$ KSI
AXIAL ↖$C_R$ ↖FIG 5-8

$F_m = (8500 + 16000)/2 = 12250$ LB

$F_a = 16000 - 12250 = 3750$ LB

AT FILLET: $r/d = 0.05/0.63 = 0.079$; $D/d = 1.00/0.63 = 1.59$ : $K_t = 2.10$

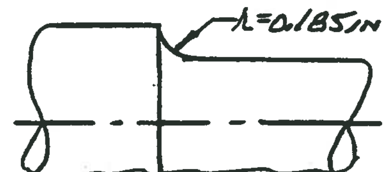
$A = \pi d^2/4 = \pi(0.63)^2/4 = 0.312\ \text{IN}^2$

$\sigma_m = F_m/A = \dfrac{12250\ \text{LB}}{0.312\ \text{IN}^2} = 39300$ PSI ; $\sigma_a = F_a/A = \dfrac{3750\ \text{LB}}{0.312\ \text{IN}^2} = 12030$ PSI

$\dfrac{1}{N} = \dfrac{\sigma_m}{S_u} + \dfrac{K_t\sigma_a}{S_n'} = \dfrac{39300}{221000} + \dfrac{2.10(12030)}{42100} = 0.778$ ; $N = 1.29$ LOW

74 SEE PROB. 73: TRY TO REDESIGN TO ACHIEVE $N \geq 3.0$.

INCREASE FILLET RADIUS TO $r = 0.185$ IN. FILLET WOULD THEN JUST BLEND WITH OUTSIDE OF 1.00 IN DIA.




$r/d = \dfrac{0.185}{0.630} = 0.29$ ; $\dfrac{D}{d} = \dfrac{1.00}{0.63} = 1.59$ ; $K_t = 1.36$

USE STRONGER MATERIAL: TRY SAE 8650 OQT 700

$S_u = 240$ KSI ; $S_y = 222$ KSI ; 12% ELONGATION

GRIND ALL CRITICAL SURFACES GENTLY. $S_n = 88$ KSI ; USE $C_R = 0.81$

THEN $S_n' = (0.81)(0.8)(88\text{KSI}) = 57.0$ KSI

$\dfrac{1}{N} = \dfrac{39300}{240000} + \dfrac{(1.36)(12030)}{57000} = 0.451$ ; $N = 2.22$ STILL TOO LOW

EVEN IF $K_t = 1.0$, $N = 2.67$ – SLIGHTLY LOW

DIAMETERS MAY HAVE TO BE INCREASED.

75 REPEATED REVERSED LOAD – CASE 4:

BENDING MOMENT AT FILLETS $= F(4.00\text{IN}) = (800)(4.0) = 3200$ LB·IN.

$S = \dfrac{BH^2}{6} = \dfrac{(2.00)(1.25)^2}{6} = 0.5208\ \text{IN}^3$ ; $\sigma_{NOM} = \dfrac{M}{S} = \dfrac{3200\ \text{LB·IN}}{0.5208\ \text{IN}^3} = 6144$ PSI

$N = \dfrac{S_n'}{\sigma_{MAX}} = \dfrac{S_n'}{K_t\,\sigma_{NOM}}$ ; THEN REQ'D $K_t = \dfrac{S_n'}{N\,\sigma_{NOM}}$

$C_S$ FOR RECTANGLE 1.25 × 2.00 IN. $D_e = 0.808\sqrt{hb} = 0.808\sqrt{(1.25)(2.00)} = 1.28$

$C_S = (1.28/0.3)^{-0.11} = 0.85$

AISI 1144 OQT 1100: $S_u = 112$ KSI, $S_n = 42$ KSI. USE $C_R = 0.81$

$S_n' = C_S C_R S_n = (0.85)(0.81)(42) = 28.9$ KSI $= 28900$ PSI

THEN $K_{t\,MAX} = \dfrac{S_n'}{N\,\sigma_{NOM}} = \dfrac{28900\ \text{PSI}}{(3)(6144)\ \text{PSI}} = 1.57$

FROM eFATIGUE : $H/h = 2.00/1.25 = 1.6$; FOR $K_t = 1.57$

THEN $r = 0.220$ IN GIVES $K_t = 1.57$

76. DESIGN SECTION AT "B" FOR $N \geq 3.0$.

$\Sigma M_{PIN} = 0 = 800(4.75) - R_1(2.25)$

$R_1 = 800(4.75)/2.25 = 1689\text{ LB} \downarrow$

MULTIPLE SOLUTIONS POSSIBLE:

LET WIDTH = 2.00 IN; SAME AS OTHER SECTIONS

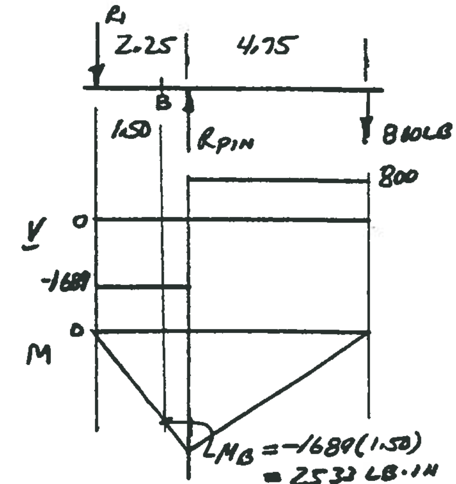
DESIGN FOR $K_t \approx 1.50$

CASE 4: $N = S_n'/\sigma_{MAX} = S_n'/K_t\sigma_{NOM}$

BUT $\sigma_{NOM} = \dfrac{M}{S} = \dfrac{M}{bh^2/6} = \dfrac{6M}{bh^2}$

THEN $N = \dfrac{S_n' bh^2}{K_t(6M)}$

$$t = \sqrt{\frac{N K_t (6M)}{S_n' b}} = \left[\frac{(3.0)(1.50)(6)(2533)}{(28900)(2.00)}\right]^{1/2} = 0.444 \text{ IN. MIN.}$$




LET $t = 0.85$ IN ; $w = 2.00$ : FOR $K_t = 1.50$, $r/t = 0.3$ EST.

THEN $r = 0.30t = 0.30(0.85) = 0.255$ IN 1ST. ESTIMATE. BY TRIAL → $\boxed{\text{FOR } r = 0.19,\ K_t = 1.50}$

PROBLEMS 77-83 ARE DESIGN PROBLEMS FOR WHICH THERE ARE MULTIPLE SOLUTIONS POSSIBLE.

# CHAPTER 6
# COLUMNS

**1.** $r = D/4 = 0.75/4 = 0.188\text{ IN}$ ; $KL/r = 1.0(32)/0.188 = 171$

$S_y = 42\,000\text{ PSI}$ ; $C_c = \sqrt{\dfrac{2\pi^2 E}{S_y}} = \sqrt{\dfrac{2\pi^2(30\times10^6)}{42000}} = 119 \rightarrow$ LONG COLUMN EULER

$A = \pi D^2/4 = 0.442\text{ IN}^2$

$$P_{CR} = \frac{\pi^2 EA}{(KL/r)^2} = \frac{\pi^2(30\times10^6)(0.442)}{(171)^2} = \underline{4473\text{ LB}}$$

**2.** $KL/r = 1.0(15)/0.188 = 79.8 < C_c \rightarrow$ SHORT – JOHNSON FORMULA

$$P_{CR} = AS_y\left[1 - \frac{S_y(KL/r)^2}{4\pi^2 E}\right] = (.442)(42000)\left[1 - \frac{42000(79.8)^2}{4\pi^2(30\times10^6)}\right] = \underline{14373\text{ LB}}$$

**3.** $r = 0.188\text{ IN}$ ; $KL/r = 171$ ; $S_y = 21\,000\text{ PSI}$ ; $E = 10\times10^6\text{ PSI}$

$C_c = \sqrt{\dfrac{2\pi^2(10\times10^6)}{21\,000}} = 97 \rightarrow$ LONG COLUMN

$$P_{CR} = \frac{\pi^2 EA}{(KL/r)^2} = \frac{\pi^2(10\times10^6)(0.442)}{(171)^2} = \underline{1492\text{ LB}}$$

**4.** $KL/r = (0.65)(32)/0.188 = 111 < C_c \rightarrow$ SHORT – JOHNSON FORMULA

$$P_{CR} = (0.442)(42000)\left[1 - \frac{42000(111)^2}{4\pi^2(30\times10^6)}\right] = \underline{10\,500\text{ LB}}$$

**5.** SQUARE: $r = S/\sqrt{12} = 0.65/\sqrt{12} = 0.188\text{ IN}$ SAME AS ROUND – PROB. 1.

$P_{CR} = 4473\text{ LB.}$

**6.** ACRYLIC : LET $S_y$ = TENSILE STRENGTH = 5400 PSI ; $E = 220\,000\text{ PSI}$

$C_c = \sqrt{\dfrac{2\pi^2 E}{S_y}} = \sqrt{\dfrac{2\pi^2(220\,000)}{5400}} = 28.4$ ; $KL/r = 171 > C_c$ – LONG

$$P_{CR} = \frac{\pi^2 EA}{(KL/r)^2} = \frac{\pi^2(220\,000)(0.442)}{(171)^2} = \underline{32.8\text{ LB}}$$

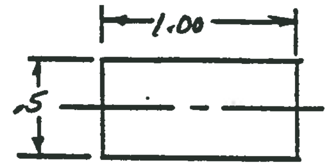
**7.**

$r = 0.5\,\text{in}/\sqrt{12} = 0.144\ \text{IN.}$

$KL/r = 1.0(8.5)/0.144 = 58.9$

$S_y = 181\,000\ \text{PSI} \rightarrow C_c = 57$ (FIG. 6-4) LONG COL.

$$P_{CR} = \frac{\pi^2 EA}{(KL/r)^2} = \frac{\pi^2(30\times10^6)(0.5)}{(58.9)^2} = \underline{42\,675\ \text{LB}}$$




**8.**

$r = \sqrt{D^2+d^2}/4 = \sqrt{(1.60)^2+(1.382)^2}/4 = 0.529\ \text{IN.};\ L = 6.25\ \text{FT}\left(\frac{12\ \text{IN}}{\text{FT}}\right) = 75\ \text{IN}$

$A = \pi(D^2-d^2)/4 = 0.5106\ \text{IN}^2;\ L/r = 75/0.529 = 142$

$S_y = 30\,000\ \text{PSI};\ C_c = 140$ (FIG. 6-5)

a) PINNED ENDS: $KL/r = 1.0(L/r) = 142 > C_c$ LONG-EULER

$$P_{CR} = \frac{\pi^2(30\times10^6)(0.5106)}{(142)^2} = \underline{7498\ \text{LB}}$$

b) FIXED-FIXED: $KL/r = 0.65(L/r) = 92.3 < C_c$ - SHORT-JOHNSON

$$P_{CR} = (0.5106)(30\,000)\left[1 - \frac{30\,000(92.3)^2}{4\pi^2(30\times10^6)}\right] = \underline{12\,000\ \text{LB}}$$

c) FIXED-PINNED: $KL/r = 0.8(142) = 114 < C_c$ - SHORT-JOHNSON

$$P_{CR} = (0.5106)(30\,000)\left[1 - \frac{30\,000(114)^2}{4\pi^2(30\times10^6)}\right] = \underline{10\,300\ \text{LB}}$$

d) FIXED/FREE: $KL/r = 2.10(142) = 298 > C_c$ - LONG

$$P_{CR} = \frac{\pi^2(30\times10^6)(0.5106)}{(298)^2} = \underline{1700\ \text{LB}}$$

**9.**

$S_y = 152\ \text{KSI}:\ C_c \cong 60$ (FIG. 6-5): ASSUME COLUMN IS LONG: $K = 1.0$

(EQ. 6-9) $$D = \left[\frac{64NP(KL)^2}{\pi^3 E}\right]^{1/4} = \left[\frac{64(3)(8500)(50)^2}{\pi^3(30\times10^6)}\right]^{1/4} = [4.39]^{1/4} = 1.45\ \text{IN.}$$

USE 1.50 IN

$r = D/4 = 1.50/4 = 0.375:\ KL/r = (1.0)(50)/.375 = 133 > C_c$ LONG, OK.

**10.**

$S_y = 30\ \text{KSI}:\ C_c = 140$ (FIG. 6-5): ASSUME COLUMN IS LONG

$D = 1.45\ \text{IN}$ (SAME AS PROB. 9) USE $D = 1.50$: $KL/r = 133 < C_c$ - JOHNSON

(EQ. 6-10) $$D = \left[\frac{4(3)(8500)}{\pi(30\,000)} + \frac{4(30\,000)(50)^2}{\pi^2(30\times10^6)}\right]^{1/2} = 1.45\ \text{IN}\ \text{USE } 1.50\ \text{IN.}$$

NO ADVANTAGE TO 4140 STEEL

**11.**

ALUM. 2014-T4: $S_y = 42\,000\ \text{PSI};\ C_c \cong 69$: ASSUME LONG COLUMN

$$D = \left[\frac{64(3)(8500)(50)^2}{\pi^3(10\times10^6)}\right]^{1/4} = 1.90\ \text{IN.}$$ — USED = 2.00 IN, $r = D/4 = 0.50\ \text{IN.}$

$KL/r = (1.0)(50)/0.50 = 100 > C_c$ LONG. D OK.

**12.**

SQUARE: $I = S^4/12;\ A = S^2$: FROM EQ. 6-8 - EULER

$$I = S^4/12 = NP(KL)^2/\pi^2 E$$

$$S = \left[\frac{12NP(KL)^2}{\pi^2 E}\right]^{1/4}$$

(CONTINUE NEXT PAGE)

$\boxed{12.}$ (CONTINUED) JOHNSON: $r^2 = S^2/12$

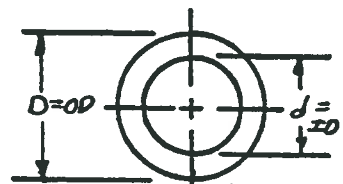
$$\text{EQ. 6-7: } P_{CR} = NP = A S_y\left[1 - \frac{S_y (KL)^2}{4\pi^2 E r^2}\right] = S^2 S_y\left[1 - \frac{S_y (KL)^2}{4\pi^2 E (S^2/12)}\right]$$

$$NP = S^2 S_y - \frac{\cancel{S^2} S_y^2 (KL)^2 \cancel{12}^{\,3}}{\cancel{4}\pi^2 E \cancel{S^2}} = S^2 S_y - \frac{3 S_y^2 (KL)^2}{\pi^2 E} \text{ : SOLVE FOR S}$$

$$S^2 S_y = NP + \frac{3 S_y^2 (KL)^2}{\pi^2 E} \quad \text{DIVIDE BY } S_y \text{ AND TAKE } \sqrt{\ }.$$

$$S = \left[\frac{NP}{S_y} + \frac{3 S_y (KL)^2}{\pi^2 E}\right]^{1/2}$$

$\boxed{13.}$ EULER: EQ. (6-8) $I = NP(KL)^2/\pi^2 E$




$R = ID/OD = d/D$

$d = RD$

$$I = \frac{\pi(D^4 - d^4)}{64} = \frac{\pi(D^4 - R^4 D^4)}{64} = \frac{\pi D^4 (1 - R^4)}{64}$$

$$\frac{\pi D^4 (1 - R^4)}{64} = \frac{NP(KL)^2}{\pi^2 E}$$

$$D = \left[\frac{64\, NP (KL)^2}{\pi^3 E (1 - R^4)}\right]^{1/4}$$

JOHNSON: EQ. (6-7)

$$A = \frac{\pi(D^2 - d^2)}{4} = \frac{\pi(D^2 - R^2 D^2)}{4}$$

$$A = \pi D^2 (1 - R^2)/4$$

$$r^2 = \frac{D^2 + d^2}{16} = \frac{D^2 + R^2 D^2}{16}$$

$$r^2 = \frac{D^2 (1 + R^2)}{16}$$

$$P_{CR} = NP = A S_y\left[1 - \frac{S_y (KL/r)^2}{4\pi^2 E}\right]$$

$$\frac{NP}{S_y} = A - \frac{A S_y (KL)^2}{4\pi^2 E r^2}$$

$$= \frac{\pi D^2 (1 - R^2)}{4} - \frac{\cancel{\pi} \cancel{D^2} (1 - R^2) S_y (KL)^2}{\cancel{4 \cdot 4}\pi^2 E\, \cancel{D^2} (1 + R^2)/\cancel{16}}$$

$$\frac{NP}{S_y} = \frac{\pi D^2 (1 - R^2)}{4} - \frac{(1 - R^2) S_y (KL)^2}{\pi E (1 + R^2)}$$

$$\frac{\pi D^2 (1 - R^2)}{4} = \frac{NP}{S_y} + \frac{(1 - R^2) S_y (KL)^2}{\pi E (1 + R^2)}$$

$$D = \left[\frac{4NP}{\pi S_y (1 - R^2)} + \frac{4 \cancel{(1 - R^2)} S_y (KL)^2}{\pi^2 E (1 + R^2) \cancel{(1 - R^2)}}\right]^{1/2}$$

$$D = \left[\frac{4NP}{\pi S_y (1 - R^2)} + \frac{4 S_y (KL)^2}{\pi^2 E (1 + R^2)}\right]^{1/2}$$

**14** ASSUME COLUMN IS LONG: FROM PROB. 12: $S = \left[\frac{12NP(KL)^2}{\pi^2 E}\right]^{1/4}$

$KL = 0.65(64) = 41.6$ IN (FIXED ENDS)

$$S = \left[\frac{12(3)(6500)(41.6)^2}{\pi^2(10\times10^6)}\right]^{1/4} = 1.423 \text{ IN — USE } \underline{S = 1.500 \text{ IN}}$$

CHECK: $r = S/\sqrt{12} = 1.50/\sqrt{12} = 0.433$ : $KL/r = 41.6/0.433 = 96.1$

FOR 6061-T6, $S_y = 40$ ksi : FROM FIG (6-6) $C_c = 70$ LONG OK

**15.** $R = ID/OD = 0.8$ : $(1-R^4) = 0.5904$

ASSUME LONG; FROM PROB. 13

$$D = \left[\frac{64NP(KL)^2}{4\pi^3 E(1-R^4)}\right]^{1/4} = \left[\frac{64(3)(6500)(41.6)^2}{4\pi^3(10\times10^6)(0.5904)}\right]^{1/4} = 1.31 \text{ IN}$$

USE $\underline{D = 1.50 \text{ IN}}$ ; $\underline{d = 0.8D = 1.20 \text{ IN}}$ : $r = \frac{\sqrt{D^2+d^2}}{4} = 0.48$ IN : $\frac{KL}{r} = 87$ LONG OK

WEIGHT COMPARISON: WT. PROPORTIONAL TO AREA

SQUARE : $A = S^2 = (1.50)^2 = 2.25$ IN$^2$

TUBE : $A = \frac{\pi}{4}(D^2-d^2) = \frac{\pi}{4}(1.50^2-1.20^2) = 0.636$ IN$^2$

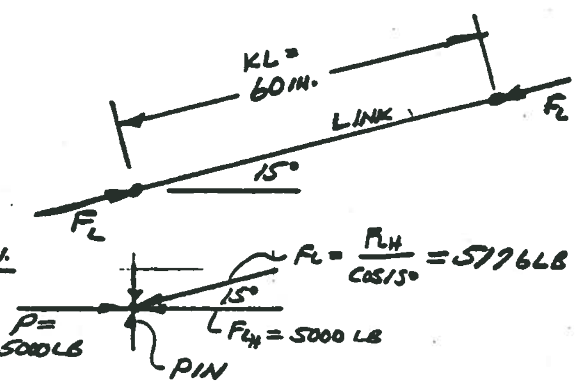
$\underline{W_S/W_T} = 2.25/0.636 = \underline{3.54}$ — TUBE MUCH MORE EFFICIENT

**16.** ASSUME COLUMN IS LONG: EQ. (6-9)

$$D = \left[\frac{64NP(KL)^2}{\pi^3 E}\right]^{1/4}$$

$$D = \left[\frac{64(3.5)(5176)(60)^2}{\pi^3(30\times10^6)}\right]^{1/4} = 1.46 \text{ IN}$$

USE $\underline{D = 1.50 \text{ IN.}}$




CHECK $r = D/4 = 0.375$

$KL/r = 60/0.375 = 160$ LONG. OK

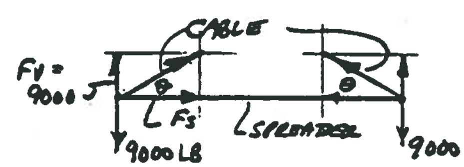
$C_c \approx 60$ FIG. (6-5) ; $S_y = 151$ ksi

NOTE: CARE WOULD HAVE TO BE USED AT CONNECTIONS TO ENSURE AXIAL LOAD.

**17.** MULTIPLE DESIGNS POSSIBLE. CONSIDER HOLLOW TUBE – ROUND OR SQUARE. CHEAPER MATERIAL MAY ALSO BE USED.

**18.** MULTIPLE DESIGNS POSSIBLE

$F_S = F_V/\tan\theta = \frac{9000}{\tan 30°} = 15588$ LB




**19.** MULTIPLE DESIGNS POSSIBLE

$F_S = \frac{9000}{\tan 15°} = 33588$ LB

**20.** $L = 10.75\ FT \times 12\ IN/FT = 129\ IN.$ : $KL = 2.1(129) = 271\ IN$ : $S_y = 68\ KSI$ ; $C_c = 93$

ASSUME COLUMN IS LONG – EULER – EQ. (6-9)

$$D = \left[\frac{64NP(KL)^2}{\pi^3 E}\right]^{1/4} = \left[\frac{64(2.5)(25000)(271)^2}{\pi^3(30\times10^6)}\right]^{1/4} = 4.21\ IN.$$

USE $D = 4.25\ IN.$

CHECK $r = D/4 = 1.063\ IN$ : $KL/r = 271/1.063 = 255$ — LONG . OK

**21.** **22.** MULTIPLE SOLUTIONS POSSIBLE

**23.** CROOKED COLUMN : $a = 0.08''$, $D = 0.75\ IN$, $c = D/2 = 0.375\ IN$,
$r = D/4 = 0.188\ IN$, $r^2 = 0.0352\ IN^2$ ; $A = 0.442\ IN^2$ ; $P_{CR} = 4473\ LB$ [PROB. 6-1]

IN EQ. 6-11 : $C_1 = -\frac{1}{N}\left[S_y A + \left(1 + \frac{ac}{r^2}\right)P_{cr}\right]$

$$C_1 = -\frac{1}{3}\left[(42000)(0.442) + \left(1 + \frac{(0.08)(0.375)}{0.0352}\right)(4473)\right] = -8951$$

$$C_2 = \frac{S_y A P_{cr}}{N^2} = \frac{42000(0.442)(4473)}{(3)^2} = 9.226\times10^6$$

$$P = 0.5\left[-(-8951) - \sqrt{(-8951)^2 - 4(9.226\times10^6)}\right] = \underline{1189\ LB}$$

**24.** CROOKED COLUMN : $a = 0.04\ IN$ ; $C = 0.5/2 = 0.25\ IN$ ; $P_{cr} = 42675\ LB$ (PROB 6-7)
$r = 0.144\ IN$ ; $r^2 = 0.0208\ IN^2$ ; $A = 0.50\ IN^2$ ; $S_y = 181\,000\ PSI$

$$C_1 = -\frac{1}{3}\left[(181\,000)(0.50) + \left(1 + \frac{(0.04)(0.25)}{0.0208}\right)(42675)\right] = -51220$$

$$C_2 = \frac{(181\,000)(0.50)(42675)}{(3)^2} = 4.291\times10^8$$

$$P = 0.5\left[-(-51220) - \sqrt{(-51220)^2 - 4(4.291\times10^8)}\right] = \underline{10552\ LB}$$

**25.** CROOKED COLUMN : $a = 0.15\ IN$ ; $C = OD/2 = 0.80\ IN$ ; $P_{CR} = 7498\ LB$ (PROB 6-8)
$r = 0.529\ IN$ ; $r^2 = 0.280\ IN^2$ ; $A = 0.5106\ IN^2$ ; $S_y = 30\,000\ PSI$

$$C_1 = \frac{-1}{3}\left[(30\,000)(0.5106) + \left(1 + \frac{(0.15)(0.80)}{0.280}\right)(7498)\right] = -8677$$

$$C_2 = \frac{(30\,000)(0.5106)(7498)}{(3)^2} = 1.276\times10^7$$

$$P = 0.5\left[-(-8677) - \sqrt{(-8677)^2 - 4(1.276\times10^7)}\right] = \underline{1877\ LB}$$

**26** ECCENTRIC COLUMN: $L = 42$ IN.; $S = 1.25$ IN.; $C = S/2 = 0.625$ IN.


$r = S/\sqrt{12} = 0.361$ IN.; $r^2 = 0.130$ IN$^2$; $A = S^2 = 1.563$ IN$^2$; $e = 0.60$ IN.

$I = S^4/12 = 0.2035$ IN$^4$; $S_y = 13\,000$ PSI FOR AL. 6063-T4.

FROM EQ(6-12): $\sigma_{MAX} = \sigma_{L/2} = \dfrac{1250}{1.563}\left[1 + \dfrac{(0.60)(0.625)}{0.130}\,\sec\left(\dfrac{42}{2(0.361)}\sqrt{\dfrac{1250}{(1.563)(10\times10^6)}}\right)\right]$

$\underline{\sigma_{MAX} = 3458 \text{ PSI}}$

EQ.(6-14): $\underline{y_{MAX}} = 0.60\left[\sec\left(\dfrac{42}{2(0.361)}\sqrt{\dfrac{1250}{(1.563)(10\times10^6)}}\right) - 1\right] = \underline{0.0915 \text{ IN.}}$

**27** ECCENTRIC COLUMN: $L = 3.2$ m $= 3200$ mm; $P = 30.5$ kN $= 30500$ N

3-IN SCH. 40: $D_o = 3.501$ IN, $C = D_o/2 = 1.75$ IN $(25.4 \text{ mm/in.}) = 44.5$ mm

$r = 1.16$ in. $(25.4) = 29.46$ mm; $r^2 = 868$ mm$^2$; $A = 2.23$ IN$^2$ $(25.4^2) = 1439$ mm$^2$

$I = 3.02$ IN$^4$ $(25.4^4) = 1.257\times10^6$ mm$^4$; $e = 150$ mm

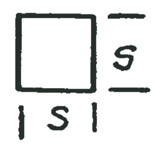
AISI 1020HR: $E = 207$ GPa $= 207\times10^9$ Pa $= 207000$ MPa $= 207000$ N/mm$^2$

EQ(6-12): $\sigma_{MAX} = \sigma_{L/2} = \dfrac{30500}{1439}\left[1 + \dfrac{(150)(44.5)}{868}\,\sec\left(\dfrac{3200}{2(29.46)}\sqrt{\dfrac{30500}{(1439)(207000)}}\right)\right]$

$\underline{\sigma_{MAX} = 212 \text{ MPa}}$: BUT $S_y = 207$ MPa, THEN $\underline{\text{STRESS IS TOO HIGH.}}$

$\underline{y_{MAX}} = (150)\left[\sec(\quad) - 1\right] = \underline{25.9 \text{ mm}}$ IF MAT'L. DOES NOT YIELD.

**28** ECCENTRIC COLUMN: $L = 14.75$ IN; $e = 0.30$ IN; $S = 0.250$ IN



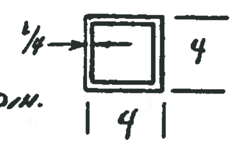

$A = S^2 = 0.0625$ IN$^2$; $r = S/\sqrt{12} = 0.25/\sqrt{12} = 0.0722$ IN; $r^2 = 0.00521$ IN$^2$

$P = 45$ LB; $E = 28\times10^6$ PSI: $c = S/2 = 0.125$ IN.

EQ(6-12): $\underline{\sigma_{MAX}} = \dfrac{45}{0.0625}\left[1 + \dfrac{(0.30)(0.125)}{0.00521}\,\sec\left(\dfrac{14.75}{2(0.0722)}\sqrt{\dfrac{45}{(0.0625)(28\times10^6)}}\right)\right] = \underline{6685 \text{ PSI}}$

EQ(6-14): $\underline{y_{MAX}} = (0.30)\left[\sec(\quad) - 1\right] = \underline{0.045 \text{ IN.}}$

**29** ECCENTRIC COLUMN: $L = 40$ IN; $e = 0.50$ IN; $P = 75000$ LB




FROM APP. 15-14: $A = 3.37$ IN$^2$; $r = 1.52$ IN; $r^2 = 2.31$ IN$^2$; $C = 4/2 = 2.00$ IN.

ASTM A242: $S_y = 50000$ PSI; $E = 30\times10^6$ PSI

LET $N = 3$, THEN $\sigma_d = S_y/3 = 16667$ PSI

LET $\sigma_d'$ = RIGHT SIDE OF EQ.(6-13) $= \dfrac{75000}{3.37}\left[1 + \dfrac{(0.50)(2.00)}{2.31}\,\sec\left(\dfrac{40}{2(1.52)}\sqrt{\dfrac{75000(3)}{(3.37)(30\times10^6)}}\right)\right]$

$\sigma_d' = 34099$ PSI $> \sigma_d$ $\underline{\text{UNSAFE}}$ / NOTE: $6\times6\times1/2$ REQ'D: $\sigma_d' = 9493$ PSI

**30** $\underline{\text{CENTRAL LOAD}}$: $L = 16.0$ FT $(12 \text{ in/ft}) = 192$ IN; ASTM A36, $S_y = 36$ KSI

FROM APP 15-9: $A = 5.54$ IN$^2$; $I_y = 9.13$ IN$^4$; $r = \sqrt{I/A} = 1.28$; $\dfrac{KL}{r} = \dfrac{(0.8)(192)}{1.28} = 119.7$

FROM FIG 6-5, $C_c = 125$; SHORT COLUMN – JOHNSON FORMULA. LET $N = 3$

$\underline{P_a} = \dfrac{P_{cr}}{N} = \dfrac{(5.54)(36000)}{3}\left[1 - \dfrac{(36000)(119.7)^2}{4\pi^2(30\times10^6)}\right] = \underline{37500 \text{ LB}}$

**31**

CENTRAL LOAD: FIXED-END, $K=0.65$, $L_e=0.65(66)=42.9$ IN.

S4×7.7: $A=2.26\ IN^2$; $r_{MIN}=r_y=0.581\ IN$; $L_e/r=73.8$; $N=3$, $P_a=P_{CR}/N$

ASTM A36: $S_y=36000$ PSI; $E=30\times10^6$ PSI; $C_c \approx 130$ — SHORT COLUMN. JOHNSON EQ (6-7)

$$P_a=[(2.26)(36000)/3]\left[1-\frac{(36000)(73.8)^2}{4\pi^2(30\times10^6)}\right]=\underline{22\ 600\ LB.}$$

**32**

ECCENTRIC LOAD: $P=1000$ LB; $e=0.50+0.80/2=0.90$ IN; $L=72$ IN.

$A=(1.60)(0.80)=1.28\ IN^2$; $C=0.80/2=0.40$ IN; $r=0.80/\sqrt{12}=0.2309$ IN.

USE STEEL — $E=30\times10^6$ PSI

$$\sigma_{MAX}=\sigma_{L/2}=\frac{1000}{1.28}\left[1+\frac{(0.90)(0.40)}{(0.2309)^2}\sec\left(\frac{72}{2(0.2309)}\sqrt{\frac{1000}{(1.28)(30\times10^6)}}\right)\right]=\underline{8315\ PSI}$$

$$y_{MAX}=0.90[\sec(.7955)-1]=\underline{0.386\ IN.}$$

SPECIFY A MATERIAL TO PROVIDE $N\geq3$.

USING EQ (6-13): $$\sigma_d=\frac{1000}{1.28}\left[1+\frac{(0.90)(0.40)}{(.2309)^2}\sec\left(\frac{72}{2(0.2309)}\sqrt{\frac{3000}{AE}}\right)\right]$$

$\sigma_d=28\ 280$ PSI $=S_y/N$: THEN $S_y=N\sigma_d=84900$ PSI

SPECIFY AISI 1040 WQT 1000, $S_y=86000$ PSI (OTHER SOLUTIONS POSSIBLE.)

**33**

CENTRAL LOAD: SPECIFY A STEEL TUBE: $S_y=36000$ PSI; LET $N=3$

ASSUME COLUMN SUPPORTS ½ TOTAL LOAD: $P_a=55000\ LB/2=27500$ LB

ASSUME COLUMN IS LONG: EQ. 6-8: $I=\dfrac{NP_a(KL)^2}{\pi^2E}$ $\qquad C_c\approx130$

ASSUME FIXED-PINNED COLUMN, $K=0.8$: $KL=(0.8)(18.5\ FT)\frac{12\ IN}{FT}=177.6$ IN. (LONG)

$$I=\frac{3(27500)(177.6)^2}{\pi^2(30\times10^6)}=8.79\ IN^4$$ ; USE SQ. TUBE 4×4×½, $I=11.9\ IN^4$

OR RECT. TUBE 6×4×¼, $I_y=11.1\ IN^4$ — LIGHTER

**34**

CENTRAL LOAD: C5×9 STEEL CHANNEL: $A=2.64\ IN^2$; $r_y=0.489$ IN; $N=3$

$KL/r=1.0(112)/0.489)=229$: ASTM A36 - $S_y=36000$ PSI, $C_c\approx130$ — LONG COL.

$$P_a=P_{CR}/N=\frac{\pi^2EA}{N(KL/r)^2}=\frac{\pi^2(30\times10^6)(2.64)}{3(229)^2}=\underline{4967\ LB}$$

**35**

CENTRAL LOAD: SAME AS 34 EXCEPT FIXED ENDS, $K=0.65$

$KL/r=(0.65)(112)/0.489=148.9$ — LONG COLUMN

$$P_a=\frac{\pi^2(30\times10^6)(2.64)}{3(148.9)^2}=\underline{11\ 750\ LB}$$

**36**

ECCENTRIC LOAD: $e=\bar{x}$ FROM APP. 15-4 $=0.478$ IN. AND $c=e$

USE EQ (6-13): $\sigma_d=S_y/N=36000/3=12000$ PSI

$$\sigma_d=\frac{P_a}{2.64}\left[1+\frac{(0.478)(0.478)}{(0.489)^2}\sec\left(\frac{112}{2(0.489)}\sqrt{\frac{3(P_a)}{(2.64)(30\times10^6)}}\right)\right]$$

BY ITERATION: FOR $\underline{P_a=4100\ LB}$, $\sigma_d=11\ 920$ PSI

# ECCENTRIC COLUMN ANALYSIS

*Data from:* Problem 6-37

Solves Equation 6-13 for design stress and Equation 6-14 for maximum deflection

| Enter data for variables in *italics in shaded boxes* | | | | Use consistent U.S. Customary units. | | |
|---|---|---|---|---|---|---|
| **Data To Be Entered:** | | | | **Computed Values:** | | |
| **Length and End Fixity:** | | | | | | |
| *Column length, L =* | *126* | *in* | | | | |
| *End fixity, K =* | *1* | | ——> | Eq. Length, $L_e$ = KL = | 126.0 | in |
| **Material Properties:** | | | | | | |
| *Yield strength, $s_y$ =* | *46000* | *psi* | | | | |
| *Modulus of Elasticity, E =* | *2.90E+07* | *psi* | ——> | Column constant, $C_c$ = | 111.6 | |
| **Cross Section Properties:** | | | | Argument for secant = | 0.777 | for strength |
| [Note: Enter *r* or compute *r* = sqrt(*I/A*) ] | | | | Value of secant = | 1.4025 | |
| [Always enter Area] | | | | | | |
| [Enter zero for *I or* r *if not used*] | | | | Argument for secant = | 0.449 | for deflection |
| *Area, A =* | *6.020* | *in*$^2$ | | Value of secant = | 1.1098 | |
| *Moment of Inertia, I =* | *0* | *in*$^4$ | | | | |
| *Radius of Gyration, r =* | *1.410* | *in* | ——> | Slenderness ratio, KL/r = | 89.4 | |
| **Values for Eqns. 6-13 and 6-14:** | | | | | | |
| *Eccentricity = e =* | *3* | *in* | | Column is: | ***short*** | |
| *Neutral axis to outside = c =* | *2* | *in* | | | | |
| *Allowable load = $P_a$ =* | *17600* | *lb* | | **Req'd yield strength =** | ***45,896*** | ***psi*** |
| **Design Factor** | | | | **Must be less than actual yield strength:** | | |
| *Design factor on load, N =* | *3* | | ——> | $s_y$ = | ***46,000*** | ***psi*** |
| | | | | **Max. deflection, $y_{max}$ =** | **0.329** | **in** |

Note: *A* and *r* from Appendix 15-14

## ECCENTRIC COLUMN ANALYSIS

*Data from:* Problem 38A US

Solves Equation 6-13 for design stress and Equation 6-14 for maximum deflection

Enter data for variables in *italics in shaded boxes*

Use consistent U.S. Customary units

| Data To Be Entered: | | Computed Values: |
|---|---|---|
| | | **NOTE: This solution considers the eccentric load with bending about the horizontal axis.** |
| **Length and End Fixity:** *Column length, L =* 40 in; *End fixity, K =* 1 | → | Eq. Length, $L_e = KL$ = 40.0 in |
| **Material Properties:** *Yield strength, $s_y$ =* 40000 psi; *Modulus of Elasticity, E =* 1.00E+07 psi | → | Column const., $C_c$ = 70.2 |
| **Cross Section Properties:** [Note: Enter *r* or compute *r* = sqrt(*I/A*)] [Always enter Area] [Enter zero for *I* or r *if not used*] | | Argument of sec = 0.855 for strength; Value of secant = 1.5236; Argument of sec = 0.494 for deflection; Value of secant = 1.1355 |
| *Area, A =* 0.600 $in^2$; *Moment of Inertia, I =* 0 $in^4$; *OR* *Radius of Gyration, r =* 0.433 in | → | Slender. ratio, KL/r = 92.4 |
| **Values for Eqns. 6-13 and 6-14:** *Eccentricity, e =* 1.75 in; *Neutral axis to outside, c =* 0.75 in; *Allowable load, $P_a$ =* 685 lb | | Column is: *long* |
| | | **FINAL RESULTS** |
| **Design Factor** *Design factor on load, N =* 3 | | **Req'd yield strength = *39,954 psi*** ***Must be less than actual yield strength:*** **$s_y$ = *40,000 psi*** |
| | | **Max Deflection, $y_{max}$ = *0.237 in*** |
| | | **See also Solution 38B for buckling about the thinner vertical axis.** |

NOTE: $A = (1.50\ IN)(0.40\ IN) = 0.600\ IN^2$

$r = \frac{H}{\sqrt{12}} = \frac{1.50\ IN}{\sqrt{12}} = 0.433\ IN$ ; $P_a = 685$ LB.

ECCENTRIC LOAD TENDS TO BUCKLE THE BAR ABOUT ITS STRONG AXIS. BUT SEE SOLUTION 38B. LIMITING LOAD IS 163 LB FOR BUCKLING ABOUT THIN AXIS.

| COLUMN ANALYSIS PROGRAM | Data from: Problem 6-38B US |
|---|---|
| Refer to Figure 6-4 for analysis logic | |
| Enter data for variables in *italics in shaded boxes* | Use consistent U.S. Customary units. |
| **Data To Be Entered:** | **Computed Values:** |
| **Length and End Fixity:** | |
| *Column length, L =* *40 in* | |
| *End fixity, K = 1* | Eq. Length, $L_e = KL =$ 40.0 in |
| **Material Properties:** | |
| *Yield strength, $s_y$ =* *40000 psi* | |
| *Modulus of Elasticity, E =* *1.00E+07 psi* | Column const., $C_c$ = 70.2 |
| **Cross Section Properties:** | **NOTE: Cross section properties taken with respect to the vertical axis because the load is central to that axis. But buckling is expected about the axis through the thin (0.40 in) section.** |
| [Note: Enter *r* or compute *r* = sqrt(*I/A*)] | |
| [Always enter Area] | |
| [Enter zero for *I* or *r* if not used] | |
| *Area, A =* *0.6 in²* | |
| *Moment of Inertia, I =* *0 in⁴* | |
| **Or** | |
| *Radius of Gyration, r =* *0.115 in* | Slender. ratio, KL/r = 347.8 |
| | Column is: ***long*** |
| | **Critical Buckling Load =** ***489 lb*** |
| **Design Factor** | |
| *Design factor on load, N = 3* | **Allowable Load =** ***163 lb*** |
| | ***This value governs the design, not solution 38A*** |

ANALYSIS AS A STRAIGHT CENTRALLY LOADED COLUMN THAT TENDS TO BUCKLE ABOUT THIN AXIS, $t = 0.40$ IN

$$r = \frac{t}{\sqrt{12}} = \frac{0.40\text{ IN}}{\sqrt{12}} = 0.115\text{ IN}$$

FROM EULER FORMULA WITH $N=3$ $\underline{P_a = 163 \text{ LB.}}$

## ECCENTRIC COLUMN ANALYSIS

*Data from:* Problem 39-SI

Solves Equation 6-13 for design stress and Equation 6-14 for maximum deflection

Enter data for variables in *italics in shaded boxes* — Use consistent SI Metric units

| Data To Be Entered: | | Computed Values: | |
|---|---|---|---|
| **Length and End Fixity:** | | | |
| *Column length, L =* | *750 mm* | | |
| *End fixity, K =* | *1* | Eq. Length, $L_e = KL$ = | 750.0 mm |
| **Material Properties:** | | | |
| *Yield strength, $s_y$ =* | *966 Mpa* | | |
| *Modulus of Elasticity, E =* | *200 Gpa* | Column const., $C_c$ = | 63.9 |
| **Cross Section Properties:** | | Argument of sec = | 0.811 for strength |
| [Note: Enter *r* or compute *r* = sqrt(*I/A*).] | | Value of secant = | 1.4512 |
| [Always enter Area] | | | |
| [Enter zero for *I* or *r* if not used] | | Argument of sec = | 0.468 for deflection |
| *Area, A =* | *314 mm²* | Value of secant = | 1.1205 |
| *Moment of Inertia, I =* | *0 mm⁴* | | |
| OR | | | |
| *Radius of Gyration, r =* | *7.289 mm* | Slender. ratio, KL/r = | 102.9 |
| **Values for Eqns. 6-13 and 6-14:** | | | |
| *Eccentricity, e =* | *20 mm* | Column is: | *long* |
| *Neutral axis to outside, c =* | *12.5 mm* | **FINAL RESULTS** | |
| *Allowable load, $P_a$ =* | *5200 N* | **Req'd yield strength =** | ***389 MPa*** |
| **Design Factor** | | ***Must be less than actual yield strength:*** | |
| *Design factor on load, N =* | *3* | $s_y$ = | ***966 MPa*** |
| | | **Max Deflection, $y_{max}$ =** | ***2.41 mm*** |
| | | **Piston rod is safe for $P_a$ = 5200 N.** | |

## COLUMN ANALYSIS PROGRAM

Data from: *Problem 40A-Straight*

Refer to Figure 6-4 for analysis logic

Enter data for variables in *italics in shaded boxes*

Use consistent U.S. Customary units.

**Data To Be Entered:**

**Length and End Fixity:**

*Column length, L =* 156 *in*

*End fixity, K =* 1

**Material Properties:**

*Yield strength, $s_y$ =* 36000 *psi*

*Modulus of Elasticity, E =* 3.00E+07 *psi*

**Cross Section Properties:**

[Note: Enter *r* or compute *r* = sqrt(*I*/*A*)]

[Always enter Area]

[Enter zero for *I* or *r* if not used]

*Area, A =* 1.075 *in*$^2$

*Moment of Inertia, I =* 0 *in*$^4$

*Or*

*Radius of Gyration, r =* 0.787 *in*

**Design Factor**

*Design factor on load, N =* 3

**Computed Values:**

**NOTE: Analysis of straight pipe. See also Solution 40B for crooked pipe.**

Eq. Length, $L_e = KL$ = 156.0 in

Column const., $C_c$ = 128.3

Slender. ratio, KL/r = 198.2

Column is: ***long***

**Critical Buckling Load = *8,101 lb***

**Straight Pipe**

**Allowable Load = *2,700 lb***

**See also Solution 40B for crooked pipe.**

# CROOKED COLUMN ANALYSIS

Solves Equation 6-11 for Allowable Load

Enter data for variables in *italics in shaded boxes*

*Data from:* Problem 40B-Crooked

Use consistent U.S. Customary units.

| Data To Be Entered: | | Computed Values: | |
|---|---|---|---|
| **Length and End Fixity:** | | | |
| *Column length, L =* | *156 in* | | |
| *End fixity, K =* | *1* | Eq. Length, $L_e = KL$ = | 156.0 in |
| **Material Properties:** | | | |
| *Yield strength, $s_y$ =* | *36000 psi* | | |
| *Modulus of Elasticity, E =* | *3.00E+07 psi* | Column const., $C_c$ = | 128.3 |
| **Cross Section Properties:** | | Euler buckling load = | 8101 lb |
| [ Note: Enter *r* or compute *r* = sqrt(*I/A*) ] | | | |
| [Always enter Area] | | $C_1$ in Eqn. 6-11 = | -22074 |
| [Enter zero for *I or* r *if not used*] | | $C_2$ in Eqn. 6-11 = | 3.483E+07 |
| *Area, A =* | *1.075 in²* | | |
| *Moment of Inertia, I =* | *0 in⁴* | | |
| *Radius of Gyration, r =* | *0.787 in* | Slender. ratio, KL/r = | 198.2 |
| **Values for Eqn. 6-11:** | | | |
| *Initial crookedness = a =* | *1.25 in* | Column is: | ***long*** |
| *Neutral axis to outside = c =* | *1.188 in* | ***Straight Column*** | |
| | | **Critical Buckling Load =** | ***8,101 lb*** |
| **Design Factor** | | ***Crooked Column*** | |
| *Design factor on load, N =* | *3* | **Allowable Load =** | ***1,711 lb*** |

**This value governs the use of the pipe.**

**See solution for straight pipe; Problem 40A.**

# CHAPTER 7
# BELT DRIVES AND CHAIN DRIVES

V-BELTS

**1.** $C \leq 24.0$ IN; $D_2 = 13.95$ IN; $D_1 = 5.25$ IN; 3V BELT

EQ. 7-3:

$$L = 2(24) + 1.57(13.95 + 5.25) + \frac{(13.95 - 5.25)^2}{4(24)} = 78.23 \text{ IN}$$

USE $\underline{L = 75 \text{ IN}}$ – STANDARD LENGTH

**2.** ACTUAL C FROM EQ. 7-4: $L = 75$

$$B = 4(75) - 6.28(13.95 + 5.25) = 179.4$$

$$C = \frac{179.4 + \sqrt{(179.4)^2 - 32(13.95 - 5.25)^2}}{16} = 22.00 \text{ IN}$$

**3.** $$\theta_1 = 180° - 2\sin^{-1}\left[\frac{13.95 - 5.25}{2(22.0)}\right] = 157.2° \quad \text{(EQ. 7-5)}$$

$$\theta_2 = 180° + 2\sin^{-1}\left[\frac{13.95 - 5.25}{2(22.0)}\right] = 202.8° \quad \text{(EQ. 7-6)}$$

**4.** $C \leq 60.0$ IN; $D_2 = 27.7$ in; $D_1 = 8.4$ IN; 5V BELT (EQ. 7-3)

COMPUTED $L = 178.2$ IN. USE $L = 170$ IN

**5.** ACTUAL $C = 55.83$ IN. (EQ. 7-4)

**6.** $\theta_1 = 160.1°$; $\theta_2 = 199.9°$ (EQ 7-5), (EQ 7-6)

**7.** $C \leq 144$ IN.; $D_2 = 94.8$ IN; $D_1 = 13.8$ IN; 8V BELT (EQ. 7-3)

COMPUTED $L = 469.9$ IN. USE $L = 450$ IN.

**8.** ACTUAL $C = 133.61$ IN (EQ. 7-4)

**9.** $\theta_1 = 144.7°$; $\theta_2 = 215.3°$ (EQNS. 7-5, 7-6)

**10.** $$v_b = R_1 \omega_1 = \frac{5.25 \text{ IN}}{2} \times \frac{1750 \text{ REV}}{\text{MIN}} \times \frac{2\pi \text{ RAD}}{\text{REV}} \times \frac{1 \text{ FT}}{12 \text{ IN}} = 2405 \text{ FT/MIN}$$

**11.** $$v_b = R_1 \omega_1 = \frac{8.4 \text{ IN}}{2} \times \frac{1160 \text{ REV}}{\text{MIN}} \times \frac{2\pi \text{ RAD}}{\text{REV}} \times \frac{1 \text{ FT}}{12 \text{ IN}} = 2551 \text{ FT/MIN}$$

**12.** $$v_b = R_1 \omega_1 = \frac{13.8 \text{ IN}}{2} \times \frac{870 \text{ REV}}{\text{MIN}} \times \frac{2\pi \text{ RAD}}{\text{REV}} \times \frac{1 \text{ FT}}{12 \text{ IN}} = 3143 \text{ FT/MIN}$$

**13.** RATIO $= 13.95/5.25 = 2.66$; $P \cong 6.25$ hp; $C_\theta = 0.94$; $C_L = 1.03$

CORRECTED POWER $= C_\theta C_L P = (.94)(1.03)(6.25) = 6.05$ hp

## V-BELTS

**14.** RATIO = 27.7/8.4 = 3.30 ; $P = 15.5 + 1.26 = 16.76$ hp ; $C_\theta = .95$ ; $C_L = 1.05$
CORRECTED POWER = $C_\theta C_L P = (.95)(1.05)(16.76) = 16.72$ hp

**15.** RATIO = 94.8/13.8 = 6.87 ; $P = 48$ hp ; $C_\theta = .904$ ; $C_L = 1.09$
CORRECTED POWER = $C_\theta C_L P = (.904)(1.09)(48) = 47.3$ hp

**16.** A <u>15N</u> BELT IS A METRIC SIZE HAVING A TOP WIDTH OF 15 mm SIMILAR TO A 5V BELT.

**17.** A <u>17A</u> BELT IS A METRIC AUTOMOTIVE BELT HAVING A TOP WIDTH OF 17 mm. SIMILAR TO A "3/4 IN." AUTOMOTIVE BELT

**18.** <u>DESIGN:</u> SERVICE FACTOR = 1.5 ; DESIGN POWER = 1.5(25) = 37.5 hp
FROM FIG. 7-9 — USE 5V BELT
RATIO = 870/310 = 2.81
FOR $v_b \approx 4000$ FT/MIN ; $D_1 = \frac{12(4000)}{\pi(870)} = 17.56$ IN
FOR $D_1 = 13.1$ IN ; $D_2 = 37.4$ IN : $n_G = 870 \times 13.1/37.4 = 305$ RPM <u>OK</u>
RATED POWER = 22.5 + .94 = 23.44 hp
CENTER DISTANCE:

$D2 < C < 3(D_2 + D_1)$

$37.4 < C < 3(37.4 + 13.1) = 151.5$

TRY $C \approx 48$ IN. → NOMINAL $L = 178$ IN EQ.(7-3)
USE $L = 180$ IN. → ACTUAL $C = 48.85$ IN (EQ. 7-4)
$\theta_1 = 151.2°$ ; $\theta_2 = 208.8°$ (EQS. 7-5, 7-6)
$C_\theta = 0.92$ ; $C_L = 1.06$ : CORR. POWER = $(.92)(1.06)(23.44) = 22.86$ hp/BELT
No. OF BELTS = 37.5 hp / 22.86 hp/BELT = 1.64 BELTS → 2 BELTS

**19.** <u>DESIGN:</u> S.F. = 1.2 ; DES. POWER = 1.2(5) = 6.0 hp → 3V BELT
RATIO = 1750/725 = 2.41 ; $D_1 \approx 12(4000)/\pi(1750) = 8.7$ IN
FOR $D_1 = 7.95$ ; $D_2 = 18.95$ ; $n_2 = 1750 \; 7.95/18.95 = 734$ RPM OK
RATED POWER ≅ 10.3 hp : $18.95 < C < 80.7$ ; TRY $C \approx 30$ IN.
$L \approx 103$ IN → USE $L = 100$ IN ; ACTUAL $C = 28.35$ IN
$\theta_1 = 157.6°$ ; $\theta_2 = 202.4°$ ; $C_\theta = .94$ ; $CL = 1.09$
CORR. POWER = $(0.94)(1.09)(10.3) = 10.55$ hp/BELT — ONE BELT REQ'D

V-BELTS

**20.** DESIGN: S.F. = 1.4 ; DES. POWER = 1.4(40) = 56 hp → 5V BELT
RATIO = 1500/550 = 2.73 ; $D_1 \approx 12(4000)/\pi(1500) = 10.2$ IN.
FOR $D_1 = 10.2$ IN ; $D_2 = 27.7$ IN ; $n_2 = 1500 \times 10.2/27.7 = 552$ RPM OK
RATED POWER ≅ 25.7 hp BY INTERPOLATION ON FIG 13-10 AT 1500 RPM
$27.7 < C < 114$ ; USE $C \approx 36$ IN. ; $L \approx 133.6$ IN. → USE $L = 132$ IN.
ACTUAL $C = 35.16$ IN. ; $\theta_1 = 151.2°$ ; $\theta_2 = 208.8°$ ; $C_\theta = .92$ ; $C_L = 1.01$
CORR. POWER = (.92)(1.01)(25.7) = 23.9 hp/BELT
NO. OF BELTS = 56/23.9 = 2.35 → USE 3 BELTS

**21.** DESIGN: S.F. = 1.4 ; DES. POWER = 1.4(20) = 28 hp → 3V BELT
RATIO = 1250/695 = 1.80 ; $D_1 \approx 12(4000)/\pi(1250) = 12.2$ IN
FOR $D_1 = 10.55$ IN ; $D_2 = 18.95$ IN ; $n_2 = 1250 \times 10.55/18.95 = 695.9$ RPM OK
RATED POWER ≈ 10.4 hp BY INTERPOLATION ON FIG. 13-9 AT 1250 RPM
$18.95 < C < 88.5$ ; USE $C \approx 20$ IN ; $L \approx 87.2$ IN → USE $L = 90$ IN.
ACTUAL $C = 21.43$ IN ; $\theta_1 = 157.4°$ ; $\theta_2 = 202.6°$ ; $C_\theta = .94$ ; $C_L = 1.07$
CORR. POWER = (.94)(1.07)(10.4) = 10.46 hp/BELT
NO. OF BELTS = 28/10.46 = 2.68 BELTS → USE 3 BELTS

**22.** DESIGN: S.F. = 2.0 (CHOKING) ; DES. P = 2.0(100) = 200 hp → 5V BELT
RATIO = 870/625 = 1.39 ; $D_1 \approx 12(4000)/\pi(870) = 17.6$ IN
FOR $D_1 = 10.8$ IN ; $D_2 = 14.9$ IN ; $n_2 = 870 \times 10.8/14.9 = 631$ RPM OK
RATED POWER = 17.6 + .77 = 18.37 hp ; $14.9 < C < 77.1$ ; USE $C \approx 48$ IN
$L \approx 136$ IN → USE $L = 132$ IN ; ACTUAL $C = 45.78$ IN
$\theta_1 = 174.9°$ ; $\theta_2 = 185.1°$ ; $C_\theta = .988$ ; $C_L = 1.01$
CORR. POWER = (.988)(1.01)(18.37) = 18.33 hp/BELT
NO. OF BELTS = 200/18.33 = 10.9 → USE 11 BELTS, NOT ACCEPTABLE

TRY 8V BELT: FOR $D_1 = 17.8$ IN ; $D_2 = 24.8$ IN ; $n_2 = 624.4$ RPM OK
RATED POWER = 66 hp ; $24.8 < C < 127.8$ ; USE $C \approx 48$ IN
$L \approx 163$ IN → USE $L = 160$ IN ; ACTUAL $C = 46.43$ IN
$\theta_1 = 171.4°$ ; $\theta_2 = 188.6°$ ; $C_\theta = .98$ ; $C_L = .94$
CORR. POWER = (.98)(.94)(66) = 60.8 hp/BELT
NO. OF BELTS = 200/60.8 = 3.3 → USE 4 BELTS

ROLLER CHAIN

23. CHAIN NO. 140: PITCH = 14/8 = $1\frac{3}{4}$ IN.

24. CHAIN NO. 60: PITCH = 6/8 = 3/4 IN

25. STATIC LOAD = 1250 LB: AVG. TENSILE STRENGTH = 10W = 12500 LB.
USE A NO. 80 CHAIN (1.00 IN. PITCH); T.S. = 14500, TABLE 7-5

26. LOAD ON EACH CHAIN = 2500 LB; T.S. = 10W = 25000 LB
USE NO. 120 CHAIN $1\frac{1}{2}$ IN. PITCH; T.S = 34000 LB

27. FATIGUE OF LINK PLATES; IMPACT OF ROLLERS ON SPROCKET TEETH; GALLING BETWEEN PINS AND BUSHINGS.

28. TABLE 7-8: GIVEN NO. 60 CHAIN, 20 TEETH, 750 RPM
RATED POWER = 21.69 HP (INTERPOLATION), TYPE B LUBE (BATH)
SERVICE FACTOR = 1.2 FOR HYDRAULIC DRIVE
DESIGN POWER RATING = 21.69/1.2 = 18.08 HP.

29. 3 STRANDS: FACTOR = 2.5
POWER RATING = 2.5(18.08) = 45.2 HP

30. TABLE 7-7: GIVEN NO. 40 CHAIN, 12 TEETH, 860 RPM
RATED POWER = 4.44 HP (INTERPOLATION), TYPE B LUBE (BATH)
S.F. = 1.2: DESIGN POWER RATING = 4.44/1.2 = 3.70 HP.

31. 4 STRANDS: FACTOR = 3.3; POWER RATING = 3.3(3.70) = 12.21 HP

32. TABLE 7-9; GIVEN NO. 80 CHAIN; 32 TEETH; 1160 RPM
RATED POWER = 78.69 HP (INTERPOLATION), TYPE C LUBE (OIL STREAM)
SF = 1.2; DESIGN POWER = 78.69/1.2 = 65.57 HP

33. 2 STRANDS: FACTOR = 1.7; POWER RATING = (1.7) 65.57 HP = 111.5 HP

34. NO. 60 CHAIN; $N_1 = 15$; $N_2 = 50$; $C \le 36$ IN - USE EQ. 7-9
$L_p$ = 3/4 IN = 0.75 IN.
C = 36 IN / 0.75 IN/PITCH = 48 PITCHES

$$L = 2(48) + \frac{50+15}{2} + \frac{(50-15)^2}{4\pi^2(48)} = 129.1 \text{ PITCHES}$$

USE 128 PITCHES (EVEN NUMBER); L = 128(0.75) = 96 IN.

$\boxed{35.}$ FOR $L = 128$ PITCHES; C FROM EQ. 7-10

$$C = \frac{1}{4}\left[128 - \frac{50+15}{2} + \sqrt{\left[128 - \frac{50+15}{2}\right]^2 - \frac{8(50-15)^2}{4\pi^2}}\right] = 47.42 \text{ PITCHES}$$

$C = 47.42$ PITCHES $\times .75$ IN/PITCH $= 35.57$ IN.

$\boxed{36.}$ NO. 40 CHAIN: $N_1 = 11$; $N_2 = 45$; $C \leq 24$ IN.

$p = 1/2$ IN. $= 0.50$ IN. : $C = 24/0.5 = 48$ PITCHES

$$L = 2(48) + \frac{45+11}{2} + \frac{(45-11)^2}{4\pi^2(48)} = 124.6 \text{ PITCHES} \rightarrow \text{USE } 124 \text{ PITCHES}$$

$124(0.5) = 62$ IN.

$\boxed{37.}$

$$C = \frac{1}{4}\left[124 - \frac{45+11}{2} + \sqrt{\left[124 - \frac{45+11}{2}\right]^2 - \frac{8(45-11)^2}{4\pi^2}}\right] = 47.69 \text{ PITCHES}$$

$C = 47.69$ PITCHES $\times 0.5$ IN/PITCH $= 23.85$ IN.

$\boxed{38.}$ DESIGN: 25 HP; $n_1 = 310$ RPM; $n_2 = 160$ RPM; NOM. RATIO $= 310/160 = 1.94$

SF $= 1.5$; DESIGN POWER $= 1.5(25) = 37.5$ hP

USE 3 STRANDS : RATING $= 37.5/2.5 = 15.0$ hP PER STRAND

NO. 80 CHAIN; 15 TEETH RATED $> 15.8$ hP AT 310 RPM; TYPE B LUBE

$N_2 = N_1 \times \text{RATIO} = 15(1.94) = 29.1 \rightarrow 29$ TEETH

$n_2 = n_1 \times N_1/N_2 = 310 \times 15/29 = 160.3$ RPM OK

$D_1 = 1.00/\sin(180/15) = 4.810$ IN.; $D_2 = 1.00/\sin(180/29) = 9.249$ IN.

USE $C \approx 40$ PITCHES WITH $P = 1.00$ IN FOR NO. 80 CHAIN

$$L = 2(40) + \frac{29+15}{2} + \frac{(29-15)^2}{4\pi^2(40)} = 102.1 \text{ PITCHES} \rightarrow \text{USE } 102 \text{ PITCHES} = L$$

$$C = \frac{1}{4}\left[102 - \frac{29+15}{2} + \sqrt{\left[102 - \frac{29+15}{2}\right]^2 - \frac{8(29-15)^2}{4\pi^2}}\right] = 39.94 \text{ PITCHES} = C$$

$C = 39.94$ IN.

Problems 38-42 are design problems for chain drives for which there are no unique solutions. The general procedure is illustrated above for one possible solution for Problem 38. This and the other design problems are shown on the following pages using the spreadsheet that is available from the publisher's website for this book. Data for design power per strand from Tables 7-7, 7-8, or 7-9 must be used to ensure that the selected chain design has sufficient capacity.

## CHAIN DRIVE DESIGN

| | | |
|---|---|---|
| **Initial Input Data:** | Problem 38 - Multiple strands | |
| Application: | Hammer Mill | |
| Drive type: | Electric motor | |
| Driven machine | Hammer Mill | |
| Power input: | 25 hp | |
| Service factor: | 1.5 | Table 7-1 |
| Input speed: | 310 rpm | |
| Desired output speed: | 160 rpm | |
| **Computed Data:** | | |
| Design power: | 37.5 hp | |
| Speed ratio: | 1.94 | |
| **Design Decisions-Chain Type and Teeth Numbers:** | | |
| Number of strands: | 3 | 1 2 3 4 |
| Strand factor: | 2.5 | 1.0 1.7 2.5 3.3 |
| Required power per strand: | 15.00 hp | |
| Chain number: | 80 | Tables 7-7, 7-8 or 7-9 |
| Chain pitch: | 1 in | |
| Nunber of teeth-Driver sprocket: | 15 | |
| Computed no. of teeth-Driven sprocket: | 29.06 | |
| Enter: Chosen number of teeth: | 29 | |
| **Computed Data:** | | |
| Actual output speed: | 160.3 rpm | |
| Pitch diameter-Driver sprocket: | 4.810 in | |
| Pitch diameter-Driven sprocket: | 9.249 in | |
| **Center Distance, Chain Length and Angle of Wrap:** | | |
| Enter: Nominal center distance: | 40 pitches | 30 to 50 pitches recommended |
| Computed nominal chain length: | 102.1 pitches | |
| Enter: Specified no. of pitches: | 102 pitches | Even number recommended |
| Actual chain length: | 102.00 in | |
| Computed actual center distance: | 39.938 pitches | |
| Actual center distance: | 39.938 in | |
| Angle of wrap-Driver sprocket: | 173.6 degrees | Should be greater than 120 degrees |
| Angle of wrap-Driven sprocket: | 186.4 degrees | |

| CHAIN DRIVE DESIGN | | |
|---|---|---|
| **Initial Input Data:** | *Problem 39 - Single strand* | |
| Application: | *Agitator* | |
| Drive type: | *Electric motor* | |
| Driven machine | *Agitator* | |
| Power input: | *5* hp | |
| Service factor: | *1* | Table 7-1 |
| Input speed: | *750* rpm | |
| Desired output speed: | *325* rpm | |
| **Computed Data:** | | |
| Design power: | 5 hp | |
| Speed ratio: | 2.31 | |
| **Design Decisions-Chain Type and Teeth Numbers:** | | |
| Number of strands: | *1* | 1 2 3 4 |
| Strand factor: | *1.0* | 1.0 1.7 2.5 3.3 |
| Required power per strand: | 5.00 hp | |
| Chain number: | *40* | Tables 7-7, 7-8 or 7-9 |
| Chain pitch: | *0.5* in | |
| Nunber of teeth-Driver sprocket: | *16* | |
| Computed no. of teeth-Driven sprocket: | 36.92 | |
| *Enter: Chosen number of teeth:* | *37* | |
| **Computed Data:** | | |
| Actual output speed: | 324.3 rpm | |
| Pitch diameter-Driver sprocket: | 2.563 in | |
| Pitch diameter-Driven sprocket: | 5.896 in | |
| **Center Distance, Chain Length and Angle of Wrap:** | | |
| *Enter: Nominal center distance:* | *32* pitches | 30 to 50 pitches recommended |
| Computed nominal chain length: | 90.8 pitches | |
| *Enter: Specified no. of pitches:* | *90* pitches | Even number recommended |
| Actual chain length: | 45.00 in | |
| Computed actual center distance: | 31.573 pitches | |
| Actual center distance: | 15.787 in | |
| Angle of wrap-Driver sprocket: | 167.9 degrees | Should be greater than 120 degrees |
| Angle of wrap-Driven sprocket: | 192.1 degrees | |

| CHAIN DRIVE DESIGN | | | |
|---|---|---|---|
| **Initial Input Data:** | Problem 40 - Multiple strands | | |
| Application: | Conveyor | | |
| Drive type: | Engine | | |
| Driven machine: | Heavy conveyor | | |
| Power input: | 40 | hp | |
| Service factor: | 1.4 | | Table 7-1 |
| Input speed: | 500 | rpm | |
| Desired output speed: | 250 | rpm | |
| **Computed Data:** | | | |
| Design power: | 56 | hp | |
| Speed ratio: | 2.00 | | |
| **Design Decisions-Chain Type and Teeth Numbers:** | | | |
| Number of strands: | 3 | | 1 2 3 4 |
| Strand factor: | 2.5 | | 1.0 1.7 2.5 3.3 |
| Required power per strand: | 22.40 | hp | |
| Chain number: | 80 | | Tables 7-7, 7-8 or 7-9 |
| Chain pitch: | 1.00 | in | |
| Nunber of teeth-Driver sprocket: | 14 | | |
| Computed no. of teeth-Driven sprocket: | 28.00 | | |
| Enter: Chosen number of teeth: | 28 | | |
| **Computed Data:** | | | |
| Actual output speed: | 250.0 | rpm | |
| Pitch diameter-Driver sprocket: | 4.494 | in | |
| Pitch diameter-Driven sprocket: | 8.931 | in | |
| **Center Distance, Chain Length and Angle of Wrap:** | | | |
| Enter: Nominal center distance: | 36 | pitches | 30 to 50 pitches recommended |
| Computed nominal chain length: | 93.1 | pitches | |
| Enter: Specified no. of pitches: | 94 | pitches | Even number recommended |
| Actual chain length: | 94.00 | in | |
| Computed actual center distance: | 36.432 | pitches | |
| Actual center distance: | 36.432 | in | |
| Angle of wrap-Driver sprocket: | 173.0 | degrees | Should be greater than 120 degrees |
| Angle of wrap-Driven sprocket: | 187.0 | degrees | |

| CHAIN DRIVE DESIGN | | |
|---|---|---|
| **Initial Input Data:** | Problem 41 - Multiple strands | |
| Application: | Pump drive | |
| Drive type: | Steam turbine | |
| Driven machine: | Centrifugal pump | |
| Power input: | 20 hp | |
| Service factor: | 1 | Table 7-1 |
| Input speed: | 2200 rpm | |
| Desired output speed: | 775 rpm | |
| **Computed Data:** | | |
| Design power: | 20 hp | |
| Speed ratio: | 2.84 | |
| **Design Decisions-Chain Type and Teeth Numbers:** | | |
| Number of strands: | 2 | 1 2 3 4 |
| Strand factor: | 1.7 | 1.0 1.7 2.5 3.3 |
| Required power per strand: | 11.76 hp | |
| Chain number: | 40 | Tables 7-7 7-8 or 7-9 |
| Chain pitch: | 0.50 in | |
| Number of teeth-Driver sprocket: | 25 | 11.95 hp rating at 2200 rpm |
| Computed no. of teeth-Driven sprocket: | 70.97 | |
| Enter: Chosen number of teeth: | 71 | Check availability from vendor |
| **Computed Data:** | | |
| Actual output speed: | 774.6 rpm | |
| Pitch diameter-Driver sprocket: | 3.989 in | |
| Pitch diameter-Driven sprocket: | 11.304 in | |
| **Center Distance, Chain Length and Angle of Wrap:** | | |
| Enter: Nominal center distance: | 30 pitches | 30 to 50 pitches recommended |
| Computed nominal chain length: | 109.8 pitches | |
| Enter: Specified no. of pitches: | 110 pitches | Even number recommended |
| Actual chain length: | 55.00 in | |
| Computed actual center distance: | 30.110 pitches | |
| Actual center distance: | 15.055 in | |
| Angle of wrap-Driver sprocket: | 151.9 degrees | Should be greater than 120 degrees |
| Angle of wrap-Driven sprocket: | 208.1 degrees | |

| CHAIN DRIVE DESIGN | | |
|---|---|---|
| **Initial Input Data:** | *Problem 42 - Multiple strands* | |
| Application: | *Rock Crusher* | |
| Drive type: | *Hydraulic drive* | |
| Driven machine | *Rock Crusher* | |
| Power input: | *100* hp | |
| Service factor: | *1.4* | Table 7-1 |
| Input speed: | *625* rpm | |
| Desired output speed: | *225* rpm | |
| **Computed Data:** | | |
| Design power: | 140 hp | |
| Speed ratio: | 2.78 | |
| **Design Decisions-Chain Type and Teeth Numbers:** | | |
| Number of strands: | *4* | 1 2 3 4 |
| Strand factor: | *3.3* | 1.0 1.7 2.5 3.3 |
| Required power per strand: | 42.42 hp | |
| Chain number: | *80* | Tables 7-7, 7-8 or 7-9 |
| Chain pitch: | *1.00* in | |
| Nunber of teeth-Driver sprocket: | *21* | >42.94 hp per strand |
| Computed no. of teeth-Driven sprocket: | 58.33 | |
| *Enter: Chosen number of teeth:* | *58* | Check availability from vendor |
| **Computed Data:** | | |
| Actual output speed: | 226.3 rpm | |
| Pitch diameter-Driver sprocket: | 6.710 in | |
| Pitch diameter-Driven sprocket: | 18.471 in | |
| **Center Distance, Chain Length and Angle of Wrap:** | | |
| *Enter: Nominal center distance:* | *40* pitches | 30 to 50 pitches recommended |
| Computed nominal chain length: | 120.4 pitches | |
| *Enter: Specified no. of pitches:* | *120* pitches | Even number recommended |
| Actual chain length: | 120.00 in | |
| Computed actual center distance: | 39.815 pitches | |
| Actual center distance: | 39.815 in | |
| Angle of wrap-Driver sprocket: | 163.0 degrees | Should be greater than 120 degrees |
| Angle of wrap-Driven sprocket: | 197.0 degrees | |

# CHAPTER 8
# KINEMATICS OF GEARS

## Gear Geometry

**1.** $N = 44\ ;\ P_d = 12$

a. $D = N/P_d = 44/12 = 3.667$ in.
b. $P_c = \pi/P_d = \pi/12 = 0.2618$ in.
c. $m = 25.4/P_d = 25.4/12 = 2.117$ mm
d. $m = 2.00$ mm
e. $a = 1/P_d = 1/12 = 0.0833$ in.
f. $b = 1.25/P_d = 1.25/12 = 0.1042$ in.
g. $c = 0.25/P_d = 0.25/12 = 0.0208$ in
h. $h_t = a + b = 2.25/P_d = 0.1875$ in
i. $h_k = 2a = 2/P_d = 2/12 = 0.1667$ in.
j. $t = \pi/2P_d = \pi/2(12) = 0.131$ in.
k. $D_o = (N+2)/P_d = 46/12 = 3.833$ in.

**2.** $N = 34\ ;\ P_d = 24$

a. $D = 34/24 = 1.417$ in.
b. $P_c = \pi/24 = 0.131$ in
c. $m = 25.4/24 = 1.058$ mm
d. $m = 1.00$ mm
e. $a = 1/24 = 0.0417$ in.
f. $b = \dfrac{1.200}{P_d} + 0.002 = 0.0520$ in
g. $c = 0.200/P_d + 0.002 = 0.0103$ in
h. $h_t = a + b = 0.0417 + 0.0520 = 0.0937$ in.
i. $h_k = 2a = 2/24 = 0.0833$ in.
j. $t = \pi/2(24) = 0.0654$ in.
k. $D_o = (N+2)/P_d = 36/24 = 1.500$ in.

**3.** $N = 45\ ;\ P_d = 2$

a. $D = 45/2 = 22.500$ in.
b. $P_c = \pi/2 = 1.571$ in.
c. $m = 25.4/2 = 12.7$ mm
d. $m = 12$ mm
e. $a = 1/2 = 0.500$ in
f. $b = 1.25/2 = 0.625$ in.
g. $c = 0.25/2 = 0.125$ in.
h. $h_t = 2.25/2 = 1.125$ in.
i. $h_k = 2/2 = 1.000$ in.
j. $t = \pi/2(2) = 0.7854$ in.
k. $D_o = 47/2 = 23.50$ in.

**4.** $N = 18\ ;\ P_d = 8$

a. $D = 18/8 = 2.250$ in.
b. $P_c = \pi/8 = 0.3927$ in.
c. $m = 25.4/8 = 3.175$ mm
d. $m = 3.0$ mm
e. $a = 1/8 = 0.125$ in.
f. $b = 1.25/8 = 0.1563$ in.
g. $c = 0.25/8 = 0.0313$ in
h. $h_t = 2.25/8 = 0.2813$ in
i. $h_k = 2/8 = 0.250$ in
j. $t = \pi/2(8) = 0.1963$ in.
k. $D_o = 20/8 = 2.500$ in.

**5.** $N = 22\ ;\ P_d = 1.75$

a. $D = 22/1.75 = 12.571$ in.
b. $P_c = \pi/1.75 = 1.795$ in.
c. $m = 25.4/1.75 = 14.514$ mm
d. $m = 16$ mm
e. $a = 1/1.75 = 0.5714$ in.
f. $b = 1.25/1.75 = 0.7143$ in
g. $c = 0.25/1.75 = 0.1429$ in.
h. $h_t = 2.25/1.75 = 1.2857$ in.
i. $h_k = 2/1.75 = 1.1429$ in.
j. $t = \pi/2(1.75) = 0.8976$ in
k. $D_o = 24/1.75 = 13.714$ in.

**6.** $N = 20;\ P_d = 64$

a. $D = 20/64 = 0.3125$ IN.
b. $P_c = \pi/64 = 0.0491$ IN.
c. $m = 25.4/64 = 0.397$ mm
d. $m = 0.40$ mm
e. $a = 1/64 = 0.0156$ IN.
f. $b = 1.200/64 + 0.002 = 0.0208$ IN.
g. $c = 0.200/64 + 0.002 = 0.0051$ IN.
h. $h_t = a + b = 0.0364$ IN.
i. $h_k = 2/64 = 0.0313$ IN.
j. $t = \pi/2(64) = 0.0245$ IN
k. $D_o = 22/64 = 0.3438$ IN.

**7.** $N = 180;\ P_d = 80$

a. $D = 180/80 = 2.250$ IN.
b. $P_c = \pi/80 = 0.0393$ IN
c. $m = 25.4/80 = 0.318$ mm
d. $m = 0.30$ mm
e. $a = 1/80 = 0.0125$ IN.
f. $b = 1.200/80 + 0.002 = 0.0170$ IN.
g. $c = 0.200/80 + 0.002 = 0.0045$ IN.
h. $h_t = a + b = 0.0295$ IN.
i. $h_k = 2/80 = 0.025$
j. $t = \pi/2(80) = 0.0196$ IN.
k. $D_o = 182/80 = 2.275$ IN.

**8.** $N = 28;\ P_d = 18$

a. $D = 28/18 = 1.556$ IN.
b. $P_c = \pi/18 = 0.1745$ IN.
c. $m = 25.4/18 = 1.411$ mm
d. $m = 1.5$ mm
e. $a = 1/18 = 0.0556$ IN.
f. $b = 1.25/18 = 0.0694$ IN.
g. $c = 0.25/18 = 0.0139$ IN.
h. $h_t = 2.25/18 = 0.125$ IN
i. $h_k = 2/18 = 0.1111$ IN.
j. $t = \pi/2(18) = 0.0873$ IN.
k. $D_o = 30/18 = 1.667$ IN.

**9.** $N = 28;\ P_d = 20$

a. $D = 28/20 = 1.400$ IN.
b. $P_c = \pi/20 = 0.1571$ IN.
c. $m = 25.4/20 = 1.27$ mm
d. $m = 1.25$ mm
e. $a = 1/20 = 0.050$ IN.
f. $b = 1.200/20 + 0.002 = 0.0620$ IN.
g. $c = 0.200/20 + 0.002 = 0.012$ IN.
h. $h_t = a + b = 0.1120$ IN.
i. $h_k = 2/20 = 0.1000$ IN.
j. $t = \pi/2(20) = 0.0785$ IN.
k. $D_o = 30/20 = 1.500$ IN.

**10.** $N = 34;\ m = 3 = D/N$

a. $D = mN = 3(34) = 102$ mm
b. $P_c = \pi D/N = \pi m = \pi(3) = 9.425$ mm
c. $P_d = 25.4/m = 25.4/3 = 8.47$
d. $P_d = 8$
e. $a = m = 3.00$ mm
f. $b = 1.25m = 1.25(3) = 3.750$ mm
g. $c = 0.25m = 0.25(3) = 0.750$ mm
h. $h_t = a + b = 2.25m = 6.750$ mm
i. $h_k = 2a = 2m = 6.00$ mm
j. $t = P_c/2 = \pi m/2 = 4.712$ mm
k. $D_o = m(N+2) = 3(36) = 108$ mm

**11.** $N = 45;\ m = 1.25$

a. $D = mN = 1.25(45) = 56.25\,mm$
b. $P_c = \pi m = \pi(1.25) = 3.927\,mm$
c. $P_d = 25.4/m = 25.4/1.25 = 20.3$
d. $P_d = 20$
e. $a = m = 1.25\,mm$
f. $b = 1.25(m) = 1.25(1.25) = 1.563\,mm$
g. $C = 0.25m = 0.25(1.25) = 0.313\,mm$
h. $h_t = 2.25m = 2.25(1.25) = 2.813\,mm$
i. $h_k = 2m = 2(1.25) = 2.500\,mm$
j. $t = P_c/2 = \pi m/2 = 1.963\,mm$
k. $D_o = m(N+2) = 1.25(47) = 58.75\,mm$

**12.** $N = 18;\ m = 12$

a. $D = mN = 12(18) = 216\,mm$
b. $P_c = \pi(12) = 37.70\,mm$
c. $P_d = 25.4/12 = 2.117$
d. $P_d = 2$
e. $a = m = 12\,mm$
f. $b = 1.25(12) = 15.00\,mm$
g. $C = 0.25(12) = 3.00\,mm$
h. $h_t = 2.25(12) = 27.00\,mm$
i. $h_k = 2m = 24.00\,mm$
j. $t = \pi(12)/2 = 18.85\,mm$
k. $D_o = m(N+2) = 12(20) = 240\,mm$

**13.** $N = 22;\ m = 20$

a. $D = mN = 20(22) = 440\,mm$
b. $P_c = \pi(20) = 62.83\,mm$
c. $P_d = 25.4/20 = 1.27$
d. $P_d = 1.25$
e. $a = m = 20\,mm$
f. $b = 1.25(20) = 25.00\,mm$
g. $C = 0.25(20) = 5.00\,mm$
h. $h_t = 2.25(20) = 45.00\,mm$
i. $h_k = 2m = 2(20) = 40.00\,mm$
j. $t = \pi(20)/2 = 31.42\,mm$
k. $D_o = 20(24) = 480\,mm$

**14.** $N = 20;\ m = 1$

a. $D = mN = 20.00\,mm$
b. $P_c = \pi m = 3.14\,mm$
c. $P_d = 25.4/1 = 25.4$
d. $P_d = 24$
e. $a = m = 1.00\,mm$
f. $b = 1.25(1) = 1.25\,mm$
g. $C = 0.25(1) = 0.25\,mm$
h. $h_t = 2.25(1) = 2.25\,mm$
i. $h_k = 2(1) = 2.00\,mm$
j. $t = \pi(1)/2 = 1.571\,mm$
k. $D_o = (1)(22) = 22.00\,mm$

**15.** $N = 180;\ m = 0.4$

a. $D = 0.4(180) = 72.00\,mm$
b. $P_c = \pi(0.4) = 1.26\,mm$
c. $P_d = 25.4/0.4 = 63.5$
d. $P_d = 64$
e. $a = m = 0.40\,mm$
f. $b = 1.25(0.4) = 0.50\,mm$
g. $C = 0.25(0.4) = 0.10\,mm$
h. $h_t = 2.25(0.4) = 0.900\,mm$
i. $h_k = 2(0.4) = 0.80\,mm$
j. $t = \pi(0.4)/2 = 0.628\,mm$
k. $D_o = 0.4(182) = 72.80\,mm$

**16.** $N = 28;\ m = 1.5$

a. $D = mN = 1.5(28) = 42.00$ mm
b. $p_c = \pi m = \pi(1.5) = 4.71$ mm
c. $P_d = 25.4/1.5 = 16.93$
d. $P_d = 16$
e. $a = m = 1.50$ mm
f. $b = 1.25(1.5) = 1.875$ mm
g. $c = 0.25(1.5) = 0.375$ mm
h. $h_t = 2.25(1.5) = 3.375$ mm
i. $h_k = 2(1.5) = 3.00$ mm
j. $t = \pi(1.5)/2 = 2.36$ mm
k. $D_o = 1.5(30) = 45.00$ mm

**17.** $N = 28;\ m = 0.8$

a. $D = 0.8(28) = 22.40$ mm
b. $p_c = \pi(0.8) = 2.51$ mm
c. $P_d = 25.4/0.8 = 31.75$
d. $P_d = 32$
e. $a = m = 0.80$ mm
f. $b = 1.25(0.8) = 1.00$ mm
g. $c = 0.25(0.8) = 0.20$ mm
h. $h_t = 2.25(0.8) = 1.80$ mm
i. $h_k = 2(0.8) = 1.60$ mm
j. $t = \pi(0.8)/2 = 1.257$ mm
k. $D_o = 0.8(30) = 24.00$ mm

**18.** BACKLASH – SEE P. 389.

**19.** GEAR FROM PROB. 1: $P_d = 12$: BACKLASH 0.006 TO 0.009 IN
GEAR FROM PROB. 12: $m = 12$: BACKLASH 0.52 TO 0.82 mm
DEPENDING ON CENTER DISTANCE.

**Velocity Ratio**

**20.** a. $C = \dfrac{N_P + N_G}{2P_d} = \dfrac{18 + 64}{2(8)} = 5.125$ IN.
b. $VR = N_G/N_P = 64/18 = 3.556$
c. $n_G = n_P(N_P/N_G) = 2450(18/64) = 689$ RPM
d. $v_t = \dfrac{\pi d n_P}{12} = \dfrac{\pi N_P n_P}{12 P_d} = \dfrac{\pi(18)(2450)}{12(8)} = 1443$ ft/min

**21.** a. $C = (20 + 92)/2(4) = 14.000$ IN.
b. $VR = 92/20 = 4.60$
c. $n_G = 225(20/92) = 48.9$ RPM
d. $v_t = \dfrac{\pi(20)(225)}{12(4)} = 294.5$ ft/min

**22.** a. $C = (30 + 68)/2(20) = 2.450$ IN.
b. $VR = 68/30 = 2.267$
c. $n_G = 850(30/68) = 375$ RPM
d. $v_t = \dfrac{\pi(30)(850)}{12(20)} = 334$ ft/min

**23.** a. $C = (40 + 250)/2(64) = 2.266$ IN.
b. $VR = 250/40 = 6.25$
c. $n_G = 3450(40/250) = 552$ RPM
d. $v_t = \dfrac{\pi(40)\,3450}{12(64)} = 565$ ft/min

**24.** a. $C = (24+88)/2(12) = 4.667\text{ IN.}$

b. $VR = 88/24 = 3.667$

c. $n_G = 1750(24/88) = 477\text{ RPM}$

d. $v_t = \dfrac{\pi(24)(1750)}{12(12)} = 916\text{ ft/min}$

**25.** a. $C = (N_G + N_P)m/2 = (68+22)(2)/2 = 90.00\text{ mm}$

b. $VR = N_G/N_P = 68/22 = 3.091$

c. $n_G = n_P \cdot N_P/N_G = 1750(22/68) = 566\text{ RPM}$

d. $v_t = R\omega = \dfrac{D\omega}{2} = \left[\dfrac{mN_P}{2}\ \dfrac{n_P}{1}\right]\dfrac{(\text{mm})(\text{rev})}{\text{min}} \times \dfrac{2\pi\text{ RAD}}{\text{rev}} \times \dfrac{1\text{ min}}{60\text{ s}} \times \dfrac{\text{m}}{10^3\text{ mm}}$

$$v_t = \frac{m\,N_P\,n_P}{19099}\ \text{m/s} = \frac{(2)(22)(1750)}{19099} = 4.03\text{ m/s}$$

**26.** a. $C = (48+18)(0.8)/2 = 26.40\text{ mm}$

b. $VR = 48/18 = 2.667$

c. $n_G = 1150(18/48) = 431\text{ RPM}$

d. $v_t = \dfrac{(0.8)(18)(1150)}{19099}$

$v_t = 0.867\text{ m/s}$

**27.** a. $C = (45+36)(4)/2 = 162\text{ mm}$

b. $VR = 45/36 = 1.25$

c. $n_G = 150(36/45) = 120\text{ RPM}$

d. $v_t = \dfrac{(4)(36)(150)}{19099} = 1.13\text{ m/s}$

**28.** a. $C = (36+15)(12)/2 = 306\text{ mm}$

b. $VR = 36/15 = 2.40$

c. $n_G = 480\ 15/36 = 200\text{ RPM}$

d. $v_t = \dfrac{(12)(15)(480)}{19099} = 4.52\text{ m/s}$

**Errors in statements for Problems 29 - 32**

**29.** PINION AND GEAR CAN'T HAVE DIFFERENT PITCHES

**30.** $C = \dfrac{N_P + N_G}{2P_d} = \dfrac{18+82}{2(6)} = 8.333\text{ IN}$; GIVEN C IS INACCURATE BY 0.033 IN

**31.** TOO FEW TEETH IN THE PINION, ASSUMING 20° F.D. TEETH INTERFERENCE WOULD OCCUR.

**32.** $C = \dfrac{N_P + N_G}{2P_d} = \dfrac{24+45}{2(16)} = 2.156\text{ IN}$: $D_o$ CANNOT BE USED TO FIND C.

**Housing Dimensions**

**33.** HOUSING MUST CLEAR ADDENDUM CIRCLE OF ALL GEARS BY 0.10 IN/SIDE

$a = 1/P_d = 1/8 = 0.125\text{ IN.}$; $D_{OG} = (N_G+2)/P_d = 66/8 = 8.25\text{ IN}$

$Y = 8.25\text{ IN} + 2(0.10) = 8.45\text{ IN.}$

$X = d + D + 2a + 2(0.10) = \dfrac{N_P}{P_d} + \dfrac{N_G}{P_d} + \dfrac{2}{P_d} + 2(0.1) = 2.250 + 8.00 + .25 + .20$

$X = 10.700\text{ IN.}$

$\boxed{34.}$ $D_{OG} = (N_G+2)/P_d = 252/64 = 3.938\,IN$ ; $Y = 3.938 + 0.2 = \underline{4.138\,IN = Y}$

$X = d + D + 2a + 2(0.1) = \frac{40}{64} + \frac{250}{64} + \frac{2}{64} + 0.20 = \underline{4.763\,IN = X}$

$\boxed{35.}$ $D_{OG} = (N_G+2)m = 50(0.8) = 40.0\,mm$ ; $\underline{Y = 40.0 + 2(2mm) = 44.00\,MM}$

$X = d + D + 2a + 2(2) = mN_P + mN_G + 2m + 2(2)$

$X = 0.8(18) + 0.8(48) + 2(0.8) + 4.0 = \underline{58.40\,mm = X}$

$\boxed{36.}$ $D_{OG} = 47(4) = 188\,mm$ ; $\underline{Y = 188 + 2(2) = 192\,mm}$

$X = d + D + 2a + 2(2) = mN_P + mN_G + 2m + 4 =$

$X = 144 + 180 + 8 + 4 = \underline{336\,mm = X}$

**Gear Trains - Analysis**

$\boxed{37.}$ $TV = -\frac{N_B}{N_A} \cdot \frac{N_D}{N_C} \cdot \frac{N_F}{N_E} = \frac{-42}{18} \cdot \frac{54}{18} \cdot \frac{54}{24} = -15.75 = n_{IN}/n_{OUT}$

$n_{OUT} = n_{IN}/TV = 1750\,RPM/(-15.75) = \underline{-111\,RPM\ CCW}$

$\boxed{38.}$ $TV = -\frac{\cancel{N_B}}{N_A} \cdot \frac{N_C}{\cancel{N_B}} \cdot \frac{N_E}{N_D} = \frac{-68}{22} \cdot \frac{68}{25} = -8.407 = n_{IN}/n_{OUT}$

$n_{OUT} = n_{IN}/TV = 1750\,RPM/-8.407 = \underline{-208\,RPM\ CCW}$

$\boxed{39.}$ $TV = +\frac{D_B}{D_A} \cdot \frac{D_D}{D_C} \cdot \frac{D_F}{D_E} \cdot \frac{N_H}{N_G} = \frac{2.875}{1.250} \cdot \frac{2.375}{1.125} \cdot \frac{2.250}{1.500} \cdot \frac{30}{18} = 12.139$

$D_A = N_A/P_d = 20/16 = 1.250\,IN$

$D_D = N_D/P_d = 38/16 = 2.375$

$D_E = N_E/P_d = 18/12 = 1.500$

$n_{OUT} = \frac{n_{IN}}{TV} = \frac{1750}{12.139} = \underline{144\,RPM\ CW}$

$\boxed{40.}$ $TV = +\frac{N_B}{N_A} \cdot \frac{N_D}{N_C} = \frac{24}{80} \cdot \frac{18}{60} = +0.09$

$n_{OUT} = n_{IN}/TV = 1750/0.09 = \underline{19\,444\,RPM\ CW}$

$\boxed{41}$ HELICAL GEAR $P_d = 8$, $\phi_t = 14\frac{1}{2}°$, $N = 45$ TEETH, $F = 2.00$ IN
HELIX ANGLE $= \psi = 30°$.
CIRCULAR PITCH $= p = \pi/P_d = \pi/8 = \underline{0.3927 \text{ IN.}}$
NORMAL CIRCULAR PITCH $= p_n = p\cos\psi = (0.3927)\cos(30°) = \underline{0.3401 \text{ IN.}}$
NORMAL DIAMETRAL PITCH $= P_{nd} = P_d/\cos\psi = 8/\cos(30°) = \underline{9.238}$
AXIAL PITCH $= P_x = \dfrac{\pi}{P_d \tan\psi} = \dfrac{\pi}{8\tan(30°)} = \underline{0.680 \text{ IN}}$
PITCH DIAMETER $= D_G = N/P_d = 45/8 = \underline{5.625 \text{ IN.}}$
NORMAL PRESSURE ANGLE $= \phi_n = \text{Tan}^{-1}[\tan\phi_t \cos\psi]$
$\phi_n = \text{Tan}^{-1}[\tan(14.5°)\cos(30°)] = \underline{12.62°}$
$F/P_x = 2.00 \text{ IN}/0.680 \text{ IN} = \underline{2.94 \text{ AXIAL PITCHES IN FACEWIDTH}}$

$\boxed{42}$ HELICAL GEAR $N = 48$, $P_{nd} = 12$, $\phi_n = 20°$; $F = 1.50$ IN, $\psi = 45°$.
$p = \pi/P_d$ — BUT $P_d = P_{nd}\cos\psi = 12 \cdot \cos(45°) = \underline{8.485}$
$p = \pi/8.485 = \underline{0.370 \text{ IN}}$ ; $p_n = p\cos\psi = \pi/P_{nd} = \pi/12 = \underline{0.2618 \text{ IN.}}$
$P_x = p/\tan\psi = \dfrac{0.370 \text{ IN}}{\tan\psi} = \underline{0.370 \text{ IN}}$ ; $D_G = N/P_d = 48/8.485 = \underline{5.657 \text{ IN}}$
$\phi_t = \text{Tan}^{-1}\left[\dfrac{\tan\phi_n}{\cos\psi}\right] = \text{Tan}^{-1}\left[\dfrac{\tan 20°}{\cos 45°}\right] = \underline{27.2°}$
$F/P_x = 1.500 \text{ IN}/0.370 \text{ IN} = \underline{4.05 \text{ AXIAL PITCHES IN FACE WIDTH}}$

$\boxed{43}$ HELICAL GEAR $N = 36$, $P_d = 6$, $\phi_t = 14\frac{1}{2}°$, $\psi = 45°$; $F = 1.00$ IN
$p = \pi/P_d = \pi/6 = \underline{0.5236 \text{ IN.}}$ ; $p_n = p\cos\psi = \frac{\pi}{6}\cdot\cos(45°) = \underline{0.370 \text{ IN.}}$
$P_{nd} = P_d/\cos\psi = 6/\cos 45° = \underline{8.485}$ ; $P_x = \dfrac{\pi}{P_d\tan\psi} = \dfrac{\pi}{6\cdot\tan 45°} = \underline{0.5236 \text{ IN.}}$
$D = N/P_d = 36/6 = \underline{6.000 \text{ IN.}}$ ; $\phi_n = \text{Tan}^{-1}[\tan\phi_t\cos\psi] = \underline{10.36°}$
$F/P_x = 1.00 \text{ IN}/0.5236 \text{ IN} = \underline{1.91 \text{ AXIAL PITCHES IN FACE WIDTH}}$ (LOW)

$\boxed{44}$ HELICAL GEAR $N = 72$; $P_{nd} = 24$; $\phi_n = 14\frac{1}{2}°$; $F = 0.25$ IN, $\psi = 45°$.
$p = \pi/P_d$, BUT $P_d = P_{nd}\cos\psi = 24\cos 45° = \underline{16.97}$
$p = \pi/16.97 = \underline{0.1851 \text{ IN.}}$ ; $p_n = p\cos\psi = 0.1851 \text{ IN}\cdot\cos 45° = \underline{0.1309 \text{ IN.}}$
$P_x = p/\tan\psi = 0.1851/\tan 45° = \underline{0.1851 \text{ IN.}}$ ; $D_G = N/P_d = 72/16.97 = \underline{4.243 \text{ IN.}}$
$\phi_t = \text{Tan}^{-1}\left[\dfrac{\tan\phi_n}{\cos\psi}\right] = \text{Tan}^{-1}\left[\dfrac{\tan(14.5°)}{\cos 45°}\right] = \underline{20.0°}$ ; $F/P_x = 0.25/0.185 = \underline{1.35}$
LOW

SEE PROBLEM 49 ON NEXT PAGE FOR FORMULAS AND SYMBOLS

## BEVEL GEAR GEOMETRY

| PROBLEM: 45 | | | EXAMPLE PROBLEM: 47 | | |
|---|---|---|---|---|---|
| **GIVEN DATA** | | | **GIVEN DATA** | | |
| No of teeth in pinion | 15 | | No of teeth in pinion | 25 | |
| No of teeth in gear | 45 | | No of teeth in gear | 50 | |
| Diametral pitch | 6 | | Diametral pitch | 10 | |
| Pressure angle | 20 | degrees | Pressure angle | 20 | degrees |
| **COMPUTED VALUES** | | | **COMPUTED VALUES** | | |
| Gear ratio | 3.000 | | Gear ratio | 2.000 | |
| Pitch diameter: Pinion | 2.500 | in | Pitch diameter: Pinion | 2.500 | in |
| Pitch diameter: Gear | 7.500 | in | Pitch diameter: Gear | 5.000 | in |
| Pitch cone angle: Pinion | 18.435 | degrees | Pitch cone angle: Pinion | 26.565 | degrees |
| Pitch cone angle: Gear | 71.565 | degrees | Pitch cone angle: Gear | 63.435 | degrees |
| Outer cone distance | 3.953 | in | Outer cone distance | 2.795 | in |
| Nominal face width | 1.186 | in | Nominal face width | 0.839 | in |
| Maximum face width (a) | 1.318 | in | Maximum face width (a) | 0.932 | in |
| Maximum face width (b) | 1.667 | in | Maximum face width (b) | 1.000 | in |
| INPUT Face width | 1.250 | in | INPUT Face width | 0.900 | in |
| Mean cone distance | 3.328 | in | Mean cone distance | 2.345 | in |
| Ratio $A_m/A_o$ | 0.842 | | Ratio $A_m/A_o$ | 0.839 | |
| Mean circular pitch | 0.441 | in | Mean circular pitch | 0.264 | in |
| mean working depth | 0.281 | in | mean working depth | 0.168 | in |
| Clearance | 0.035 | in | Clearance | 0.021 | in |
| Mean whole depth | 0.316 | in | Mean whole depth | 0.189 | in |
| mean addendum factor | 0.242 | | mean addendum factor | 0.283 | |
| Gear mean addendum | 0.068 | in | Gear mean addendum | 0.047 | in |
| Pinion mean addendum | 0.213 | in | Pinion mean addendum | 0.120 | in |
| Gear mean dedendum | 0.248 | in | Gear mean dedendum | 0.141 | in |
| Pinion mean dedendum | 0.103 | in | Pinion mean dedendum | 0.068 | in |
| Gear dedendum angle | 4.257 | degrees | Gear dedendum angle | 3.450 | degrees |
| Pinion dedendum angle | 1.774 | degrees | Pinion dedendum angle | 1.670 | degrees |
| Gear outer addendum | 0.087 | in | Gear outer addendum | 0.061 | in |
| Pinion outer addendum | 0.259 | in | Pinion outer addendum | 0.148 | in |
| Gear outside diameter | 7.555 | in | Gear outside diameter | 5.054 | in |
| Pinion outside diameter | 2.992 | in | Pinion outside diameter | 2.764 | in |

NOTE: Maximum face width is the smallest of (a) or (b)

49

**Given:** $N_P = 18; N_G = 72; P_d = 12$; 20° pressure angle.
**Computed values:**

| | |
|---|---|
| Gear ratio | $m_G = N_G/N_P = 72/18 = 4.000$ |
| Pitch diameter: Pinion | $d = N_P/P_d = 18/12 = 1.500$ in |
| Pitch diameter: Gear | $D = N_G/P_d = 72/12 = 6.000$ in |
| Pitch cone angle: Pinion | $\gamma = \tan^{-1}(N_P/N_G) = \tan^{-1}(18/72) = 14.03°$ |
| Pitch cone angle: Gear | $\Gamma = \tan^{-1}(N_G/N_P) = \tan^{-1}(72/18) = 75.96°$ |
| Outer cone distance | $A_o = 0.5D/\sin(\Gamma) = 0.5(6.00\text{ in})/\sin(75.96°) = 3.092$ in |

Face width must be specified: $F = 0.800$ in Based on the following guidelines:

Nominal face width: $F_{nom} = 0.30\,A_o = 0.30(3.092\text{ in}) = 0.928$ in
Maximum face width: $F_{max} = A_o/3 = (3.092\text{ in})/3 = 1.031$ in
or $F_{max} = 10/P_d = 10/12 = 0.833$in

Mean cone distance $A_m = A_{mG} = A_o - 0.5F = 3.092\text{ in} - 0.5(0.80\text{ in}) = 2.692$ in

Ratio $(A_m/A_o) = (2.692/3.092) = 0.871$ [This ratio occurs in several following calculations]

| | |
|---|---|
| Mean circular pitch | $p_m = (\pi/P_d)(A_m/A_o) = (\pi/12)(0.871) = 0.228$ in |
| Mean working depth | $h = (2.00/P_d)(A_m/A_o) = (2.00/12)(0.871) = 0.145$ in |
| Clearance | $c = 0.125h = 0.125(0.145\text{ in}) = 0.018$ in |
| Mean whole depth | $h_m = h + c = 0.145\text{ in} + 0.018\text{ in} = 0.163$ in |
| Mean addendum factor | $c_1 = 0.210 + 0.290/(m_G)^2 = 0.210 + 0.290/(4.00)^2 = 0.228$ |
| Gear mean addendum | $a_G = c_1\,h = (0.228)(0.145\text{ in}) = 0.033$ in |
| Pinion mean addendum | $a_P = h - a_G = 0.145\text{ in} - 0.033\text{ in} = 0.112$ in |
| Gear mean dedendum | $b_G = h_m - a_G = 0.163\text{ in} - 0.033\text{ in} = 0.130$ in |
| Pinion mean dedendum | $b_P = h_m - a_P = 0.163\text{ in} - 0.112\text{ in} = 0.051$ in |
| Gear dedendum angle | $\delta_G = \tan^{-1}(b_G/A_{mG}) = \tan^{-1}(0.130/2.692) = 2.76°$ |
| Pinion dedendum angle | $\delta_P = \tan^{-1}(b_P/A_{mG}) = \tan^{-1}(0.051/2.692) = 1.09°$ |
| Gear outer addendum | $a_{oG} = a_G + 0.5F\tan\delta_P$<br>$a_{oG} = (0.033\text{ in}) + (0.5)(0.80\text{ in})\tan(1.09°) = 0.0406$ in |
| Pinion outer addendum | $a_{oP} = a_P + 0.5F\tan\delta_G$<br>$a_{oP} = (0.112\text{ in}) + (0.5)(0.80\text{ in})\tan(2.76°) = 0.1313$ in |
| Gear outside diameter | $D_o = D + 2a_{oG}\cos\Gamma$<br>$D_o = 6.000\text{ in} + 2(0.0406\text{ in})\cos(75.96°) = 6.020$ in |
| Pinion outside diameter | $d_o = d + 2a_{oP}\cos\gamma$<br>$d_o = 1.500\text{ in} + 2(0.1313\text{ in})\cos(14.04°) = 1.755$ in |

## BEVEL GEAR GEOMETRY

### PROBLEM: 49

| GIVEN DATA | | |
|---|---|---|
| No of teeth in pinion | 18 | |
| No of teeth in gear | 72 | |
| Diametral pitch | 12 | |
| Pressure angle | 20 | degrees |
| **COMPUTED VALUES** | | |
| Gear ratio | 4.000 | |
| Pitch diameter: Pinion | 1.500 | in |
| Pitch diameter: Gear | 6.000 | in |
| Pitch cone angle: Pinion | 14.036 | degrees |
| Pitch cone angle: Gear | 75.964 | degrees |
| Outer cone distance | 3.092 | in |
| Nominal face width | 0.928 | in |
| Maximum face width (a) | 1.031 | in |
| Maximum face width (b) | 0.833 | in |
| INPUT Face width | 0.800 | in |
| Mean cone distance | 2.692 | in |
| Ratio $A_m/A_o$ | 0.871 | |
| Mean circular pitch | 0.228 | in |
| mean working depth | 0.145 | in |
| Clearance | 0.018 | in |
| Mean whole depth | 0.163 | in |
| mean addendum factor | 0.228 | |
| Gear mean addendum | 0.033 | in |
| Pinion mean addendum | 0.112 | in |
| Gear mean dedendum | 0.130 | in |
| Pinion mean dedendum | 0.051 | in |
| Gear dedendum angle | 2.767 | degrees |
| Pinion dedendum angle | 1.090 | degrees |
| Gear outer addendum | 0.041 | in |
| Pinion outer addendum | 0.131 | in |
| Gear outside diameter | 6.020 | in |
| Pinion outside diameter | 1.755 | in |

### PROBLEM: 50

| GIVEN DATA | | |
|---|---|---|
| No of teeth in pinion | 16 | |
| No of teeth in gear | 64 | |
| Diametral pitch | 32 | |
| Pressure angle | 20 | degrees |
| **COMPUTED VALUES** | | |
| Gear ratio | 4.000 | |
| Pitch diameter: Pinion | 0.500 | in |
| Pitch diameter: Gear | 2.000 | in |
| Pitch cone angle: Pinion | 14.036 | degrees |
| Pitch cone angle: Gear | 75.964 | degrees |
| Outer cone distance | 1.031 | in |
| Nominal face width | 0.309 | in |
| Maximum face width (a) | 0.344 | in |
| Maximum face width (b) | 0.313 | in |
| INPUT Face width | 0.300 | in |
| Mean cone distance | 0.881 | in |
| Ratio $A_m/A_o$ | 0.854 | |
| Mean circular pitch | 0.084 | in |
| mean working depth | 0.053 | in |
| Clearance | 0.007 | in |
| Mean whole depth | 0.060 | in |
| mean addendum factor | 0.228 | |
| Gear mean addendum | 0.012 | in |
| Pinion mean addendum | 0.041 | in |
| Gear mean dedendum | 0.048 | in |
| Pinion mean dedendum | 0.019 | in |
| Gear dedendum angle | 3.113 | degrees |
| Pinion dedendum angle | 1.227 | degrees |
| Gear outer addendum | 0.015 | in |
| Pinion outer addendum | 0.049 | in |
| Gear outside diameter | 2.007 | in |
| Pinion outside diameter | 0.596 | in |

NOTE: Maximum face width is the smallest of (a) or (b)

## BEVEL GEAR GEOMETRY

| PROBLEM: 51 | | |
|---|---|---|
| **GIVEN DATA** | | |
| *No of teeth in pinion* | 12 | |
| *No of teeth in gear* | 36 | |
| *Diametral pitch* | 48 | |
| *Pressure angle* | 20 | degrees |
| **COMPUTED VALUES** | | |
| Gear ratio | 3.000 | |
| Pitch diameter: Pinion | 0.250 | in |
| Pitch diameter: Gear | 0.750 | in |
| Pitch cone angle: Pinion | 18.435 | degrees |
| Pitch cone angle: Gear | 71.565 | degrees |
| Outer cone distance | 0.395 | in |
| Nominal face width | 0.119 | in |
| Maximum face width (a) | 0.132 | in |
| Maximum face width (b) | 0.208 | in |
| INPUT Face width | 0.125 | in |
| Mean cone distance | 0.333 | in |
| Ratio $A_m/A_o$ | 0.842 | |
| Mean circular pitch | 0.055 | in |
| mean working depth | 0.035 | in |
| Clearance | 0.004 | in |
| Mean whole depth | 0.039 | in |
| mean addendum factor | 0.242 | |
| Gear mean addendum | 0.008 | in |
| Pinion mean addendum | 0.027 | in |
| Gear mean dedendum | 0.031 | in |
| Pinion mean dedendum | 0.013 | in |
| Gear dedendum angle | 5.316 | degrees |
| Pinion dedendum angle | 2.217 | degrees |
| Gear outer addendum | 0.011 | in |
| Pinion outer addendum | 0.032 | in |
| Gear outside diameter | 0.757 | in |
| Pinion outside diameter | 0.311 | in |

| EXAMPLE PROBLEM: 8-8 | | |
|---|---|---|
| **GIVEN DATA** | | |
| No of teeth in pinion | 16 | |
| No of teeth in gear | 48 | |
| Diametral pitch | 8 | |
| Pressure angle | 20 | degrees |
| **COMPUTED VALUES** | | |
| Gear ratio | 3.000 | |
| Pitch diameter: Pinion | 2.000 | in |
| Pitch diameter: Gear | 6.000 | in |
| Pitch cone angle: Pinion | 18.435 | degrees |
| Pitch cone angle: Gear | 71.565 | degrees |
| Outer cone distance | 3.162 | in |
| Nominal face width | 0.949 | in |
| Maximum face width (a) | 1.054 | in |
| Maximum face width (b) | 1.250 | in |
| INPUT Face width | 1.000 | in |
| Mean cone distance | 2.662 | in |
| Ratio $A_m/A_o$ | 0.842 | |
| Mean circular pitch | 0.331 | in |
| mean working depth | 0.210 | in |
| Clearance | 0.026 | in |
| Mean whole depth | 0.237 | in |
| mean addendum factor | 0.242 | |
| Gear mean addendum | 0.051 | in |
| Pinion mean addendum | 0.159 | in |
| Gear mean dedendum | 0.186 | in |
| Pinion mean dedendum | 0.077 | in |
| Gear dedendum angle | 3.992 | degrees |
| Pinion dedendum angle | 1.663 | degrees |
| Gear outer addendum | 0.065 | in |
| Pinion outer addendum | 0.194 | in |
| Gear outside diameter | 6.041 | in |
| Pinion outside diameter | 2.369 | in |

**NOTE: Maximum face width is the smallest of (a) or (b)**

$\boxed{52}$ WORMGEARING : $D_W = 1.250$ IN, $N_W = 1$, $P_d = 10$ ; $\phi_n = 14.5°$
$N_G = 40$ ; $F = 0.625$ IN. └ SINGLE THREAD

LEAD = AXIAL PITCH = CIRCULAR PITCH = $\pi/P_d = \pi/10 = 0.3142$ IN.

LEAD ANGLE = $\lambda = \tan^{-1}\left(\frac{L}{\pi D_W}\right) = \tan^{-1}\left(\frac{0.3142}{\pi(1.250)}\right) = 4.57°$

ADDENDUM $= a = 1/P_d = 1/10 = 0.100$ IN. ; DEDENDUM $= \frac{1.157}{P_d} = 0.1157$ IN

WORM OUTSIDE DIA. $= D_{OW} = D_W + 2a = 1.250 + 2(0.100) = 1.450$ IN.

WORM ROOT DIA. $= D_{RW} = D_W - 2b = 1.250 - 2(0.1157) = 1.0186$ IN.

GEAR PITCH DIA. $= D_G = N_G/P_d = 40/10 = 4.000$ IN.

CENTER DISTANCE $= C = (D_G + D_W)/2 = (4.000 + 1.250)/2 = 2.625$ IN.

VELOCITY RATIO $= VR = N_G/N_W = 40/1 = 40$

NOTE: On the following two pages are the results of Problems 52-57 giving pertinent geometric properties of worms and wormgears and their velocity ratios. The detailed calculations follow the pattern illustrated above for Problem 52. The equations come from Section 8-9, Equations 8-20 to 8-25.

Compare the results to discern how variations in geometry such as diametral pitch and the number of threads in the worm affect the overall results. This is especially pertinent to Problem 53 in which three different designs for worm/wormgear sets provide the same velocity ratio. The single threaded worm produces the smallest center distance and overall size of the reducer. But note, also, that it has the smallest lead angle. The lead angle increases as the number of threads is increased. On the positive side, the small lead angle makes the reducer self-locking. On the negative side, the small lead angle results in lower mechanical efficiency as will be shown in Chapter 10, Section 10-11. The designer must balance these advantages and disadvantages for each application.

| WORMGEARING | PROBLEM: 52 |
|---|---|
| ***INPUT DATA*** | |
| Worm pitch diameter = | 1.250 in |
| Diametral pitch = | 10 |
| No. of worm threads = | 1 |
| No. of gear teeth = | 40 |
| Face width of gear = | 0.625 in |
| ***COMPUTED RESULTS*** | |
| Circular pitch of gear = | 0.3142 in |
| Axial pitch of worm = | 0.3142 in |
| Lead of the worm = | 0.3142 in |
| Lead angle = | 4.574 deg |
| Addendum = | 0.100 in |
| Dedendum = | 0.116 in |
| Worm outside diameter = | 1.450 in |
| Worm root diameter = | 1.019 in |
| Gear pitch diameter = | 4.000 in |
| Center distance = | 2.625 in |
| Velocity ratio = | 40.00 |

| WORMGEARING | PROBLEM: 53A |
|---|---|
| ***INPUT DATA*** | |
| Worm pitch diameter = | 1.000 in |
| Diametral pitch = | 12 |
| No. of worm threads = | 1 |
| No. of gear teeth = | 20 |
| Face width of gear = | 0.500 in |
| ***COMPUTED RESULTS*** | |
| Circular pitch of gear = | 0.2618 in |
| Axial pitch of worm = | 0.2618 in |
| Lead of the worm = | 0.2618 in |
| Lead angle = | 4.764 deg |
| Addendum = | 0.083 in |
| Dedendum = | 0.096 in |
| Worm outside diameter = | 1.167 in |
| Worm root diameter = | 0.807 in |
| Gear pitch diameter = | 1.667 in |
| Center distance = | 1.333 in |
| Velocity ratio = | 20.00 |

| WORMGEARING | PROBLEM: 53B |
|---|---|
| ***INPUT DATA*** | |
| Worm pitch diameter = | 1.000 in |
| Diametral pitch = | 12 |
| No. of worm threads = | 2 |
| No. of gear teeth = | 40 |
| Face width of gear = | 0.500 in |
| ***COMPUTED RESULTS*** | |
| Circular pitch of gear = | 0.2618 in |
| Axial pitch of worm = | 0.2618 in |
| Lead of the worm = | 0.5236 in |
| Lead angle = | 9.462 deg |
| Addendum = | 0.083 in |
| Dedendum = | 0.096 in |
| Worm outside diameter = | 1.167 in |
| Worm root diameter = | 0.807 in |
| Gear pitch diameter = | 3.333 in |
| Center distance = | 2.167 in |
| Velocity ratio = | 20.00 |

| WORMGEARING | PROBLEM: 53C |
|---|---|
| ***INPUT DATA*** | |
| Worm pitch diameter = | 1.000 in |
| Diametral pitch = | 12 |
| No. of worm threads = | 4 |
| No. of gear teeth = | 80 |
| Face width of gear = | 0.500 in |
| ***COMPUTED RESULTS*** | |
| Circular pitch of gear = | 0.2618 in |
| Axial pitch of worm = | 0.2618 in |
| Lead of the worm = | 1.0472 in |
| Lead angle = | 18.435 deg |
| Addendum = | 0.083 in |
| Dedendum = | 0.096 in |
| Worm outside diameter = | 1.167 in |
| Worm root diameter = | 0.807 in |
| Gear pitch diameter = | 6.667 in |
| Center distance = | 3.833 in |
| Velocity ratio = | 20.00 |

**WORMGEARING** PROBLEM: 54

***INPUT DATA***

| | |
|---|---|
| Worm pitch diameter = | 0.625 in |
| Diametral pitch = | 16 |
| No. of worm threads = | 2 |
| No. of gear teeth = | 100 |
| Face width of gear = | 0.313 in |

***COMPUTED RESULTS***

| | |
|---|---|
| Circular pitch of gear = | 0.1963 in |
| Axial pitch of worm = | 0.1963 in |
| Lead of the worm = | 0.3927 in |
| Lead angle = | 11.310 deg |
| Addendum = | 0.063 in |
| Dedendum = | 0.072 in |
| Worm outside diameter = | 0.750 in |
| Worm root diameter = | 0.480 in |
| Gear pitch diameter = | 6.250 in |
| Center distance = | 3.438 in |
| Velocity ratio = | 50.00 |

**WORMGEARING** PROBLEM: 55

***INPUT DATA***

| | |
|---|---|
| Worm pitch diameter = | 2.000 in |
| Diametral pitch = | 6 |
| No. of worm threads = | 4 |
| No. of gear teeth = | 72 |
| Face width of gear = | 1.000 in |

***COMPUTED RESULTS***

| | |
|---|---|
| Circular pitch of gear = | 0.5236 in |
| Axial pitch of worm = | 0.5236 in |
| Lead of the worm = | 2.0944 in |
| Lead angle = | 18.435 deg |
| Addendum = | 0.167 in |
| Dedendum = | 0.193 in |
| Worm outside diameter = | 2.333 in |
| Worm root diameter = | 1.614 in |
| Gear pitch diameter = | 12.000 in |
| Center distance = | 7.000 in |
| Velocity ratio = | 18.00 |

**WORMGEARING** PROBLEM: 56

***INPUT DATA***

| | |
|---|---|
| Worm pitch diameter = | 4.000 in |
| Diametral pitch = | 3 |
| No. of worm threads = | 1 |
| No. of gear teeth = | 54 |
| Face width of gear = | 2.000 in |

***COMPUTED RESULTS***

| | |
|---|---|
| Circular pitch of gear = | 1.0472 in |
| Axial pitch of worm = | 1.0472 in |
| Lead of the worm = | 1.0472 in |
| Lead angle = | 4.764 deg |
| Addendum = | 0.333 in |
| Dedendum = | 0.386 in |
| Worm outside diameter = | 4.667 in |
| Worm root diameter = | 3.229 in |
| Gear pitch diameter = | 18.000 in |
| Center distance = | 11.000 in |
| Velocity ratio = | 54.00 |

**WORMGEARING** PROBLEM: 57

***INPUT DATA***

| | |
|---|---|
| Worm pitch diameter = | 0.333 in |
| Diametral pitch = | 48 |
| No. of worm threads = | 4 |
| No. of gear teeth = | 80 |
| Face width of gear = | 0.156 in |

***COMPUTED RESULTS***

| | |
|---|---|
| Circular pitch of gear = | 0.0654 in |
| Axial pitch of worm = | 0.0654 in |
| Lead of the worm = | 0.2618 in |
| Lead angle = | 14.050 deg |
| Addendum = | 0.021 in |
| Dedendum = | 0.024 in |
| Worm outside diameter = | 0.375 in |
| Worm root diameter = | 0.285 in |
| Gear pitch diameter = | 1.667 in |
| Center distance = | 1.000 in |
| Velocity ratio = | 20.00 |

FOR PROBLEM 58 – ASSUME THAT THE INPUT SHAFT ROTATES CLOCKWISE.

$\boxed{58}$

TRAIN VALUE $= TV = n_1/n_6$ ; $n_1 = 3450$ RPM

$$TV = \frac{-N_B}{N_A} \cdot \frac{N_D}{N_C} \cdot \frac{N_F}{N_E} \cdot \frac{N_H}{N_G} \cdot \frac{N_I}{N_H} = \frac{-82}{18}\ \frac{64}{17}\ \frac{110}{20}\ \frac{18}{18}\ \frac{38}{18} = -119.1$$

$$n_6 = \frac{n_1}{TV} = \frac{3450\ RPM}{-199.1} = \underline{-17.32\ RPM}\quad \text{COUNTERCLOCKWISE}$$

GEAR H IS AN IDLER. IT DOES NOT AFFECT THE TV BUT CHANGES THE DIRECTION OF THE OUTPUT SHAFT.

$\boxed{59}$

$n_1 = 12\,200$ RPM ; FIND $n_5$ : $TV = n_1/n_5$

$$TV = \frac{N_B}{N_A}\ \frac{N_D}{N_C}\ \frac{N_F}{N_E}\ \frac{N_H}{N_G} = \frac{50}{12}\ \frac{40}{12}\ \frac{60}{1}\ \frac{72}{2} = 30\,000$$

$$n_5 = \frac{n_1}{TV} = \frac{12\,200\ RPM}{30\,000} = \underline{0.4067\ RPM}$$

$\boxed{60}$

$n_1 = 6840$ RPM ; FIND $n_4$ ; $TV = n_1/n_4$

$$TV = \frac{N_B}{N_A}\ \frac{N_D}{N_C}\ \frac{N_F}{N_E} = \frac{48}{16}\ \frac{48}{18}\ \frac{60}{12} = 40 \text{ EXACTLY}$$

$$n_4 = \frac{n_1}{TV} = \frac{6840}{40} = \underline{171\ RPM}\ \text{EXACTLY}$$

$\boxed{61}$

$n_1 = 2875$ RPM ; FIND $n_4$ : $TV = n_1/n_4$

$$TV = \frac{N_B}{N_A}\ \frac{N_D}{N_C}\ \frac{N_F}{N_E} = \frac{100}{3}\ \frac{80}{2}\ \frac{85}{20} = 5666.7$$

$$n_4 = \frac{n_1}{TV} = \frac{2875\ RPM}{5666.7} = \underline{0.5074\ RPM}$$

## Gear Trains - Kinematic Design

**VELOCITY RATIO FOR GEARS** *PROBLEM 62*

DESIRED VR = 3.1416 = $\pi$

| | NP | NG | NG Act | VR-Act | DIFF = Des VR - VR Act | |
|---|---|---|---|---|---|---|
| | 16 | 50.27 | 50 | 3.1250 | 0.01659 | |
| | 17 | 53.41 | 53 | 3.1176 | 0.02395 | |
| | 18 | 56.55 | 57 | 3.1667 | 0.02507 | |
| | 19 | 59.69 | 60 | 3.1579 | 0.01630 | |
| | 20 | 62.83 | 63 | 3.1500 | 0.00841 | |
| XX | 21 | 65.97 | 66 | 3.1429 | 0.00126 | XX |
| | 22 | 69.12 | 69 | 3.1364 | 0.00523 | |
| | 23 | 72.26 | 72 | 3.1304 | 0.01116 | |
| | 24 | 75.40 | 75 | 3.1250 | 0.01659 | |
| | | | | Min diff = | 0.00126 | |

**VELOCITY RATIO FOR GEARS** *PROBLEM 63*

DESIRED VR = 1.7321 = $\sqrt{3}$

| | NP | NG | NG Actual | VR Actual | DIFF = Des VR - VR Act | |
|---|---|---|---|---|---|---|
| | 16 | 27.71 | 28 | 1.7500 | 0.01795 | |
| | 17 | 29.44 | 29 | 1.7059 | 0.02617 | |
| | 18 | 31.18 | 31 | 1.7222 | 0.00983 | |
| | 19 | 32.91 | 33 | 1.7368 | 0.00479 | |
| | 20 | 34.64 | 35 | 1.7500 | 0.01795 | |
| | 21 | 36.37 | 36 | 1.7143 | 0.01777 | |
| XX | 22 | 38.11 | 38 | 1.7273 | 0.00478 | XX |
| | 23 | 39.84 | 40 | 1.7391 | 0.00708 | |
| | 24 | 41.57 | 42 | 1.7500 | 0.01795 | |
| | | | | Min diff = | 0.00478 | |

**VELOCITY RATIO FOR GEARS** *PROBLEM 64*

DESIRED VR = 6.1644 = $\sqrt{38}$

| | NP | NG | NG Act | VR-Act | DIFF = Des VR - VR Act | |
|---|---|---|---|---|---|---|
| | 16 | 98.63 | 99 | 6.1875 | 0.02309 | |
| | 17 | 104.80 | 105 | 6.1765 | 0.01206 | |
| XX | 18 | 110.96 | 111 | 6.1667 | 0.00225 | XX |
| | 19 | 117.12 | 117 | 6.1579 | 0.00652 | |
| | 20 | 123.29 | 123 | 6.1500 | 0.01441 | |
| | 21 | 129.45 | 129 | 6.1429 | 0.02156 | |
| | 22 | 135.62 | 136 | 6.1818 | 0.01740 | |
| | 23 | 141.78 | 142 | 6.1739 | 0.00950 | |
| XX | 24 | 147.95 | 148 | 6.1667 | 0.00225 | XX |
| TWO EQUAL SOLUTIONS | | | | Min diff = | 0.00225 | |

**VELOCITY RATIO FOR GEARS** *PROBLEM 65*

DESIRED VR = 7.42

| | NP | NG | NG Actual | VR Actual | DIFF = Des VR - VR Act | |
|---|---|---|---|---|---|---|
| | 16 | 118.72 | 119 | 7.4375 | 0.01750 | |
| | 17 | 126.14 | 126 | 7.4118 | 0.00824 | |
| | 18 | 133.56 | 134 | 7.4444 | 0.02444 | |
| XX | 19 | 140.98 | 141 | 7.4211 | 0.00105 | XX |
| | 20 | 148.40 | 148 | 7.4000 | 0.02000 | |
| | 21 | 155.82 | 156 | 7.4286 | 0.00857 | |
| | 22 | 163.24 | 163 | 7.4091 | 0.01091 | |
| | 23 | 170.66 | 171 | 7.4348 | 0.01478 | |
| | 24 | 178.08 | 178 | 7.4167 | 0.00333 | |
| | | | | Min diff = | 0.00105 | |

**66**

**DESIGN:** $n_{IN} = 1800$ RPM  $n_{OUT} = 2$ RPM  EXACT RATIO REQ'D.

$TV = 1800/2 = 900$ EXACT ; USE FACTORING : $N_{MAX} = 150$

2 | 900
2 | 450
5 | 225
5 | 45
3 | 9
3

FACTORS ARE: $2 \cdot 2 \cdot 5 \cdot 5 \cdot 3 \cdot 3$

SEE TABLE 8-6 FOR INTERFERENCE DATA

FOR 20° F.D. TEETH USE $N_{MIN} = 16$ OR 17

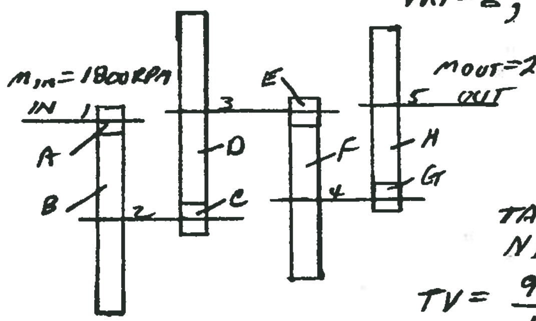
NOMINAL $VR_{MAX}$ PER PAIR $= 150/17 = 8.82$  TOO SMALL

TWO PAIRS : $(8.82)^2 = 77.8$  SMALL

THREE PAIRS : $(8.82)^3 = 687$  SMALL

FOUR PAIRS : $(8.82)^4 = 6061$  REQ'D.

RECOMBINE FACTORS: $VR1 = 6$, $VR2 = 6$, $VR3 = 5$, $VR4 = 5$




$N_A = 16$, $N_B = 96$ — 6

$N_C = 16$, $N_D = 96$ — 6

$N_E = 16$, $N_F = 80$ — 5

$N_G = 16$, $N_H = 80$ — 5

TABLE 8-6 SAYS NO INTERFERENCE FOR $N_P = 16$ IF $N_G < 101$.

$$TV = \frac{96}{16} \cdot \frac{96}{16} \cdot \frac{80}{16} \cdot \frac{80}{16} = 900 \text{ EXACT}$$

**67**

**DESIGN:** $n_{IN} = 1800$ RPM EXACT ; $21 < n_{OUT} < 22$ : $N_{MAX} = 150$ & 20° F.D.

$TV_{NOM} = 1800/21.5 = 83.7$  $TV_{MIN} = 1800/22 = 81.8$  $TV_{MAX} = 1800/21 = 85.7$

FROM TABLE 8-6, NO INTERFERENCE WITH $N_P \geq 17$ FOR 20° F.D. TEETH

$VR_{MAX}$ PER PAIR $= 150/17 = 8.82$ SMALL ; TWO PAIRS $VR_{MAX} = (8.82)^2 = 77.9$ LOW

LAYOUT AS IN FIG. 8-31 IN TEXT — TRIPLE REDUCTION.

TRY EQUAL REDUCTION RATIO: $VR1 \cong VR2 \cong VR3 \cong \sqrt[3]{83.7} = 4.37$

LET $N_A = N_C = N_E = 17$ : LET $VR1 = 5$, $VR2 = 4$ ; THEN $VR3 = 83.7/20 = 4.19$

$N_F = (17)(4.19) = 71.2 \Rightarrow$ SPECIFY $N_F = 71$

FINAL $TV = \frac{85}{17} \cdot \frac{68}{17} \cdot \frac{71}{17} = 83.53$  $n_{OUT} = \frac{n_{IN}}{TV} = \frac{1800}{83.53} = 21.55$ RPM OK

**68**

**DESIGN:** $n_{IN} = 3360$ RPM EXACT : $n_{OUT} = 12$ RPM EXACT : $N_{MAX} = 150$

20° F.D. TEETH. FROM TABLE 8-6 LET $N_{MIN} = 17$ FOR NO INTERFERENCE

$VR_{MAX}$ PER PAIR $= 150/17 = 8.82$ : 2 PAIRS $VR_{MAX} = (8.82)^2 = 77.8$ : 3 PAIRS $= 686$

$TV = 3360/12 = 280$ EXACT : USE 3 PAIRS SIMILAR TO FIG 8-31 IN TEXT

FACTORING:
2 | 280
2 | 140
5 | 70
2 | 14
7

$2 \cdot 2 \cdot 5 \cdot 2 \cdot 7 = 280$. RECOMBINE $8 \cdot 7 \cdot 5 = 280$

| | |
|---|---|
| $VR1 = 8$ ; | $N_A = 17$, $N_B = 136$ |
| $VR2 = 7$ : | $N_C = 17$, $N_D = 119$ |
| $VR3 = 5$ : | $N_E = 17$, $N_F = 85$ |

$$TV = \frac{136}{17} \cdot \frac{119}{17} \cdot \frac{85}{17} = 280 \text{ EXACTLY}$$

(OTHER DESIGNS POSSIBLE)

**69** DESIGN: $m_{IN} = 4200$ RPM CW EXACTLY: $13.0 < m_{OUT} < 13.5$ RPM CW : POSITIVE TV

$TV_{MIN} = \frac{4200}{13.5} = 311.1$ : $TV_{NOM} = \frac{4200}{13.25} = 316.98$ : $TV_{MAX} = \frac{4200}{13.0} = 323.08$

FROM PROB 68, 3 PAIRS REQ'D. LAYOUT IN FIG 8-31 PRODUCES A NEGATIVE TV. USE IDLER IN ANY PAIR.

TRY RESIDUAL RATIO METHOD. NOMINAL $VR = \sqrt[3]{317} = 6.82$ PER PAIR

TRY $VR1 = 7$; $VR2 = 6$; THEN $VR3 \cong 317/42 \cong 7.55$ : USE $VR3 = 7.50$

FINAL $TV = VR1 \cdot VR2 \cdot VR3 = (7)(6)(7.50) = 315$ OK

IT IS PREFERRED TO PLACE HIGHER RATIOS EARLY IN THE TRAIN. LET $VR1 = 7.5$, $VR2 = 7$, $VR3 = 6$.

LET $N_A = 18$; $N_B = 7.5(18) = 135$; LET $N_C = 17$, $N_D = 7(17) = 119$

LET $N_E = 17$, $N_F = 17(6) = 102$.

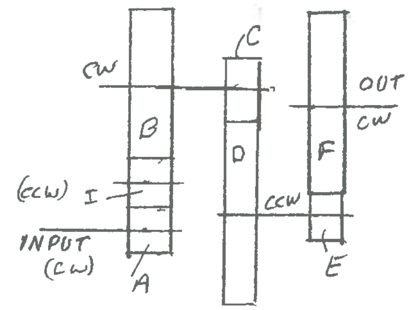
IDLER NEEDED FOR POSITIVE TRAIN

LET $N_I = 17$. PLACE IN FIRST PAIR.

FINAL TRAIN VALUE:

$$TV = \frac{135}{18} \times \frac{119}{17} \times \frac{102}{17} = 315$$

$$m_{OUT} = \frac{m_{IN}}{TV} = \frac{4200}{315} = 13.3\overline{3} \text{ RPM CW}$$ OK




NOTE: CHAPTER 9 GIVES INFORMATION ON SELECTION OF $P_d$ - DIAMETRAL PITCH. BECAUSE OF SPEED/TORQUE CHANGES, $P_{d1} > P_{d2} > P_{d3}$. LARGER $P_d$ GIVES SMALLER GEARS. THIS IS THE REASON THAT LARGER RATIOS SHOULD BE PLACED EARLIER IN THE TRAIN.

**70** DESIGN: $m_{IN} = 5500$ RPM EXACTLY:

$221 < m_{OUT} < 225$

DESIGN: TWO DOUBLE REDUCTION WITH ALL EXTERNAL GEARS

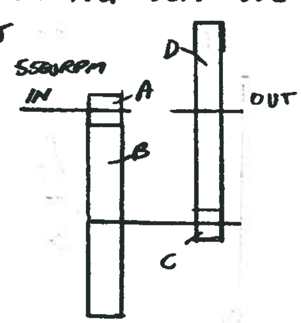
$TV_{MAX} = 5500/221 = 24.88$ : $TV_{MIN} = 5500/225 = 24.44$ : $TV_{NOM} = 5500/223 = 24.66$

NOMINAL RATIO FOR EACH PAIR $= \sqrt{24.66} = 4.97$. TRY $VR1 = 5$

THEN $VR2 \cong 24.66/5 = 4.93$ : FOR $N_P = 16$ $N_G = 78.9$ — USE $N_G = 79$

FINAL $TV1 = \boxed{\frac{N_B}{N_A} \cdot \frac{N_D}{N_C} = \frac{80}{16} \cdot \frac{79}{16}} = 24.6875$

$m_{OUT} = \frac{5500}{TV1} = 222.78$ RPM OK



**71** DESIGN: $n_{IN} = 5500$ RPM     $13.0 < n_{OUT} < 14.0$ RPM

$TV_{NOM} = 5500/13.5 = 407.4$     SKETCH AS IN **20**.

MAX RATIO FOR ONE PAIR $= 150/17 = 8.82$

TWO PAIRS - MAX $= 77.85$; THREE PAIRS 687 - OK

NOMINAL RATIO PER PAIR: $\sqrt[3]{407.4} = 7.41$

TRY $VR_1 \cong 8$, $VR_2 = 8$ - BUT USE HUNTING TOOTH APPROACH.

$VR = 8$: $N_A = 17$, $N_B = 17(8) = 136$: $\underline{\text{USE } N_B = 135}$

SAME FOR $N_C, N_D$.

$(VR_1)(VR_2) = \left(\frac{135}{17}\right)^2 = 7.94^2 = 63.06$

RESIDUAL RATIO: $407.4/63.06 = 6.46 = N_F/N_E$

LET $N_E = 17$; $N_F = 6.46(17) = 109.82 \Rightarrow$ USE 110 TEETH

FINAL TV $= \underline{\frac{135}{17} \times \frac{135}{17} \times \frac{110}{17}} = 408.05$

FINAL OUTPUT SPEED $= 5500/408.05 = \underline{13.48 \text{ RPM} - \text{OK}}$

**72** DESIGN: $n_{IN} = 1750$;     $146 < n_{OUT} < 150$

$TV_{NOM} = 1750/148 = 11.82$

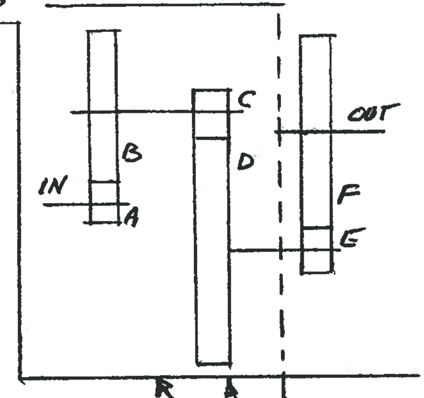
LET $VR_1 = N_B/N_A = 75/18 = 4.167$

RESIDUAL RATIO $= 11.82/4.167 = 2.837$

LET $N_C = 18$: $N_D = 18(2.837) = 51.06 \Rightarrow 51$

$\underline{N_A = 18,\ N_B = 75,\ N_C = 18,\ N_D = 51}$

$n_{OUT} = 1750 \times \frac{18}{75} \times \frac{18}{51} = \underline{148.2 \text{ RPM}}$ OK




SKETCH SAME AS **71** WITH ONLY TWO PAIRS

[THESE RESULTS USED IN PROBLEM 9-74.]

**73** DESIGN: $n_{IN} = 850$ RPM;     $40 < n_{OUT} < 44$; USE 2 PAIRS

$TV_{NOM} = 850/42 = 20.24$. LET $VR_1 = N_B/N_A = 81/18 = 4.50$

RESIDUAL RATIO $= VR_2 = 20.24/4.50 = 4.50$: $\underline{N_C = 18;\ N_D = 81}$

$\underline{n_{OUT}} = 850 \times \frac{18}{81} \times \frac{18}{81} = \underline{41.98 \text{ RPM}}$ OK

[THESE RESULTS USED IN PROBLEM 9-75.]

**74** DESIGN: USE TWO PAIRS: $n_{IN} = 3000$ RPM; $548 < n_{OUT} < 552$

$TV_{NOM} = 3000/550 = 5.4545$: LET $VR_1 = VR_2 = \sqrt{5.4545} = 2.335$

LET $\underline{N_A = 15}$; $\underline{N_B} = 15(2.335) = 35.03 \Rightarrow \underline{35}$. LET $\underline{N_C = 15,\ N_D = 35}$

$\underline{n_{OUT}} = 3000 \times \frac{15}{35} \times \frac{15}{35} = \underline{551 \text{ RPM}}$ $\underline{\text{OK}}$

[THESE RESULTS USED IN PROBLEM 9-76.]

$\boxed{75}$ DESIGN: $n_{IN} = 3600$ RPM $\quad 3.0 < n_{OUT} < 5.0$

$TV_{Nom} = 3600/4.0 = 900$: USE 4 PAIRS

FACTORING:

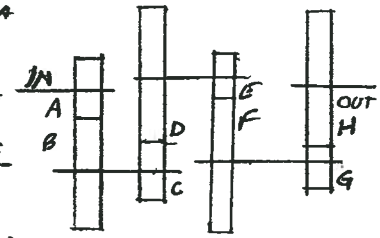
$$\begin{array}{r|l} 2 & 900 \\ 2 & 450 \\ 5 & 225 \\ 5 & 45 \\ 3 & 9 \\ & 3 \end{array}$$

USE $VR_1 = 6 = 96/16 = N_B/N_A$

$VR_2 = 6 = 96/16 = N_D/N_C$

$VR_3 = 5 = 80/16 = N_F/N_E$

$VR_4 = 5 = 80/16 = N_H/N_G$




ALTERNATE SOLUTION USING HUNTING TOOTH:

LET $N_A = N_C = N_E = N_G = 16$. LET $N_B = N_D = 95$. LET $N_F = 81$

$VR_1 = VR_2 = 95/16 = 5.9375$; $VR_3 = 81/16 = 5.0625$

$VR_1 \times VR_2 \times VR_3 = 178.47$. RESIDUAL RATIO $= 900/178.47 = 5.043$

LET $N_H = 81$. $VR_4 = VR_3 = 81/16 = 5.0625$

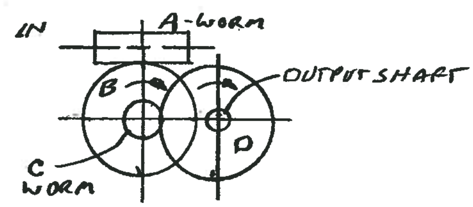
TOTAL $TV = (178.47)(5.0625) = 903.5$

FINAL $n_{OUT} = 3600/903.5 = 3.984$ RPM OK

$\boxed{76}$ DESIGN: $n_{IN} = 3600$ $\quad 3.0 < n_{OUT} < 5.0$ RPM

$TV_{NOM} = 3600/4.0 = 900$: USE TWO PAIRS OF WORM/WORMGEARS

$N_A = N_C = 1$; $N_B = N_D = 30$



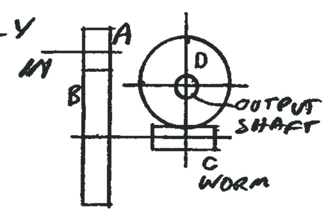

$\boxed{77}$ DESIGN: $n_{IN} = 1800$ RPM $\quad n_{OUT} = 8.0$ EXACTLY

LET $VR_2 = 50$ - WORMGEAR DRIVE.

$TV = 1800/8 = 225$: $VR_1 = 225/50 = 4.50$
HELICAL GEARS

LET $N_A = 16$, $N_B = 72$, $N_C = 1$, $N_D = 50$

$$n_{OUT} = 1800 \times \frac{16}{72} \times \frac{1}{50} = 8.0 \text{ RPM}$$




$\boxed{78}$ DESIGN: $n_{IN} = 3360$ RPM. $n_{OUT} = 12.0$ EXACTLY,

USE TWO PAIRS OF WORMGEAR DRIVES AS IN PROBLEM $\boxed{76}$.

LET $VR_1 = 20$, $VR_2 = 14$: $N_A = 2$, $N_B = 40$, $N_C = 2$, $N_D = 28$.

$\boxed{79}$ DESIGN: $n_{IN} = 4200$ RPM, $13.0 < n_{OUT} < 13.5$

USE COMBINED HELICAL WITH WORM GEAR AS IN PROBLEM 77.

LET $VR_2 = 50$. WORM GEAR DRIVE, $N_C = 1$, $N_D = 50$

$TV_{NOM} = 4200/13.25 = 316.98$. $VR_1 = 316.98/50 = 6.34$

LET $N_A = 18$. $N_B = 18(6.34) = 114.1 \Rightarrow$ USE $114 = N_B$

FINAL OUTPUT SPEED $= 4200 \times \frac{18}{114} \times \frac{1}{50} = 13.26$ RPM OK

$\boxed{80.}$ DESIGN: $n_{IN} = 5500$ RPM $13.0 < n_{OUT} < 14.0$ RPM

USE TWO WORM GEAR DRIVES AS IN PROBLEM 76.

$TV_{NOM} = 5500/13.5 = 407.4$.

LET $VR_1 = 20$. THEN $VR_2 = 407.4/20 = 20.37$

TRY $N_C = 3$. $N_D = 3(20.37) = 61.11 \Rightarrow$ USE 61

$N_A = 3$, $N_B = 60$, $N_C = 3$, $N_D = 61$

FINAL OUTPUT SPEED $= 5500 \times \frac{3}{60} \times \frac{3}{61} = 13.52$ RPM OK

CAN ALSO USE $N_D = 62$. THEN $n_{OUT} = 13.30$ RPM

$N_D = 60$. THEN $n_{OUT} = 13.75$ RPM

# CHAPTER 9
# SPUR GEAR DESIGN

### Forces on Spur Gear Teeth

**1**

GIVEN: $\phi = 20°$, $P = 7.5\ hp$, $n_P = 1750\ RPM$, $N_P = 20$, $N_G = 72$, $P_d = 12$

a) $n_G = n_P \cdot \dfrac{N_P}{N_G} = 1750\ RPM \cdot \dfrac{20}{72} = \underline{486.1\ RPM}$

b) $VR = m_G = N_G/N_P = 72/20 = \underline{3.600}$

c) $D_P = N_P/P_d = 20/12 = \underline{1.667\ IN}$; $D_G = N_G/P_d = 72/12 = \underline{6.000\ IN}$

d) $C = \dfrac{N_P + N_G}{2\ P_d} = \dfrac{20 + 72}{2(12)} = \underline{3.833\ IN}$

e) $v_t = \pi D_P n_P / 12 = \pi(1.667)(1750)/12 = \underline{764\ FT/MIN}$

f) $T_P = \dfrac{63000\ (P)}{n_P} = \dfrac{63000(7.5)}{1750} = \underline{270\ LB \cdot IN}$

$T_G = \dfrac{63000(P)}{n_G} = \dfrac{63000\ (7.5)}{486.1} = \underline{972\ LB \cdot IN}$

g) $W_t = \dfrac{T_P}{D_P/2} = \dfrac{270\ LB \cdot IN}{1.667\ IN/2} = \underline{324\ LB}$

OR $W_t = \dfrac{33000\ (P)}{v_t} = \dfrac{33000(7.5)}{764} = \underline{324\ LB}$

h) $W_R = W_t \tan\phi = (324\ lb)\tan 20° = \underline{118\ LB}$

i) $W_N = W_t/\cos\phi = 324\ LB/\cos 20° = \underline{345\ LB}$

A SIMILAR METHOD IS USED FOR PROBLEMS 2-6.
SPREADSHEET SOLUTIONS ARE SHOWN ON THE FOLLOWING PAGES. THE SOLUTION FOR PROBLEM 1 IS ALSO SHOWN FOR COMPARISON TO THE SOLUTION SHOWN ABOVE.

| Forces on Spur Gear Teeth | | | |
|---|---|---|---|

| Problem 1 | Pressure angle = | 20 | degrees |
|---|---|---|---|
| Chapter 9 | Power = | 7.5 | hp |
| | pinion speed = | 1750 | rpm |
| | teeth in pinion = | 20 | |
| | teeth in gear = | 72 | |
| | diametral pitch = | 12 | |
| | ***RESULTS:*** | | |
| a | Gear speed = | 486.1 | rpm |
| b | VR = $m_G$ = | 3.600 | |
| c | pinion PD = | 1.667 | in |
| | gear PD = | 6.000 | in |
| d | center distance = C = | 3.833 | in |
| e | pitch line speed = | 764 | ft/min |
| f | torque on pinion shaft = | 270 | lb in |
| | torque on gear shaft = | 972 | lb in |
| g | tangential force = | 324 | lb |
| h | radial force = | 118 | lb |
| i | normal force = | 345 | lb |

| Problem 2 | Pressure angle = | 20 | degrees |
|---|---|---|---|
| Chapter 9 | Power = | 50 | hp |
| | pinion speed = | 1150 | rpm |
| | teeth in pinion = | 18 | |
| | teeth in gear = | 68 | |
| | diametral pitch = | 5 | |
| | ***RESULTS:*** | | |
| a | Gear speed = | 304.4 | rpm |
| b | VR = $m_G$ = | 3.778 | |
| c | pinion PD = | 3.600 | in |
| | gear PD = | 13.600 | in |
| d | center distance = C = | 8.600 | in |
| e | pitch line speed = | 1084 | ft/min |
| f | torque on pinion shaft = | 2739 | lb in |
| | torque on gear shaft = | 10348 | lb in |
| g | tangential force = | 1522 | lb |
| h | radial force = | 554 | lb |
| i | normal force = | 1620 | lb |

## Forces on Spur Gear Teeth

| | | | |
|---|---|---|---|
| **Problem 3** | *Pressure angle =* | *20* | *degrees* |
| **Chapter 9** | *Power =* | *0.75* | *hp* |
| | *pinion speed =* | *3450* | *rpm* |
| | *teeth in pinion =* | *24* | |
| | *teeth in gear =* | *110* | |
| | *diametral pitch =* | *24* | |
| | ***RESULTS:*** | | |
| a | Gear speed = | 752.7 | rpm |
| b | VR = $m_G$ = | 4.583 | |
| c | pinion PD = | 1.000 | in |
| | gear PD = | 4.583 | in |
| d | center distance = C = | 2.792 | in |
| e | pitch line speed = | 903 | ft/min |
| f | torque on pinion shaft = | 13.70 | lb in |
| | torque on gear shaft = | 62.77 | lb in |
| g | tangential force = | 27.40 | lb |
| h | radial force = | 9.97 | lb |
| I | normal force = | 29.16 | lb |

| | | | |
|---|---|---|---|
| **Problem 4** | *Pressure angle =* | *25* | *degrees* |
| **Chapter 9** | *Power =* | *7.5* | *hp* |
| | *pinion speed =* | *1750* | *rpm* |
| | *teeth in pinion =* | *20* | |
| | *teeth in gear =* | *72* | |
| | *diametral pitch =* | *12* | |
| | ***RESULTS:*** | | |
| a | Gear speed = | 486.1 | rpm |
| b | VR = $m_G$ = | 3.600 | |
| c | pinion PD = | 1.667 | in |
| | gear PD = | 6.000 | in |
| d | center distance = C = | 3.833 | in |
| e | pitch line speed = | 764 | ft/min |
| f | torque on pinion shaft = | 270 | lb in |
| | torque on gear shaft = | 972 | lb in |
| g | tangential force = | 324 | lb |
| h | radial force = | 151 | lb |
| I | normal force = | 358 | lb |

## Forces on Spur Gear Teeth

| | | | |
|---|---|---|---|
| Problem 5 | Pressure angle = | 25 | degrees |
| Chapter 9 | Power = | 50 | hp |
| | pinion speed = | 1150 | rpm |
| | teeth in pinion = | 18 | |
| | teeth in gear = | 68 | |
| | diametral pitch = | 5 | |
| | **RESULTS:** | | |
| a | Gear speed = | 304.4 | rpm |
| b | VR = $m_G$ = | 3.778 | |
| c | pinion PD = | 3.600 | in |
| | gear PD = | 13.600 | in |
| d | center distance = C = | 8.600 | in |
| e | pitch line speed = | 1084 | ft/min |
| f | torque on pinion shaft = | 2739 | lb in |
| | torque on gear shaft = | 10348 | lb in |
| g | tangential force = | 1522 | lb |
| h | radial force = | 710 | lb |
| I | normal force = | 1680 | lb |

| | | | |
|---|---|---|---|
| Problem 6 | Pressure angle = | 25 | degrees |
| Chapter 9 | Power = | 0.75 | hp |
| | pinion speed = | 3450 | rpm |
| | teeth in pinion = | 24 | |
| | teeth in gear = | 110 | |
| | diametral pitch = | 24 | |
| | **RESULTS:** | | |
| a | Gear speed = | 752.7 | rpm |
| b | VR = $m_G$ = | 4.583 | |
| c | pinion PD = | 1.000 | in |
| | gear PD = | 4.583 | in |
| d | center distance = C = | 2.792 | in |
| e | pitch line speed = | 903 | ft/min |
| f | torque on pinion shaft = | 13.70 | lb in |
| | torque on gear shaft = | 62.77 | lb in |
| g | tangential force = | 27.40 | lb |
| h | radial force = | 12.78 | lb |
| I | normal force = | 30.24 | lb |

**Gear Manufacture and Quality**

7. See Section 9-4. Form milling, shaping, hobbing, grinding.

For Problems 8-16, refer to Section 9-5 and Table 9-3 for recommended quality numbers in the $A_v$ system according to AGMA Standard 2015. Grain harvester: $A_v = 10$.

8. Grain harvester: $A_v = 10$.
9. Printing press: $A_v = 7$.
10. Auto transmission: $A_v = 6$.
11. Gyroscope: $A_v = 2$.
12. Analytical quality measurements include *index variation, tooth alignment, tooth profile, root radius*, and *runout*.
13. AGMA Standard 2015 is currently used. See Table 9-2 for the range of quality numbers in this system and the comparisons with prior systems.

For Problems 14-16, for precision machinery, use the recommendations for machine tool drives in the lower part of Table 9-3. The choice of quality number is based on the pitch line speed of the gears.

14. (From Problem 1). Pitch line speed = 764 ft/min Use $A_v = 10$.
15. (From Problem 2). Pitch line speed = 1084 ft/min Use $A_v = 8$.
16. (From Problem 3). Pitch line speed = 903 ft/min Use $A_v = 8$.

**Gear Materials**

Answers for Problems 17 – 25 are found in Sections 9-6 and 9-7. Only brief statements are given here.

17. Bending stresses are created by the tangential force on the gear teeth acting in a manner similar to that on a cantilever. The maximum bending stress occurs in the root of the tooth where it blends with the involute tooth form. High levels of contact stress, called Hertz stress, occur in the face of the teeth near the pitch line as forces are exerted between the pinion and the gear teeth. The probable mode of failure is pitting of the tooth surface.
18. AGMA standards give allowable bending stress numbers and allowable contact stress numbers related to the hardness of the material of the teeth. See Figures 9-11 and 9-12.
19. Gear steels are typically medium carbon plain or alloy steels that are heat treated by through-hardening using a quenching and tempering process. For examples, see Table 9-4,Section 9-7.
20. The AGMA recommends hardness values from HB 180 to HB 400. See Figures 9-11 and 9-12.
21. Grade 1 steel is typical commercial quality and is recommended for use in this book. Grades 2 and 3 require progressively more stringent quality controls on the alloy content and cleanliness of the materials. Cost increases dramatically for the higher grades. See AGMA Standard 2004-C08 or the latest revision.
22. Grades 2 and 3 may be specified for high-speed aerospace applications, turbine engine driven systems, ship propulsion drives, and high-capacity industrial drives such as those in steel rolling mills.

23. Case hardening by flame hardening, induction hardening, and carburizing are three processes that produce harder surfaces than typical through-hardening.

24. See AGMA Standard 2001-D04 or the latest revision.

25. AGMA Standard 2001-D04 provides data for gray cast iron, ductile iron, and bronze. Table 9-6.

26. From Figures 9-11 and 9-12:

    a. Grade 1; 200 HB: $s_{at}$ = 28.26 ksi; $s_{ac}$ = 93.50 ksi – U.S.: $s_{at}$ = 194.9 MPa; $s_{ac}$ = 644.6 MPa – SI

    b. Grade 1; 300 HB: $s_{at}$ = 36.0 ksi; $s_{ac}$ = 125.7 ksi – U.S.: $s_{at}$ = 248.1 MPa; $s_{ac}$ = 866.6 MPa – SI

    c. Grade 1; 400 HB: $s_{at}$ = 43.72 ksi; $s_{ac}$ = 157.9 ksi – U.S.: $s_{at}$ = 301.5 MPa; $s_{ac}$ = 1088.6 MPa – SI

    d. Using HB > 400 is not recommended.

    e. Grade 2; 200 HB: $s_{at}$ = 36.80 ksi; $s_{ac}$ = 104.1 ksi – U.S.: $s_{at}$ = 253.7 MPa; $s_{ac}$ = 718.5 MPa – SI

    f. Grade 2; 300 HB: $s_{at}$ = 47.0 ksi; $s_{ac}$ = 139.0 ksi – U.S.: $s_{at}$ = 324.0 MPa; $s_{ac}$ = 959.5 MPa – SI

    g. Grade 2; 400 HB: $s_{at}$ = 57.20 ksi; $s_{ac}$ = 173.9 ksi – U.S.: $s_{at}$ = 394.3 MPa; $s_{ac}$ = 1200.5 MPa – SI

27. From Figure 9-11: Grade 1: 300 HB. Grade 2: 192 HB

28. From Table 9-5: Case hardening by carburizing produces 55-64 HRC

29. From Appendix 5: SAE 1020, 4118, 8620, and others

30. From Table 9-5: Flame or induction hardening produces 50-54 HRC with materials having high hardenability

31. SAE 4140, 4340, 6150. All have good hardenability

32. ASTM A536, Grade 80-55-06 has a minimum hardness of 179 HB.

33. a. $s_{at}$ = 45.0 ksi; $s_{ac}$ = 170.0 ksi – U.S.: $s_{at}$ = 310 MPa; $s_{ac}$ = 1172 MPa – SI [Table 9-5]

    b. $s_{at}$ = 45.0 ksi; $s_{ac}$ = 175.0 ksi – U.S.: $s_{at}$ = 310 MPa; $s_{ac}$ = 1207 MPa – SI [Table 9-5]

    c. $s_{at}$ = 55.0 ksi; $s_{ac}$ = 180.0 ksi – U.S.: $s_{at}$ = 379 MPa; $s_{ac}$ = 1241 MPa – SI [Table 9-5]

    d. Not listed

    e. $s_{at}$ = 55.0 ksi; $s_{ac}$ = 180.0 ksi – U.S.: $s_{at}$ = 379 MPa; $s_{ac}$ = 1241 MPa – SI [Table 9-5]

    f. $s_{at}$ = 5.00 ksi; $s_{ac}$ = 50.0 ksi – U.S.: $s_{at}$ = 35.0 MPa; $s_{ac}$ = 345 MPa – SI [Table 9-6]

    g. $s_{at}$ = 13.0 ksi; $s_{ac}$ = 75.0 ksi – U.S.: $s_{at}$ = 90.0 MPa; $s_{ac}$ = 517 MPa – SI [Table 9-6]

    h. $s_{at}$ = 27.0 ksi; $s_{ac}$ = 92.0 ksi – U.S.: $s_{at}$ = 186 MPa; $s_{ac}$ = 634 MPa – SI [Table 9-6]

    i. $s_{at}$ = 5.70 ksi; $s_{ac}$ = 30.0 ksi – U.S.: $s_{at}$ = 39.0 MPa; $s_{ac}$ = 207 MPa – SI [Table 9-6]

    j. $s_{at}$ = 23.6 ksi; $s_{ac}$ = 65.0 ksi – U.S.: $s_{at}$ = 163 MPa; $s_{ac}$ = 448 MPa – SI [Table 9-6]

    k. $s_{at}$ = 12.0 ksi; $s_{ac}$ not listed: $s_{at}$ = 83.0 MPa; $s_{ac}$ not listed [[Table 9-14]

    l. $s_{at}$ = 9.0 ksi; $s_{ac}$ not listed: $s_{at}$ = 62.0 MPa; $s_{ac}$ not listed [Table 9-14]

34. Depth = 0.027 in [Figure 9-13.]

35. Depth = 0.90 mm. [Figure 9-13.]

# DESIGN OF SPUR GEARS

**APPLICATION:** *Problems 36, 42, 48, 54*

*Industrial conveyor driven by an electric motor*

| | | | |
|---|---|---|---|
| ***Initial Input Data:*** | | | |
| Overload Factor: | $K_o$ = | *1.50* | Table 9-7 |
| Transmitted Power: | $P$ = | *10* | hp |
| Design Power | $P_{des}$ = | 15 | hp |
| Diametral Pitch: | $P_d$ = | *12* | Fig. 9-24 |
| Input Speed: | $n_P$ = | *1750* | rpm |
| Number of Pinion Teeth: | $N_P$ = | *18* | |
| Desired Output Speed: | $n_G$ = | *370* | rpm |
| Computed number of gear teeth: | | 85.1 | |
| *Enter:* Chosen No. of Gear Teeth: | $N_G$ = | *85* | |
| ***Computed data:*** | | | |
| Actual Output Speed: | $n_G$ = | 370.6 | rpm |
| Gear Ratio: | $m_G$ = | 4.72 | |
| Pitch Diameter - Pinion: | $D_P$ = | 1.500 | in |
| Pitch Diameter - Gear: | $D_G$ = | 7.083 | in |
| Center Distance: | C = | 4.292 | in |
| Pitch Line Speed: | $v_t$ = | 687 | ft/min |
| Transmitted Load: | $W_t$ = | 480 | lb |

***Secondary Input Data:***

| | Min | Nom | Max |
|---|---|---|---|
| Face Width Guidelines (in): | 0.667 | 1.000 | 1.333 |
| *Enter:* Face Width: | $F$ = | *1.250* | in |
| Ratio: Face width/pinion diameter: | $F/D_P$ = | 0.83 | |
| Recommended range of ratio: | $0.50 < F/D_P < 2.00$ | | |
| *Enter:* Elastic Coefficient: | $C_p$ = *2300* | | Table 9-10 |
| *Enter:* Quality Number: | $A_v$ = *11* | | Table 9-3 |
| Dynamic Factor: | $K_v$ = 1.35 | | Table 9-9 |
| [Factors for computing $K_v$:] | $B$ = 0.826 | $C$ = 59.75 | |
| *Reference:* | $N_P$ = 18 | $N_G$ = 85 | |
| Bending Geometry Factor-Pinion: | $J_P$ = *0.320* | | Fig. 9-15 |
| Bending Geometry Factor-Gear: | $J_G$ = *0.410* | | Fig. 9-15 |
| *Reference:* | $m_G$ = 4.72 | | |
| *Enter:* Pitting Geometry Factor: | $I$ = *0.108* | | Fig. 9-21 |

***Factors in Design Analysis:***

| | | | | | |
|---|---|---|---|---|---|
| Alignment Factor, $K_m = 1.0 + C_{pf} + C_{ma}$ | | If F<1.0 | If F>1.0 | | |
| Pinion Proportion Factor, $C_{pf}$ = | | 0.058 | 0.061 | [0.50 < $F/D_P$ < 2.00] | |
| *Enter:* $C_{pf}$ = | | *0.061* | Figure 9-16 | | |
| Type of gearing: | | Open | Commer. | Precision | Ex. Prec. |
| Mesh Alignment Factor, $C_{ma}$ = | | 0.268 | 0.147 | 0.083 | 0.051 |
| *Enter:* $C_{ma}$ = | | *0.147* | Figure 9-17 | | |
| Alignment Factor: $K_m$ = | | 1.21 | [Computed] | | |
| Size Factor: $K_s$ = | | *1.00* | Table 9-8: Use 1.00 if $P_d$ >= 5 | | |
| Pinion Rim Thickness Factor: $K_{BP}$ = | | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | |
| Gear Rim Thickness Factor: $K_{BG}$ = | | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | |
| Service Factor: $SF$ = | | *1.00* | Use 1.00 if no unusual conditions | | |
| Reliability Factor: $K_R$ = | | *1.00* | Table 9-11 Use 1.00 for $R$ = .99 | | |
| *Enter:* *Design Life:* | | *20000* | hours | See Table 9-12 | |
| Pinion - Number of load cycles: $N_P$ = | | 2.1E+09 | Guidelines: $Y_N$, $Z_N$ | | |
| Gear - Number of load cycles: $N_G$ = | | 4.4E+08 | $10^7$ cycles | $>10^7$ | $<10^7$ |
| Bending Stress Cycle Factor: $Y_{NP}$ = | | *0.93* | 1.00 | 0.93 | Fig. 9-22 |
| Bending Stress Cycle Factor: $Y_{NG}$ = | | *0.95* | 1.00 | 0.95 | Fig. 9-22 |
| Pitting Stress Cycle Factor: $Z_{NP}$ = | | *0.88* | 1.00 | 0.88 | Fig. 9-23 |
| Pitting Stress Cycle Factor: $Z_{NG}$ = | | *0.92* | 1.00 | 0.92 | Fig. 9-23 |
| ***Stress Analysis: Bending*** | | | | | |
| Pinion: Required $s_{at}$ = | | 37,906 | psi | See Fig. 9-11 or | |
| Gear: Required $s_{at}$ = | | 28,963 | psi | Table 9-5 | |
| ***Stress Analysis: Pitting*** | | | | | |
| Pinion: Required $s_{ac}$ = | | 199,099 | psi | See Fig. 9-12 or | |
| Gear: Required $s_{ac}$ = | | 190,443 | psi | Table 9-5 | |
| Required hardness of pinion HB: | | 528 | Equations in Fig. 9-12-Grade 1 | | |
| Required hardness of gear HB: | | 501 | Equations in Fig. 9-12-Grade 1 | | |

***Specify materials, alloy and heat treatment, for most severe requirement.***

***One possible material specification:***

Requires Grade 2 carburized. Suggest redesign to lower stress levels.

| | | | | | |
|---|---|---|---|---|---|
| Ans. Problem: | 36 | Computed bending stress number, $s_t$ = | 35253 | psi | Pinion |
| Ans. Problem: | 36 | Computed bending stress number, $s_t$ = | 27514 | psi | Gear |
| Ans. Problem: | 48 | Computed contact stress number, $s_c$ = | 175,207 | psi | Pinion |
| Ans. Problem: | 48 | Computed contact stress number, $s_c$ = | 175,207 | psi | Gear |

# DESIGN OF SPUR GEARS

**APPLICATION:** Problems 37, 43, 49, 55

*Cement kiln driven by an electric motor*

| | | | |
|---|---|---|---|
| ***Initial Input Data:*** | | | |
| Overload Factor: | $K_o$ = | *1.75* | Table 9-7 |
| Transmitted Power: | $P$ = | *40* | hp |
| Design Power | $P_{des}$ = | 70 | hp |
| Diametral Pitch: | $P_d$ = | *6* | Fig. 9-24 |
| Input Speed: | $n_P$ = | *1150* | rpm |
| Number of Pinion Teeth: | $N_P$ = | *20* | |
| Desired Output Speed: | $n_G$ = | *479* | rpm |
| Computed number of gear teeth: | | 48.0 | |
| *Enter:* Chosen No. of Gear Teeth: | $N_G$ = | *48* | |
| ***Computed data:*** | | | |
| Actual Output Speed: | $n_G$ = | 479.2 | rpm |
| Gear Ratio: | $m_G$ = | 2.40 | |
| Pitch Diameter - Pinion: | $D_P$ = | 3.333 | in |
| Pitch Diameter - Gear: | $D_G$ = | 8.000 | in |
| Center Distance: | C = | 5.667 | in |
| Pitch Line Speed: | $v_t$ = | 1004 | ft/min |
| Transmitted Load: | $W_t$ = | 1315 | lb |

| | | | |
|---|---|---|---|
| ***Secondary Input Data:*** | | | |
| | Min | Nom | Max |
| Face Width Guidelines (in): | 1.333 | 2.000 | 2.667 |
| *Enter:* Face Width: | $F$ = | *2.250* | in |
| Ratio: Face width/pinion diameter: | $F/D_P$ = | 0.68 | |
| Recommended range of ratio: | $0.50 < F/D_P < 2.00$ | | |
| *Enter:* Elastic Coefficient: | $C_p$ = *2300* | | Table 9-10 |
| *Enter:* Quality Number: | $A_v$ = *11* | | Table 9-3 |
| Dynamic Factor: | $K_v$ = 1.42 | | Table 9-9 |
| [Factors for computing $K_v$:] | $B$ = 0.826 | | $C$ = 59.75 |
| *Reference:* | $N_P$ = 20 | | $N_G$ = 48 |
| Bending Geometry Factor-Pinion: | $J_P$ = *0.325* | | Fig. 9-15 |
| Bending Geometry Factor-Gear: | $J_G$ = *0.395* | | Fig. 9-15 |
| *Reference:* | $m_G$ = 2.40 | | |
| *Enter:* Pitting Geometry Factor: | $I$ = *0.095* | | Fig. 9-21 |

***Factors in Design Analysis:***

| | | | | |
|---|---|---|---|---|
| Alignment Factor, $K_m = 1.0 + C_{pf} + C_{ma}$ | If F<1.0 | If F>1.0 | | |
| Pinion Proportion Factor, $C_{pf}$ = | 0.043 | 0.058 | $[0.50 < F/D_P < 2.00]$ | |
| *Enter:* $C_{pf}$ = | *0.058* | Figure 9-16 | | |
| Type of gearing: | Open | Commer. | Precision | Ex. Prec. |
| Mesh Alignment Factor, $C_{ma}$ = | 0.284 | 0.162 | 0.096 | 0.061 |
| *Enter:* $C_{ma}$ = | *0.162* | Figure 9-17 | | |
| Alignment Factor: $K_m$ = | 1.22 | [Computed] | | |
| Size Factor: $K_s$ = | *1.00* | Table 9-8: Use 1.00 if $P_d$ >= 5 | | |
| Pinion Rim Thickness Factor: $K_{BP}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | |
| Gear Rim Thickness Factor: $K_{BG}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | |
| Service Factor: $SF$ = | *1.00* | Use 1.00 if no unusual conditions | | |
| Reliability Factor: $K_R$ = | *1.00* | Table 9-11 Use 1.00 for $R$ = .99 | | |
| *Enter:* Design Life: | *8000* | hours | See Table 9-12 | |

| | | | | |
|---|---|---|---|---|
| | | Guidelines: $Y_N$, $Z_N$ | | |
| Pinion - Number of load cycles: $N_P$ = | 5.5E+08 | | | |
| Gear - Number of load cycles: $N_G$ = | 2.3E+08 | $10^7$ cycles | $>10^7$ | $<10^7$ |
| Bending Stress Cycle Factor: $Y_{NP}$ = | *0.95* | 1.00 | 0.95 | Fig. 9-22 |
| Bending Stress Cycle Factor: $Y_{NG}$ = | *0.96* | 1.00 | 0.96 | Fig. 9-22 |
| Pitting Stress Cycle Factor: $Z_{NP}$ = | *0.91* | 1.00 | 0.91 | Fig. 9-23 |
| Pitting Stress Cycle Factor: $Z_{NG}$ = | *0.93* | 1.00 | 0.93 | Fig. 9-23 |

***Stress Analysis: Bending***

| | | | |
|---|---|---|---|
| Pinion: Required $s_{at}$ = | 34,466 | psi | See Fig. 9-11 or |
| Gear: Required $s_{at}$ = | 28,063 | psi | Table 9-5 |

***Stress Analysis: Pitting***

| | | | |
|---|---|---|---|
| Pinion: Required $s_{ac}$ = | 189,152 | psi | See Fig. 9-12 or |
| Gear: Required $s_{ac}$ = | 185,084 | psi | Table 9-5 |

| | | |
|---|---|---|
| Required hardness of pinion HB: | 497 | Equations in Fig. 9-12-Grade 1 |
| Required hardness of gear HB: | 484 | Equations in Fig. 9-12-Grade 1 |

***Specify materials, alloy and heat treatment, for most severe requirement.***

***One possible material specification:***

Requires Grade 2 flame or induction harden. Suggest redesign to lower stress levels.

| | | | | | |
|---|---|---|---|---|---|
| Ans. Problem: | 37 | Computed bending stress number, $s_t$ = | 32743 | psi | Pinion |
| Ans. Problem: | 37 | Computed bending stress number, $s_t$ = | 26940 | psi | Gear |
| Ans. Problem: | 49 | Computed contact stress number, $s_c$ = | 172,128 | psi | Pinion |
| Ans. Problem: | 49 | Computed contact stress number, $s_c$ = | 172,128 | psi | Gear |

## DESIGN OF SPUR GEARS

**APPLICATION:** *Problems 38, 44, 50, 56*

*Small machine tool driven by an electric motor*

### *Initial Input Data:*

| | | | |
|---|---|---|---|
| Overload Factor: | $K_o$ = | *1.50* | Table 9-7 |
| Transmitted Power: | $P$ = | *0.5* | hp |
| Design Power | $P_{des}$ = | 0.75 | hp |
| Diametral Pitch: | $P_d$ = | *32* | Fig. 9-24 |
| Input Speed: | $n_P$ = | *3450* | rpm |
| Number of Pinion Teeth: | $N_P$ = | *24* | |
| Desired Output Speed: | $n_G$ = | *690* | rpm |
| Computed number of gear teeth: | | 120.0 | |
| *Enter:* Chosen No. of Gear Teeth: | $N_G$ = | *120* | |

### *Computed data:*

| | | | |
|---|---|---|---|
| Actual Output Speed: | $n_G$ = | 690.0 | rpm |
| Gear Ratio: | $m_G$ = | 5.00 | |
| Pitch Diameter - Pinion: | $D_P$ = | 0.750 | in |
| Pitch Diameter - Gear: | $D_G$ = | 3.750 | in |
| Center Distance: | C = | 2.250 | in |
| Pitch Line Speed: | $v_t$ = | 677 | ft/min |
| Transmitted Load: | $W_t$ = | 24 | lb |

### *Secondary Input Data:*

| | Min | Nom | Max |
|---|---|---|---|
| Face Width Guidelines (in): | 0.250 | 0.375 | 0.500 |
| *Enter:* Face Width: $F$ = | *0.500* | in | |
| Ratio: Face width/pinion diameter: $F/D_P$ = | 0.67 | | |
| Recommended range of ratio: | $0.50 < F/D_P < 2.00$ | | |

| | | | |
|---|---|---|---|
| *Enter:* Elastic Coefficient: | $C_p$ = *2300* | | Table 9-10 |
| *Enter:* Quality Number: | $A_v$ = *7* | | Table 9-3 |
| Dynamic Factor: | $K_v$ = 1.11 | | Table 9-9 |
| [Factors for computing $K_v$:] | $B$ = 0.397 | $C$ = 83.77 | |
| *Reference:* | $N_P$ = 24 | $N_G$ = 120 | |
| Bending Geometry Factor-Pinion: | $J_P$ = *0.360* | | Fig. 9-15 |
| Bending Geometry Factor-Gear: | $J_G$ = *0.440* | | Fig. 9-15 |
| *Reference:* | $m_G$ = 5.00 | | |
| *Enter:* Pitting Geometry Factor: | $I$ = *0.118* | | Fig. 9-21 |

### *Factors in Design Analysis:*

| | | | | |
|---|---|---|---|---|
| Alignment Factor, $K_m = 1.0 + C_{pf} + C_{ma}$ | If F<1.0 | If F>1.0 | | |
| Pinion Proportion Factor, $C_{pf}$ = | 0.042 | 0.035 | $[0.50 < F/D_P < 2.00]$ | |
| *Enter:* $C_{pf}$ = | *0.042* | Figure 9-16 | | |
| Type of gearing: | Open | Commer. | Precision | Ex. Prec. |
| Mesh Alignment Factor, $C_{ma}$ = | 0.255 | 0.135 | 0.074 | 0.043 |
| *Enter:* $C_{ma}$ = | *0.074* | Figure 9-17 | | |
| Alignment Factor: $K_m$ = | 1.12 | [Computed] | | |
| Size Factor: $K_s$ = | *1.00* | Table 9-8: Use 1.00 if $P_d$ >= 5 | | |
| Pinion Rim Thickness Factor: $K_{BP}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | |
| Gear Rim Thickness Factor: $K_{BG}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | |
| Service Factor: $SF$ = | *1.25* | Use 1.00 if no unusual conditions | | |
| Reliability Factor: $K_R$ = | *1.50* | Table 9-11 Use 1.00 for $R$ = .99 | | |
| *Enter: Design Life:* | *12000* | hours | See Table 9-12 | |

| | | Guidelines: $Y_N$, $Z_N$ | | |
|---|---|---|---|---|
| | | $10^7$ cycles | $>10^7$ | $<10^7$ |
| Pinion - Number of load cycles: $N_P$ = | 2.5E+09 | | | |
| Gear - Number of load cycles: $N_G$ = | 5.0E+08 | | | |
| Bending Stress Cycle Factor: $Y_{NP}$ = | *0.92* | 1.00 | 0.92 | Fig. 9-22 |
| Bending Stress Cycle Factor: $Y_{NG}$ = | *0.95* | 1.00 | 0.95 | Fig. 9-22 |
| Pitting Stress Cycle Factor: $Z_{NP}$ = | *0.88* | 1.00 | 0.88 | Fig. 9-23 |
| Pitting Stress Cycle Factor: $Z_{NG}$ = | *0.91* | 1.00 | 0.91 | Fig. 9-23 |

***Stress Analysis: Bending***

| | | | |
|---|---|---|---|
| Pinion: Required $s_{at}$ = | 16,448 | psi | See Fig. 9-11 or |
| Gear: Required $s_{at}$ = | 13,033 | psi | Table 9-5 |

***Stress Analysis: Pitting***

| | | | |
|---|---|---|---|
| Pinion: Required $s_{ac}$ = | 156,966 | psi | See Fig. 9-12 or |
| Gear: Required $s_{ac}$ = | 151,791 | psi | Table 9-5 |

| | | |
|---|---|---|
| Required hardness of pinion HB: | 397 | Equations in Fig. 9-12-Grade 1 |
| Required hardness of gear HB: | 381 | Equations in Fig. 9-12-Grade 1 |

***Specify materials, alloy and heat treatment, for most severe requirement.***

***One possible material specification:***

SAE 4140 OQT 800, 429 HB, 14% elongation, good ductility

SAE 4140 OQT 900, 429 HB, 14% elongation, good ductility

| | | | | | |
|---|---|---|---|---|---|
| Ans. Problem: | 38 | Computed bending stress number, $s_t$ = | 8071 | psi | Pinion |
| Ans. Problem: | 38 | Computed bending stress number, $s_t$ = | 6603 | psi | Gear |
| Ans. Problem: | 50 | Computed contact stress number, $s_c$ = | 73,669 | psi | Pinion |
| Ans. Problem: | 50 | Computed contact stress number, $s_c$ = | 73,669 | psi | Gear |

## DESIGN OF SPUR GEARS

**APPLICATION:** *Problems 39, 45, 51, 57*

*Aircraft actuator driven by a universal electric motor*

### *Initial Input Data:*

| | | | |
|---|---|---|---|
| Overload Factor: | $K_o$ = | *1.50* | Table 9-7 |
| Transmitted Power: | $P$ = | *15* | hp |
| Design Power | $P_{des}$ = | 22.5 | hp |
| Diametral Pitch: | $P_d$ = | *10* | Fig. 9-24 |
| Input Speed: | $n_P$ = | *6500* | rpm |
| Number of Pinion Teeth: | $N_P$ = | *30* | |
| Desired Output Speed: | $n_G$ = | *2216* | rpm |
| Computed number of gear teeth: | | 88.0 | |
| *Enter:* Chosen No. of Gear Teeth: | $N_G$ = | *88* | |

### *Computed data:*

| | | | |
|---|---|---|---|
| Actual Output Speed: | $n_G$ = | 2215.9 | rpm |
| Gear Ratio: | $m_G$ = | 2.93 | |
| Pitch Diameter - Pinion: | $D_P$ = | 3.000 | in |
| Pitch Diameter - Gear: | $D_G$ = | 8.800 | in |
| Center Distance: | $C$ = | 5.900 | in |
| Pitch Line Speed: | $v_t$ = | 5105 | ft/min |
| Transmitted Load: | $W_t$ = | 97 | lb |

### *Secondary Input Data:*

| | Min | Nom | Max |
|---|---|---|---|
| Face Width Guidelines (in): | 0.800 | 1.200 | 1.600 |
| *Enter:* Face Width: | $F$ = | *1.500* | in |
| Ratio: Face width/pinion diameter: | $F/D_P$ = | 0.50 | |
| Recommended range of ratio: | $0.50 < F/D_P < 2.00$ | | |
| *Enter:* Elastic Coefficient: | $C_p$ = *2300* | | Table 9-10 |
| *Enter:* Quality Number: | $A_v$ = *5* | | Table 9-3 |
| Dynamic Factor: | $K_v$ = 1.00 | | Table 9-9 |
| [Factors for computing $K_v$:] | $B$ = 0.000 | | $C$ = 106.00 |
| *Reference:* | $N_P$ = 30 | | $N_G$ = 88 |
| Bending Geometry Factor-Pinion: | $J_P$ = *0.460* | | Fig. 9-15 |
| Bending Geometry Factor-Gear: | $J_G$ = *0.530* | | Fig. 9-15 |
| *Reference:* | $m_G$ = 2.93 | | |
| *Enter:* Pitting Geometry Factor: | $I$ = *0.130* | | Fig. 9-21 |

### *Factors in Design Analysis:*

| | | | | |
|---|---|---|---|---|
| Alignment Factor, $K_m=1.0+C_{pf}+C_{ma}$ | If F<1.0 | If F>1.0 | | |
| Pinion Proportion Factor, $C_{pf}$ = | 0.025 | 0.031 | [$0.50 < F/D_P < 2.00$] | |
| *Enter:* $C_{pf}$ = | *0.031* | Figure 9-16 | | |
| Type of gearing: | Open | Commer. | Precision | Ex. Prec. |
| Mesh Alignment Factor, $C_{ma}$ = | 0.272 | 0.150 | 0.086 | 0.053 |
| *Enter:* $C_{ma}$ = | *0.053* | Figure 9-17 | | |
| Alignment Factor: $K_m$ = | 1.08 | [Computed] | | |
| Size Factor: $K_s$ = | *1.00* | Table 9-8: Use 1.00 if $P_d >= 5$ | | |
| Pinion Rim Thickness Factor: $K_{BP}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | |
| Gear Rim Thickness Factor: $K_{BG}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | |
| Service Factor: $SF$ = | *1.00* | Use 1.00 if no unusual conditions | | |
| Reliability Factor: $K_R$ = | *1.50* | Table 9-11 Use 1.00 for $R$ = .99 | | |
| *Enter: Design Life:* | *4000* | hours | See Table 9-12 | |
| | | Guidelines: $Y_N$, $Z_N$ | | |
| Pinion - Number of load cycles: $N_P$ = | 1.6E+09 | $10^7$ cycles | $>10^7$ | $<10^7$ |
| Gear - Number of load cycles: $N_G$ = | 5.3E+08 | | | |
| Bending Stress Cycle Factor: $Y_{NP}$ = | *0.93* | 1.00 | 0.93 | Fig. 9-22 |
| Bending Stress Cycle Factor: $Y_{NG}$ = | *0.95* | 1.00 | 0.95 | Fig. 9-22 |
| Pitting Stress Cycle Factor: $Z_{NP}$ = | *0.89* | 1.00 | 0.89 | Fig. 9-23 |
| Pitting Stress Cycle Factor: $Z_{NG}$ = | *0.91* | 1.00 | 0.91 | Fig. 9-23 |

### *Stress Analysis: Bending*

| | | | |
|---|---|---|---|
| Pinion: Required $s_{at}$ = | 3,685 | psi | See Fig. 9-11 or |
| Gear: Required $s_{at}$ = | 3,131 | psi | Table 9-5 |

### *Stress Analysis: Pitting*

| | | | |
|---|---|---|---|
| Pinion: Required $s_{ac}$ = | 63,637 | psi | See Fig. 9-12 or |
| Gear: Required $s_{ac}$ = | 62,239 | psi | Table 9-5 |
| Required hardness of pinion HB: | 107 | | Equations in Fig. 9-12-Grade 1 |
| Required hardness of gear HB: | 103 | | Equations in Fig. 9-12-Grade 1 |

***Specify materials, alloy and heat treatment, for most severe requirement.***

***One possible material specification:***

Stresses are quite low for steel gears. Suggest redesign.

| | | | | | |
|---|---|---|---|---|---|
| Ans. Problem: | 39 | Computed bending stress number, $s_t$ = | 2285 | psi | Pinion |
| Ans. Problem: | 39 | Computed bending stress number, $s_t$ = | 1983 | psi | Gear |
| Ans. Problem: | 51 | Computed contact stress number, $s_c$ = | 37,758 | psi | Pinion |
| Ans. Problem: | 51 | Computed contact stress number, $s_c$ = | 37,758 | psi | Gear |

## DESIGN OF SPUR GEARS

| | | | |
|---|---|---|---|
| ***APPLICATION:*** | *Problems 40, 46, 52, 58* | | |
| *Portable industrial water pump driven by a gasoline engine* | | | |
| ***Initial Input Data:*** | | | |
| Overload Factor: | $K_o$ = | *1.70* | Table 9-7 |
| Transmitted Power: | $P$ = | *125* | hp |
| Design Power | $P_{des}$ = | 212.5 | hp |
| Diametral Pitch: | $P_d$ = | *4* | Fig. 9-24 |
| Input Speed: | $n_P$ = | *2500* | rpm |
| Number of Pinion Teeth: | $N_P$ = | *32* | |
| Desired Output Speed: | $n_G$ = | *1050* | rpm |
| Computed number of gear teeth: | | 76.2 | |
| *Enter:* Chosen No. of Gear Teeth: | $N_G$ = | *76* | |
| ***Computed data:*** | | | |
| Actual Output Speed: | $n_G$ = | 1052.6 | rpm |
| Gear Ratio: | $m_G$ = | 2.38 | |
| Pitch Diameter - Pinion: | $D_P$ = | 8.000 | in |
| Pitch Diameter - Gear: | $D_G$ = | 19.000 | in |
| Center Distance: | C = | 13.500 | in |
| Pitch Line Speed: | $v_t$ = | 5236 | ft/min |
| Transmitted Load: | $W_t$ = | 788 | lb |

| ***Secondary Input Data:*** | | | | |
|---|---|---|---|---|
| | Min | Nom | Max | |
| Face Width Guidelines (in): | 2.000 | 3.000 | 4.000 | |
| *Enter:* Face Width: | $F$ = | *1.500* | in | |
| Ratio: Face width/pinion diameter: | $F/D_P$ = | 0.50 | Entered | |
| Recommended range of ratio: | $0.50 < F/D_P < 2.00$ | | | |
| *Enter:* Elastic Coefficient: | $Cp$ = | *2300* | Table 9-10 | |
| *Enter:* Quality Number: | $A_v$ = | *9* | Table 9-3 | |
| Dynamic Factor: | $K_v$ = | 1.56 | Table 9-9 | |
| [Factors for computing $K_v$:] | $B$ = 0.630 | $C$ = 70.71 | | |
| *Reference:* | $N_P$ = 32 | $N_G$ = 76 | | |
| Bending Geometry Factor-Pinion: | $J_P$ = | *0.465* | Fig. 9-15 | |
| Bending Geometry Factor-Gear: | $J_G$ = | *0.520* | Fig. 9-15 | |
| *Reference:* | $m_G$ = | 2.38 | | |
| *Enter:* Pitting Geometry Factor: | $I$ = | *0.124* | Fig. 9-21 | |

| ***Factors in Design Analysis:*** | | | | | |
|---|---|---|---|---|---|
| Alignment Factor, $K_m = 1.0 + C_{pf} + C_{ma}$ | | If F<1.0 | If F>1.0 | | |
| Pinion Proportion Factor, $C_{pf}$ = | | 0.025 | 0.031 | [$0.50 < F/D_P < 2.00$] | |
| *Enter:* $C_{pf}$ = | | *0.031* | Figure 9-16 | | |
| Type of gearing: | | Open | Commer. | Precision | Ex. Prec. |
| Mesh Alignment Factor, $C_{ma}$ = | | 0.272 | 0.150 | 0.086 | 0.053 |
| *Enter:* $C_{ma}$ = | | *0.15* | Figure 9-17 | | |
| Alignment Factor: $K_m$ = | | 1.18 | [Computed] | | |
| Size Factor: $K_s$ = | | *1.05* | Table 9-8: Use 1.00 if $P_d$ >= 5 | | |
| Pinion Rim Thickness Factor: $K_{BP}$ = | | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | |
| Gear Rim Thickness Factor: $K_{BG}$ = | | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | |
| Service Factor: $SF$ = | | *1.00* | Use 1.00 if no unusual conditions | | |
| Reliability Factor: $K_R$ = | | *1.00* | Table 9-11 Use 1.00 for $R$ = .99 | | |
| *Enter: Design Life:* | | *8000* | hours | See Table 9-12 | |
| Pinion - Number of load cycles: $N_P$ = | | 1.2E+09 | Guidelines: $Y_N$, $Z_N$ | | |
| Gear - Number of load cycles: $N_G$ = | | 5.1E+08 | $10^7$ cycles | $>10^7$ | $<10^7$ |
| Bending Stress Cycle Factor: $Y_{NP}$ = | | *0.93* | 1.00 | 0.93 | Fig. 9-22 |
| Bending Stress Cycle Factor: $Y_{NG}$ = | | *0.95* | 1.00 | 0.95 | Fig. 9-22 |
| Pitting Stress Cycle Factor: $Z_{NP}$ = | | *0.90* | 1.00 | 0.90 | Fig. 9-23 |
| Pitting Stress Cycle Factor: $Z_{NG}$ = | | *0.91* | 1.00 | 0.91 | Fig. 9-23 |
| ***Stress Analysis: Bending*** | | | | | |
| Pinion: Required $s_{at}$ = | | 15,968 | psi | See Fig. 9-11 or | |
| Gear: Required $s_{at}$ = | | 13,979 | psi | Table 9-5 | |
| ***Stress Analysis: Pitting*** | | | | | |
| Pinion: Required $s_{ac}$ = | | 106,609 | psi | See Fig. 9-12 or | |
| Gear: Required $s_{ac}$ = | | 105,438 | psi | Table 9-5 | |
| Required hardness of pinion HB: | | 241 | Equations in Fig. 9-12-Grade 1 | | |
| Required hardness of gear HB: | | 237 | Equations in Fig. 9-12-Grade 1 | | |
| ***Specify materials, alloy and heat treatment, for most severe requirement.*** | | | | | |
| ***One possible material specification:*** | | | | | |
| SAE 1040 WQT 1000, 269 HB, 22% elongation | | | | | |
| SAE 1040 WQT 1000, 269 HB, 22% elongation | | | | | |

| | | | | | |
|---|---|---|---|---|---|
| Ans. Problem: | 40 | Computed bending stress number, $s_t$ = | 14850 | psi | Pinion |
| Ans. Problem: | 40 | Computed bending stress number, $s_t$ = | 13280 | psi | Gear |
| Ans. Problem: | 52 | Computed contact stress number, $s_c$ = | 95,948 | psi | Pinion |
| Ans. Problem: | 52 | Computed contact stress number, $s_c$ = | 95,948 | psi | Gear |

# DESIGN OF SPUR GEARS

| APPLICATION: | Problems 41, 47, 53, 59 | | |
|---|---|---|---|
| *Lawn and garden tractor with fluid moror drive* | | | |
| ***Initial Input Data:*** | | | |
| Overload Factor: | $K_o$ = | *1.75* | Table 9-7 |
| Transmitted Power: | $P$ = | 2.5 | hp |
| Design Power | $P_{des}$ = | 4.375 | hp |
| Diametral Pitch: | $P_d$ = | *10* | Fig. 9-24 |
| Input Speed: | $n_P$ = | *680* | rpm |
| Number of Pinion Teeth: | $N_P$ = | *24* | |
| Desired Output Speed: | $n_G$ = | *263* | rpm |
| Computed number of gear teeth: | | 62.1 | |
| *Enter:* Chosen No. of Gear Teeth: | $N_G$ = | *62* | |
| ***Computed data:*** | | | |
| Actual Output Speed: | $n_G$ = | 263.2 | rpm |
| Gear Ratio: | $m_G$ = | 2.58 | |
| Pitch Diameter - Pinion: | $D_P$ = | 2.400 | in |
| Pitch Diameter - Gear: | $D_G$ = | 6.200 | in |
| Center Distance: | C = | 4.300 | in |
| Pitch Line Speed: | $v_t$ = | 427 | ft/min |
| Transmitted Load: | $W_t$ = | 193 | lb |

| ***Secondary Input Data:*** | Min | Nom | Max |
|---|---|---|---|
| Face Width Guidelines (in): | 0.800 | 1.200 | 1.600 |
| *Enter:* Face Width: $F$ = | | *1.250* | in |
| Ratio: Face width/pinion diameter: $F/D_P$ = | 0.52 | | |
| Recommended range of ratio: | $0.50 < F/D_P < 2.00$ | | |
| *Enter:* Elastic Coef. (Ductile iron) | $C_p$ = *2100* | | Table 9-10 |
| *Enter:* Quality Number: | $A_v$ = *11* | | Table 9-3 |
| Dynamic Factor: | $K_v$ = 1.28 | | Table 9-9 |
| [Factors for computing $K_v$:] | $B$ = 0.826 | | $C$ = 59.75 |
| *Reference:* | $N_P$ = 24 | | $N_G$ = 62 |
| Bending Geometry Factor-Pinion: | $J_P$ = *0.430* | | Fig. 9-15 |
| Bending Geometry Factor-Gear: | $J_G$ = *0.500* | | Fig. 9-15 |
| *Reference:* | $m_G$ = 2.58 | | |
| *Enter:* Pitting Geometry Factor: | $I$ = *0.122* | | Fig. 9-21 |

| ***Factors in Design Analysis:*** | | | | |
|---|---|---|---|---|
| Alignment Factor, $K_m = 1.0 + C_{pf} + C_{ma}$ | If F<1.0 | If F>1.0 | | |
| Pinion Proportion Factor, $C_{pf}$ = | 0.027 | 0.030 | [$0.50 < F/D_P < 2.00$] | |
| *Enter:* $C_{pf}$ = | *0.030* | Figure 9-16 | | |
| Type of gearing: | Open | Commer. | Precision | Ex. Prec. |
| Mesh Alignment Factor, $C_{ma}$ = | 0.268 | 0.147 | 0.083 | 0.051 |
| *Enter:* $C_{ma}$ = | *0.147* | Figure 9-17 | | |
| Alignment Factor: $K_m$ = | 1.18 | [Computed] | | |
| Size Factor: $K_s$ = | *1.00* | Table 9-8: Use 1.00 if $P_d$ >= 5 | | |
| Pinion Rim Thickness Factor: $K_{BP}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | |
| Gear Rim Thickness Factor: $K_{BG}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | |
| Service Factor: $SF$ = | *1.25* | Use 1.00 if no unusual conditions | | |
| Reliability Factor: $K_R$ = | *0.85* | Table 9-11 Use 1.00 for $R$ = .99 | | |
| *Enter: Design Life:* | *2000* | hours | See Table 9-12 | |
| Pinion - Number of load cycles: $N_P$ = | 8.2E+07 | Guidelines: $Y_N$, $Z_N$ | | |
| Gear - Number of load cycles: $N_G$ = | 3.2E+07 | $10^7$ cycles | $>10^7$ | $<10^7$ |
| Bending Stress Cycle Factor: $Y_{NP}$ = | *0.98* | 1.00 | 0.98 | Fig. 9-22 |
| Bending Stress Cycle Factor: $Y_{NG}$ = | *1.00* | 1.00 | 1.00 | Fig. 9-22 |
| Pitting Stress Cycle Factor: $Z_{NP}$ = | *0.95* | 1.00 | 0.95 | Fig. 9-23 |
| Pitting Stress Cycle Factor: $Z_{NG}$ = | *0.97* | 1.00 | 0.97 | Fig. 9-23 |
| ***Stress Analysis: Bending*** | | | | |
| Pinion: Required $s_{at}$ = | 10,254 | psi | See Fig. 9-11 or | |
| Gear: Required $s_{at}$ = | 8,642 | psi | Table 9-5 | |
| ***Stress Analysis: Pitting*** | | | | |
| Pinion: Required $s_{ac}$ = | 87,531 | psi | See Fig. 9-12 or | |
| Gear: Required $s_{ac}$ = | 85,727 | psi | Table 9-5 | |
| Required hardness of pinion HB: | 181 | Equations in Fig. 9-12-Grade 1 | | |
| Required hardness of gear HB: | 176 | Equations in Fig. 9-12-Grade 1 | | |
| ***Specify materials, alloy and heat treatment, for most severe requirement.*** | | | | |
| ***One possible material specification:*** | | | | |
| Pinion: Ductile iron 100-70-03 Q&T; $s_{at}$ = 27,000 psi; $s_{ac}$ = 92,000 psi | | | | |
| Gear: Ductile iron 100-70-03 Q&T; $s_{at}$ = 27,000 psi; $s_{ac}$ = 92,000 psi | | | | |

| | | | | | |
|---|---|---|---|---|---|
| Ans. Problem: | 41 | Computed bending stress number, $s_t$ = | 9458 | psi | Pinion |
| Ans. Problem: | 41 | Computed bending stress number, $s_t$ = | 8134 | psi | Gear |
| Ans. Problem: | 53 | Computed contact stress number, $s_c$ = | 78,263 | psi | Pinion |
| Ans. Problem: | 53 | Computed contact stress number, $s_c$ = | 78,263 | psi | Gear |

## DESIGN OF SPUR GEARS

**APPLICATION:** *Problem 60*

*Reciprocating compressor driven by an electric motor*

| *Initial Input Data:* | | | |
|---|---|---|---|
| Overload Factor: | $K_o$ = | *1.50* | Table 9-7 |
| Transmitted Power: | $P$ = | *5* | hp |
| Design Power | $P_{des}$ = | 7.5 | hp |
| Diametral Pitch: | $P_d$ = | *10* | Fig. 9-24 |
| Input Speed: | $n_P$ = | *1200* | rpm |
| Number of Pinion Teeth: | $N_P$ = | *18* | |
| Desired Output Speed: | $n_G$ = | *387.5* | rpm |
| Computed number of gear teeth: | | 55.7 | |
| *Enter:* Chosen No. of Gear Teeth: | $N_G$ = | *56* | |

| *Computed data:* | | | |
|---|---|---|---|
| Actual Output Speed: | $n_G$ = | 385.7 | rpm |
| Gear Ratio: | $m_G$ = | 3.11 | |
| Pitch Diameter - Pinion: | $D_P$ = | 1.800 | in |
| Pitch Diameter - Gear: | $D_G$ = | 5.600 | in |
| Center Distance: | C = | 3.700 | in |
| Pitch Line Speed: | $v_t$ = | 565 | ft/min |
| Transmitted Load: | $W_t$ = | 292 | lb |

| *Secondary Input Data:* | Min | Nom | Max |
|---|---|---|---|
| Face Width Guidelines (in): | 0.800 | 1.200 | 1.600 |
| *Enter:* Face Width: $F$ = | | *1.250* | in |
| Ratio: Face width/pinion diameter: $F/D_P$ = | | 0.69 | |
| Recommended range of ratio: | $0.50 < F/D_P < 2.00$ | | |

| | | | |
|---|---|---|---|
| *Enter:* Elastic Coefficient: | $Cp$ = *2300* | | Table 9-10 |
| *Enter:* Quality Number: | $A_v$ = *11* | | Table 9-3 |
| Dynamic Factor: | $K_v$ = 1.32 | | Table 9-9 |
| [Factors for computing $K_v$:] | $B$ = 0.826 | $C$ = 59.75 | |
| *Reference:* | $N_P$ = 18 | $N_G$ = 56 | |
| Bending Geometry Factor-Pinion: | $J_P$ = *0.320* | | Fig. 9-15 |
| Bending Geometry Factor-Gear: | $J_G$ = *0.400* | | Fig. 9-15 |
| *Reference:* | $m_G$ = 3.11 | | |
| *Enter:* Pitting Geometry Factor: | $I$ = *0.100* | | Fig. 9-21 |

### *Factors in Design Analysis:*

| Alignment Factor, $K_m$=1.0+$C_{pf}$+$C_{ma}$ | If F<1.0 | If F>1.0 | | |
|---|---|---|---|---|
| Pinion Proportion Factor, $C_{pf}$ = | 0.044 | 0.048 | [0.50 < $F/D_P$ < 2.00] | |
| *Enter:* $C_{pf}$ = | *0.048* | Figure 9-16 | | |
| Type of gearing: | Open | Commer. | Precision | Ex. Prec. |
| Mesh Alignment Factor, $C_{ma}$ = | 0.268 | 0.147 | 0.083 | 0.051 |
| *Enter:* $C_{ma}$ = | *0.147* | Figure 9-17 | | |
| Alignment Factor: $K_m$ = | 1.20 | [Computed] | | |

| | | |
|---|---|---|
| Size Factor: $K_s$ = | *1.00* | Table 9-8: Use 1.00 if $P_d$ >= 5 |
| Pinion Rim Thickness Factor: $K_{BP}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank |
| Gear Rim Thickness Factor: $K_{BG}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank |
| Service Factor: $SF$ = | *1.00* | Use 1.00 if no unusual conditions |
| Reliability Factor: $K_R$ = | *1.00* | Table 9-11 Use 1.00 for $R$ = .99 |

| *Enter: Design Life:* | *20000* | hours | See Table 9-12 | |
|---|---|---|---|---|
| | | Guidelines: $Y_N$, $Z_N$ | | |
| Pinion - Number of load cycles: $N_P$ = | 1.4E+09 | | | |
| Gear - Number of load cycles: $N_G$ = | 4.6E+08 | $10^7$ cycles | $>10^7$ | $<10^7$ |
| Bending Stress Cycle Factor: $Y_{NP}$ = | *0.93* | 1.00 | 0.93 | Fig. 9-22 |
| Bending Stress Cycle Factor: $Y_{NG}$ = | *0.95* | 1.00 | 0.95 | Fig. 9-22 |
| Pitting Stress Cycle Factor: $Z_{NP}$ = | *0.89* | 1.00 | 0.89 | Fig. 9-23 |
| Pitting Stress Cycle Factor: $Z_{NG}$ = | *0.92* | 1.00 | 0.92 | Fig. 9-23 |

***Stress Analysis: Bending***

| | | |
|---|---|---|
| Pinion: Required $s_{at}$ = | 18,543 psi | See Fig. 9-11 or |
| Gear: Required $s_{at}$ = | 14,522 psi | Table 9-5 |

***Stress Analysis: Pitting***

| | | |
|---|---|---|
| Pinion: Required $s_{ac}$ = | 143,088 psi | See Fig. 9-12 or |
| Gear: Required $s_{ac}$ = | 138,422 psi | Table 9-5 |

| | | |
|---|---|---|
| Required hardness of pinion HB: | 354 | Equations in Fig. 9-12-Grade 1 |
| Required hardness of gear HB: | 340 | Equations in Fig. 9-12-Grade 1 |

***Specify materials, alloy and heat treatment, for most severe requirement.***

***One possible material specification:***

Pinion requires HB 354: SAE 4140 OQT 900; HB 388, 16% Elongation

Gear requires HB 340: SAE 4140 OQT 1000; HB 340, 18% Elongation

***Comments:***

It would be reasonable to specify the same heat treatment for both the pinion and the gear because their contact stresses are very similar.

## DESIGN OF SPUR GEARS

**APPLICATION:** *Problem 61*

*Milling machine driven by an electric motor*

| **Initial Input Data:** | | | |
|---|---|---|---|
| Overload Factor: | $K_o$ = | *1.50* | Table 9-7 |
| Transmitted Power: | $P$ = | *20* | hp |
| Design Power | $P_{des}$ = | 30 | hp |
| Diametral Pitch: | $P_d$ = | *6* | Fig. 9-24 |
| Input Speed: | $n_P$ = | *550* | rpm |
| Number of Pinion Teeth: | $N_P$ = | *24* | |
| Desired Output Speed: | $n_G$ = | *185* | rpm |
| Computed number of gear teeth: | | 71.4 | |
| *Enter:* Chosen No. of Gear Teeth: | $N_G$ = | *71* | |

| **Computed data:** | | | |
|---|---|---|---|
| Actual Output Speed: | $n_G$ = | 185.9 | rpm |
| Gear Ratio: | $m_G$ = | 2.96 | |
| Pitch Diameter - Pinion: | $D_P$ = | 4.000 | in |
| Pitch Diameter - Gear: | $D_G$ = | 11.833 | in |
| Center Distance: | C = | 7.917 | in |
| Pitch Line Speed: | $v_t$ = | 576 | ft/min |
| Transmitted Load: | $W_t$ = | 1146 | lb |

| **Secondary Input Data:** | Min | Nom | Max |
|---|---|---|---|
| Face Width Guidelines (in): | 1.333 | 2.000 | 2.667 |
| *Enter:* Face Width: | $F$ = | *2.000* | in |
| Ratio: Face width/pinion diameter: | $F/D_P$ = | 0.50 | |
| Recommended range of ratio: | $0.50 < F/D_P < 2.00$ | | |

| | | | |
|---|---|---|---|
| *Enter:* Elastic Coefficient: | $C_p$ = *2300* | | Table 9-10 |
| *Enter:* Quality Number: | $A_v$ = *9* | | Table 9-3 |
| Dynamic Factor: | $K_v$ = 1.20 | | Table 9-9 |
| [Factors for computing $K_v$:] | $B$ = 0.630 | $C$ = 70.71 | |
| *Reference:* | $N_P$ = 24 | $N_G$ = 71 | |
| Bending Geometry Factor-Pinion: | $J_P$ = *0.350* | | Fig. 9-15 |
| Bending Geometry Factor-Gear: | $J_G$ = *0.420* | | Fig. 9-15 |
| *Reference:* | $m_G$ = 2.96 | | |
| *Enter:* Pitting Geometry Factor: | $I$ = *0.108* | | Fig. 9-21 |

**Factors in Design Analysis:**

| Alignment Factor, $K_m = 1.0 + C_{pf} + C_{ma}$ | If F<1.0 | If F>1.0 | | |
|---|---|---|---|---|
| Pinion Proportion Factor, $C_{pf}$ = | 0.025 | 0.038 | [$0.50 < F/D_P < 2.00$] | |
| *Enter:* $C_{pf}$ = | *0.038* | Figure 9-16 | | |
| Type of gearing: | Open | Commer. | Precision | Ex. Prec. |
| Mesh Alignment Factor, $C_{ma}$ = | 0.280 | 0.158 | 0.093 | 0.058 |
| *Enter:* $C_{ma}$ = | *0.093* | Figure 9-17 | | |
| Alignment Factor: $K_m$ = | 1.13 | [Computed] | | |

| | | |
|---|---|---|
| Size Factor: $K_s$ = | *1.00* | Table 9-8: Use 1.00 if $P_d$ >= 5 |
| Pinion Rim Thickness Factor: $K_{BP}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank |
| Gear Rim Thickness Factor: $K_{BG}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank |
| Service Factor: $SF$ = | *1.00* | Use 1.00 if no unusual conditions |
| Reliability Factor: $K_R$ = | *1.25* | Table 9-11 Use 1.00 for $R$ = .99 |

| | | | | |
|---|---|---|---|---|
| *Enter: Design Life:* | *20000* | hours | See Table 9-12 | |
| Pinion - Number of load cycles: $N_P$ = | 6.6E+08 | Guidelines: $Y_N$, $Z_N$ | | |
| Gear - Number of load cycles: $N_G$ = | 2.2E+08 | $10^7$ cycles | >$10^7$ | <$10^7$ |
| Bending Stress Cycle Factor: $Y_{NP}$ = | *0.94* | 1.00 | 0.94 | Fig. 9-22 |
| Bending Stress Cycle Factor: $Y_{NG}$ = | *0.96* | 1.00 | 0.96 | Fig. 9-22 |
| Pitting Stress Cycle Factor: $Z_{NP}$ = | *0.91* | 1.00 | 0.91 | Fig. 9-23 |
| Pitting Stress Cycle Factor: $Z_{NG}$ = | *0.93* | 1.00 | 0.93 | Fig. 9-23 |

***Stress Analysis: Bending***

| | | |
|---|---|---|
| Pinion: Required $s_{at}$ = | 26,640 psi | See Fig. 9-11 or |
| Gear: Required $s_{at}$ = | 21,737 psi | Table 9-5 |

***Stress Analysis: Pitting***

| | | |
|---|---|---|
| Pinion: Required $s_{ac}$ = | 164,319 psi | See Fig. 9-12 or |
| Gear: Required $s_{ac}$ = | 160,785 psi | Table 9-5 |

| | | |
|---|---|---|
| Required hardness of pinion HB: | 420 | Equations in Fig. 9-12-Grade 1 |
| Required hardness of gear HB: | 409 | Equations in Fig. 9-12-Grade 1 |

***Specify materials, alloy and heat treatment, for most severe requirement.***

***One possible material specification:***

Pinion and gear require flame or induction hardening

SAE 4140 OQT 800; HB 352, 21% Elongation-Core: Case harden to HRC 50 min.

***Comments:***

It would be reasonable to specify the same heat treatment for both the pinion and the gear because their contact stresses are very similar.

## DESIGN OF SPUR GEARS

**APPLICATION:** Problem 62

*Punch press driven by an electric motor*

### *Initial Input Data:*

| | | | |
|---|---|---|---|
| Overload Factor: | $K_o$ = | *1.75* | Table 9-7 |
| Transmitted Power: | $P$ = | *50* hp | |
| Design Power | $P_{des}$ = | 87.5 hp | |
| Diametral Pitch: | $P_d$ = | *4* | Fig. 9-24 |
| Input Speed: | $n_P$ = | *900* rpm | |
| Number of Pinion Teeth: | $N_P$ = | *24* | |
| Desired Output Speed: | $n_G$ = | *227.5* rpm | |
| Computed number of gear teeth: | | 94.9 | |
| *Enter:* Chosen No. of Gear Teeth: | $N_G$ = | *95* | |

### *Computed data:*

| | | |
|---|---|---|
| Actual Output Speed: | $n_G$ = | 227.4 rpm |
| Gear Ratio: | $m_G$ = | 3.96 |
| Pitch Diameter - Pinion: | $D_P$ = | 6.000 in |
| Pitch Diameter - Gear: | $D_G$ = | 23.750 in |
| Center Distance: | C = | 14.875 in |
| Pitch Line Speed: | $v_t$ = | 1414 ft/min |
| Transmitted Load: | $W_t$ = | 1167 lb |

### *Secondary Input Data:*

| | Min | Nom | Max |
|---|---|---|---|
| Face Width Guidelines (in): | 2.000 | 3.000 | 4.000 |
| *Enter:* Face Width: | $F$ = | *3.000* | in |

Ratio: Face width/pinion diameter: $F/D_P$ = 0.50

Recommended range of ratio: $0.50 < F/D_P < 2.00$

| | | |
|---|---|---|
| *Enter:* Elastic Coefficient: | $C_p$ = *2300* | Table 9-10 |
| *Enter:* Quality Number: | $A_v$ = *11* | Table 9-3 |
| Dynamic Factor: | $K_v$ = 1.50 | Table 9-9 |
| [Factors for computing $K_v$:] | $B$ = 0.826 | $C$ = 59.75 |
| *Reference:* | $N_P$ = 24 | $N_G$ = 95 |
| Bending Geometry Factor-Pinion: | $J_P$ = *0.360* | Fig. 9-15 |
| Bending Geometry Factor-Gear: | $J_G$ = *0.420* | Fig. 9-15 |
| *Reference:* | $m_G$ = 3.96 | |
| *Enter:* Pitting Geometry Factor: | $I$ = *0.114* | Fig. 9-21 |

### *Factors in Design Analysis:*

| Alignment Factor, $K_m=1.0+C_{pf}+C_{ma}$ | | If F<1.0 | If F>1.0 | | |
|---|---|---|---|---|---|
| Pinion Proportion Factor, $C_{pf}$ = | | 0.025 | 0.050 | [$0.50 < F/D_P < 2.00$] | |
| *Enter:* $C_{pf}$ = | | *0.05* | Figure 9-16 | | |
| Type of gearing: | | Open | Commer. | Precision | Ex. Prec. |
| Mesh Alignment Factor, $C_{ma}$ = | | 0.296 | 0.173 | 0.105 | 0.068 |
| *Enter:* $C_{ma}$ = | | *0.173* | Figure 9-17 | | |
| Alignment Factor: $K_m$ = | | 1.22 | [Computed] | | |

| | | |
|---|---|---|
| Size Factor: $K_s$ = | *1.05* | Table 9-8: Use 1.00 if $P_d$ >= 5 |
| Pinion Rim Thickness Factor: $K_{BP}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank |
| Gear Rim Thickness Factor: $K_{BG}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank |
| Service Factor: $SF$ = | *1.00* | Use 1.00 if no unusual conditions |
| Reliability Factor: $K_R$ = | *1.25* | Table 9-11 Use 1.00 for $R$ = .99 |
| *Enter: Design Life:* | *20000* | hours See Table 9-12 |

| | | Guidelines: $Y_N$, $Z_N$ | | |
|---|---|---|---|---|
| Pinion - Number of load cycles: $N_P$ = | 1.1E+09 | | | |
| Gear - Number of load cycles: $N_G$ = | 2.7E+08 | $10^7$ cycles | $>10^7$ | $<10^7$ |
| Bending Stress Cycle Factor: $Y_{NP}$ = | *0.94* | 1.00 | 0.94 | Fig. 9-22 |
| Bending Stress Cycle Factor: $Y_{NG}$ = | *0.96* | 1.00 | 0.96 | Fig. 9-22 |
| Pitting Stress Cycle Factor: $Z_{NP}$ = | *0.90* | 1.00 | 0.90 | Fig. 9-23 |
| Pitting Stress Cycle Factor: $Z_{NG}$ = | *0.93* | 1.00 | 0.93 | Fig. 9-23 |

***Stress Analysis: Bending***

| | | |
|---|---|---|
| Pinion: Required $s_{at}$ = | 19,333 psi | See Fig. 9-11 or |
| Gear: Required $s_{at}$ = | 16,226 psi | Table 9-5 |

***Stress Analysis: Pitting***

| | | |
|---|---|---|
| Pinion: Required $s_{ac}$ = | 139,718 psi | See Fig. 9-12 or |
| Gear: Required $s_{ac}$ = | 135,211 psi | Table 9-5 |

| | | |
|---|---|---|
| Required hardness of pinion HB: | 344 | Equations in Fig. 9-12-Grade 1 |
| Required hardness of gear HB: | 330 | Equations in Fig. 9-12-Grade 1 |

***Specify materials, alloy and heat treatment, for most severe requirement.***

***One possible material specification:***

Pinion requires HB 344: SAE 1040 WQT 800; HB 352; 21% elongation

Gear requires HB 330: SAE 1040 WQT 800; HB 352; 21% elongation

***Comments:***

It would be reasonable to specify the same heat treatment for both the pinion and the gear because their contact stresses are very similar.

## DESIGN OF SPUR GEARS

| | | | |
|---|---|---|---|
| ***APPLICATION:*** | *Problem 63* | | |
| *Cement mixer driven by a gasoline engine* | | | |
| ***Initial Input Data:*** | | | |
| Overload Factor: | $K_o$ = | *2.00* | Table 9-7 |
| Transmitted Power: | $P$ = | *2.5* | hp |
| Design Power | $P_{des}$ = | 5 | hp |
| Diametral Pitch: | $P_d$ = | *8* | Fig. 9-24 |
| Input Speed: | $n_P$ = | *900* | rpm |
| Number of Pinion Teeth: | $N_P$ = | *18* | |
| Desired Output Speed: | $n_G$ = | *75* | rpm |
| Computed number of gear teeth: | | 216.0 | |
| *Enter:* Chosen No. of Gear Teeth: | $N_G$ = | *216* | |
| ***Computed data:*** | | | |
| Actual Output Speed: | $n_G$ = | 75.0 | rpm |
| Gear Ratio: | $m_G$ = | 12.00 | |
| Pitch Diameter - Pinion: | $D_P$ = | 2.250 | in |
| Pitch Diameter - Gear: | $D_G$ = | 27.000 | in |
| Center Distance: | C = | 14.625 | in |
| Pitch Line Speed: | $v_t$ = | 530 | ft/min |
| Transmitted Load: | $W_t$ = | 156 | lb |

| | | | |
|---|---|---|---|
| ***Secondary Input Data:*** | | | |
| | Min | Nom | Max |
| Face Width Guidelines (in): | 1.000 | 1.500 | 2.000 |
| *Enter:* Face Width: | $F$ = | *2.250* | in |
| Ratio: Face width/pinion diameter: | $F/D_P$ = | 1.00 | |
| Recommended range of ratio: | $0.50 < F/D_P < 2.00$ | | |
| *Enter:* Elastic Coefficient: | $Cp$ = *2100* | | Table 9-10 |
| *Enter:* Quality Number: | $A_v$ = *12* | | Table 9-3 |
| Dynamic Factor: | $K_v$ = 1.38 | | Table 9-9 |
| [Factors for computing $K_v$:] | $B$ = 0.915 | $C$ = 54.74 | |
| *Reference:* | $N_P$ = 18 | $N_G$ = 216 | |
| Bending Geometry Factor-Pinion: | $J_P$ = *0.325* | | Fig. 9-15 |
| Bending Geometry Factor-Gear: | $J_G$ = *0.430* | | Fig. 9-15 |
| *Reference:* | $m_G$ = 12.00 | | |
| *Enter:* Pitting Geometry Factor: | $I$ = *0.116* | | Fig. 9-21 |

| | | | | |
|---|---|---|---|---|
| ***Factors in Design Analysis:*** | | | | |
| Alignment Factor, $K_m$=1.0+$C_{pf}$+$C_{ma}$ | If F<1.0 | If F>1.0 | | |
| Pinion Proportion Factor, $C_{pf}$ = | 0.075 | 0.091 | [0.50 < $F/D_P$ < 2.00] | |
| *Enter:* $C_{pf}$ = | *0.091* | Figure 9-16 | | |
| Type of gearing: | Open | Commer. | Precision | Ex. Prec. |
| Mesh Alignment Factor, $C_{ma}$ = | 0.284 | 0.162 | 0.096 | 0.061 |
| *Enter:* $C_{ma}$ = | *0.284* | Figure 9-17 | | |
| Alignment Factor: $K_m$ = | 1.38 | [Computed] | | |
| Size Factor: $K_s$ = | *1.00* | Table 9-8: Use 1.00 if $P_d$ >= 5 | | |
| Pinion Rim Thickness Factor: $K_{BP}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | |
| Gear Rim Thickness Factor: $K_{BG}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | |
| Service Factor: $SF$ = | *1.00* | Use 1.00 if no unusual conditions | | |
| Reliability Factor: $K_R$ = | *1.00* | Table 9-11 Use 1.00 for $R$ = .99 | | |
| *Enter: Design Life:* | *8000* | hours | See Table 9-12 | |
| Pinion - Number of load cycles: $N_P$ = | 4.3E+08 | Guidelines: $Y_N$, $Z_N$ | | |
| Gear - Number of load cycles: $N_G$ = | 3.6E+07 | $10^7$ cycles | >$10^7$ | <$10^7$ |
| Bending Stress Cycle Factor: $Y_{NP}$ = | *0.95* | 1.00 | 0.95 | Fig. 9-22 |
| Bending Stress Cycle Factor: $Y_{NG}$ = | *0.99* | 1.00 | 0.99 | Fig. 9-22 |
| Pitting Stress Cycle Factor: $Z_{NP}$ = | *0.92* | 1.00 | 0.92 | Fig. 9-23 |
| Pitting Stress Cycle Factor: $Z_{NG}$ = | *0.97* | 1.00 | 0.97 | Fig. 9-23 |

| | | | |
|---|---|---|---|
| ***Stress Analysis: Bending*** | | | |
| Pinion: Required $s_{at}$ = | 6,796 | psi | See Fig. 9-11 or |
| Gear: Required $s_{at}$ = | 4,929 | psi | Table 9-5 |
| ***Stress Analysis: Pitting*** | | | |
| Pinion: Required $s_{ac}$ = | 72,362 | psi | See Fig. 9-12 or |
| Gear: Required $s_{ac}$ = | 68,632 | psi | Table 9-5 |
| Required hardness of pinion HB: | 134 | Equations in Fig. 9-12-Grade 1 | |
| Required hardness of gear HB: | 123 | Equations in Fig. 9-12-Grade 1 | |

***Specify materials, alloy and heat treatment, for most severe requirement.***

***One possible material specification:***

Pinion requires less than HB 180: SAE 1040 CD; HB 160; 12% elongation

Gear requires grey cast iron, ASTM A48, Class 40 [Table 9-6]

***Comments:***

Large gear can be conveniently cast and affixed to the drum of the cement mixer.

Steel gear can be mounted on engine shaft.

## DESIGN OF SPUR GEARS

**APPLICATION:** Problem 64

***Wood chipper driven by a gasoline engine:*** ***Speed Increaser***

***Initial Input Data:***

| | | | |
|---|---|---|---|
| Overload Factor: | $K_o$ = | *2.75* | Table 9-7 |
| Transmitted Power: | $P$ = | *75* | hp |
| Design Power | $P_{des}$ = | 206.25 | hp |
| Diametral Pitch: | $P_d$ = | *6* | Fig. 9-24 |
| Input Speed: | $n_P$ = | *2200* | rpm |
| Number of Pinion Teeth: | $N_P$ = | *41* | |
| Desired Output Speed: | $n_G$ = | *4550* | rpm |
| Computed number of gear teeth: | | 19.8 | |
| *Enter:* Chosen No. of Gear Teeth: | $N_G$ = | *20* | |

***Computed data: Note - Gear drives pinion***

| | | | |
|---|---|---|---|
| Actual Output Speed: | $n_G$ = | 4510.0 | rpm |
| Gear Ratio: | $m_G$ = | 2.05 | |
| Pitch Diameter - Pinion: | $D_P$ = | 6.833 | in |
| Pitch Diameter - Gear: | $D_G$ = | 3.333 | in |
| Center Distance: | C = | 5.083 | in |
| Pitch Line Speed: | $v_t$ = | 3936 | ft/min |
| Transmitted Load: | $W_t$ = | 629 | lb |

***Secondary Input Data:***

| | Min | Nom | Max |
|---|---|---|---|
| Face Width Guidelines (in): | 1.333 | 2.000 | 2.667 |
| *Enter:* Face Width: $F$ = | | *3.000* | in |
| Ratio: Face width/pinion diameter: $F/D_P$ = | | 0.90 | |
| Recommended range of ratio: | $0.50 < F/D_P < 2.00$ | | |

| | | | |
|---|---|---|---|
| *Enter:* Elastic Coefficient: | $Cp$ = *2300* | | Table 9-10 |
| *Enter:* Quality Number: | $A_v$ = *11* | | Table 9-3 |
| Dynamic Factor: | $K_v$ = 1.81 | | Table 9-9 |
| [Factors for computing $K_v$:] | $B$ = 0.826 | $C$ = 59.75 | |
| *Reference:* | $N_P$ = 41 | $N_G$ = 20 | |
| Bending Geometry Factor-Pinion: | $J_P$ = *0.380* | | Fig. 9-15 |
| Bending Geometry Factor-Gear: | $J_G$ = *0.330* | | Fig. 9-15 |
| *Reference:* | $m_G$ = 2.05 | | |
| *Enter:* Pitting Geometry Factor: | $I$ = *0.096* | | Fig. 9-21 |

***Factors in Design Analysis:***

| | | | | | |
|---|---|---|---|---|---|
| Alignment Factor, $K_m=1.0+C_{pf}+C_{ma}$ | | If F<1.0 | If F>1.0 | | |
| Pinion Proportion Factor, $C_{pf}$ = | | 0.065 | 0.090 | [$0.50 < F/D_P < 2.00$] | |
| *Enter:* $C_{pf}$ = | | *0.09* | Figure 9-16 | | |
| Type of gearing: | | Open | Commer. | Precision | Ex. Prec. |
| Mesh Alignment Factor, $C_{ma}$ = | | 0.296 | 0.173 | 0.105 | 0.068 |
| *Enter:* $C_{ma}$ = | | *0.296* | Figure 9-17 | | |
| Alignment Factor: $K_m$ = | | 1.39 | [Computed] | | |

| | | |
|---|---|---|
| Size Factor: $K_s$ = | *1.00* | Table 9-8: Use 1.00 if $P_d$ >= 5 |
| Pinion Rim Thickness Factor: $K_{BP}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank |
| Gear Rim Thickness Factor: $K_{BG}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank |
| Service Factor: $SF$ = | *1.00* | Use 1.00 if no unusual conditions |
| Reliability Factor: $K_R$ = | *1.00* | Table 9-11 Use 1.00 for $R$ = .99 |
| *Enter: Design Life:* | *8000* | hours See Table 9-12 |

| | | Guidelines: $Y_N$, $Z_N$ | | |
|---|---|---|---|---|
| | | $10^7$ cycles | $>10^7$ | $<10^7$ |
| Pinion - Number of load cycles: $N_P$ = | 1.1E+09 | | | |
| Gear - Number of load cycles: $N_G$ = | 5.2E+08 | | | |
| Bending Stress Cycle Factor: $Y_{NP}$ = | *0.94* | 1.00 | 0.94 | Fig. 9-22 |
| Bending Stress Cycle Factor: $Y_{NG}$ = | *0.95* | 1.00 | 0.95 | Fig. 9-22 |
| Pitting Stress Cycle Factor: $Z_{NP}$ = | *0.90* | 1.00 | 0.90 | Fig. 9-23 |
| Pitting Stress Cycle Factor: $Z_{NG}$ = | *0.91* | 1.00 | 0.91 | Fig. 9-23 |

***Stress Analysis: Bending - Note - Gear drives pinion***

| | | |
|---|---|---|
| Gear: Required $s_{at}$ = | 24,281 psi | See Fig. 9-11 or |
| Pinion: Required $s_{at}$ = | 27,666 psi | Table 9-5 |

***Stress Analysis: Pitting - Adjusted equation to use $D_g$ in place of $D_p$***

| | | |
|---|---|---|
| Pinion: Required $s_{ac}$ = | 171,762 psi | See Fig. 9-12 or |
| Gear: Required $s_{ac}$ = | 169,875 psi | Table 9-5 |

| | | |
|---|---|---|
| Required hardness of pinion HB: | 443 | Equations in Fig. 9-12-Grade 1 |
| Required hardness of gear HB: | 437 | Equations in Fig. 9-12-Grade 1 |

***Specify materials, alloy and heat treatment, for most severe requirement.***

***One possible material specification:***

Pinion and gear require flame or induction hardening

SAE 4140 OQT 800; HB 352, 21% Elongation-Core: Case harden to HRC 54 min.

***Comments:***

It would be reasonable to specify the same heat treatment for both the pinion and the gear because their contact stresses are very similar.

**DESIGN OF SPUR GEARS** — *APPLICATION:* *Small tractor driven by a gasoline engine*

**NOTE: SI Metric data** — *Problem 65*

| | | | |
|---|---|---|---|
| ***Initial Input Data:*** | | | |
| Input Power: | $P$ = | *3.0* | kW |
| Input Speed: | $n_P$ = | *600* | rpm |
| [See Table 8-3] Module: | m = | *3.00* | mm |
| Number of Pinion Teeth: | $N_P$ = | *20* | |
| Desired Output Speed: | $n_G$ = | *175* | rpm |
| Computed number of gear teeth: | | 68.6 | |
| *Enter:* Chosen No. of Gear Teeth: | $N_G$ = | *68* | |
| ***Computed data:*** | | | |
| Actual Output Speed: | $n_G$ = | 176.5 | rpm |
| Gear Ratio: | $m_G$ = | 3.40 | |
| Pitch Diameter - Pinion: | $D_P$ = | 60.00 | mm |
| Pitch Diameter - Gear: | $D_G$ = | 204.00 | mm |
| Center Distance: | C = | 132.00 | mm |
| Pitch Line Speed: | $v_t$ = | 1.88 | m/s |
| Transmitted Load: | $W_t$ = | 1592 | N |

***Secondary Input Data:***

| | Min | Nom | Max |
|---|---|---|---|
| mm | 24 | 36 | 48 |
| *Enter:* Face Width: | $F$ = | *40.0* | mm |
| Ratio: Face width/pinion diameter: | $F/D_P$ = | 0.67 | |
| Recommended range of ratio: | $0.50 < F/D_P < 2.00$ | | |
| *Enter:* Elastic Coefficient: | $C_p$ = *191* | | Table 9-10 |
| *Enter:* Quality Number: | $A_v$ = *12* | | Table 9-3 |
| REF: $N_P$, $N_G$ = | 20 | 68 | |
| *Enter:* Bending Geometry Factors: | Press. angle = 20 deg | | |
| Pinion: | $J_P$ = *0.330* | | Fig. 9-15 |
| Gear: | $J_G$ = *0.415* | | Fig. 9-15 |
| *Enter:* Pitting Geometry Factor: | $I$ = *0.104* | | Fig. 9-21 |
| REF: $m_G$ = | 3.40 | | |

***Factors in Design Analysis:***

| | | | | |
|---|---|---|---|---|
| Alignment Factor, $K_m$=1.0+$C_{pf}$+$C_{ma}$ | If F<25 | If F>25 mm | | |
| Pinion Proportion Factor, $C_{pf}$ = | 0.042 | 0.049 | [0.50 < $F/D_P$ < 2.00] | |
| *Enter:* $C_{pf}$ = | *0.049* | Figure 9-16 | | |
| Type of gearing: | Open | Commer. | Precision | Ex. Prec. |
| Mesh Alignment Factor, $C_{ma}$ = | 0.274 | 0.152 | 0.088 | 0.054 |
| *Enter:* $C_{ma}$ = | *0.274* | Figure 9-17 | | |
| Alignment Factor: $K_m$ = | 1.32 | [Computed] | | |
| Overload Factor: $K_o$ = | *2.00* | Table 9-7 | | |
| Size Factor: $K_s$ = | *1.00* | Table 9-8: Use 1.00 if $P_d$ >= 5 | | |
| Pinion Rim Thickness Factor: $K_{BP}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | |
| Gear Rim Thickness Factor: $K_{BG}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | |
| Dynamic Factor: $K_v$ = | 1.32 | [Computed: See Fig. 9-19] | | |
| Service Factor: $SF$ = | *1.00* | Use 1.00 if no unusual conditions | | |
| Reliability Factor: $K_R$ = | *1.00* | Table 9-11 | Use 1.00 for $R$ = .99 | |
| *Enter: Design Life:* | *5000* | hours | See Table 9-12 | |

| For $K_v$: | |
|---|---|
| $B$ | 0.915 |
| $C$ | 3.90 |

| | | Guidelines: $Y_N$, $Z_N$ | | |
|---|---|---|---|---|
| Pinion - Number of load cycles: $N_P$ = | 1.8E+08 | | | |
| Gear - Number of load cycles: $N_G$ = | 5.3E+07 | $10^7$ cycles | $>10^7$ | $<10^7$ |
| Bending Stress Cycle Factor: $Y_{NP}$ = | *0.97* | 1.00 | 0.97 | Fig. 9-22 |
| Bending Stress Cycle Factor: $Y_{NG}$ = | *0.99* | 1.00 | 0.99 | Fig. 9-22 |
| Pitting Stress Cycle Factor: $Z_{NP}$ = | *0.94* | 1.00 | 0.94 | Fig. 9-23 |
| Pitting Stress Cycle Factor: $Z_{NG}$ = | *0.96* | 1.00 | 0.96 | Fig. 9-23 |

| | | | Through-Hardened Grade 1 Steel | |
|---|---|---|---|---|
| ***Stress Analysis: Bending*** | | | | |
| Pinion: Required $s_{at}$ = | 144 MPa | See Fig. 9-11 or | HB 105 | Fig. 9-11 |
| Gear: Required $s_{at}$ = | 113 MPa | Table 9-5 | HB 46 | Fig. 9-11 |
| ***Stress Analysis: Pitting*** | | | | |
| Pinion: Required $s_{ac}$ = | 958 MPa | See Fig. 9-12 or | HB 341 | Fig. 9-12 |
| Gear: Required $s_{ac}$ = | 938 MPa | Table 9-5 | HB 332 | Fig. 9-12 |

***Specify materials, alloy and heat treatment, for most severe requirement.***

***One possible material specification:*** Steel pinion, Steel gear

Pinion requires HB 341: SAE 4340 OQT 1000; HB 363

Gear requires HB 332: SAE 4340 OQT 1000; HB 363 (Same as pinion)

**DESIGN OF SPUR GEARS** — *APPLICATION:* *Electric power generator driven by a water turbine*

**NOTE: SI Metric data** — *Problem 66*

***Initial Input Data:***

| | | | |
|---|---|---|---|
| Input Power: | $P$ = | *75.0* | kW |
| Input Speed: | $n_P$ = | *4500* | rpm |
| [See Table 8-3] Module: | m = | *4.00* | mm |
| Number of Pinion Teeth: | $N_P$ = | *24* | |
| Desired Output Speed: | $n_G$ = | *3600* | rpm |
| Computed number of gear teeth: | | 30.0 | |
| *Enter:* Chosen No. of Gear Teeth: | $N_G$ = | *30* | |

***Computed data:***

| | | | |
|---|---|---|---|
| Actual Output Speed: | $n_G$ = | 3600.0 | rpm |
| Gear Ratio: | $m_G$ = | 1.25 | |
| Pitch Diameter - Pinion: | $D_P$ = | 96.00 | mm |
| Pitch Diameter - Gear: | $D_G$ = | 120.00 | mm |
| Center Distance: | C = | 108.00 | mm |
| Pitch Line Speed: | $v_t$ = | 22.62 | m/s |
| Transmitted Load: | $W_t$ = | 3316 | N |

***Secondary Input Data:***

| | | Min | Nom | Max |
|---|---|---|---|---|
| | mm | 32 | 48 | 64 |
| *Enter:* Face Width: | $F$ = | *50.0* | mm | |
| Ratio: Face width/pinion diameter: | $F/D_P$ = | 0.52 | | |
| Recommended range of ratio: | $0.50 < F/D_P < 2.00$ | | | |
| *Enter:* Elastic Coefficient: | $C_p$ = *191* | | Table 9-10 | |
| *Enter:* Quality Number: | $A_v$ = *7* | | Table 9-3 | |
| REF: $N_P$, $N_G$ = | 24 | 30 | | |
| *Enter:* Bending Geometry Factors: | Press. angle = 20 deg | | | |
| Pinion: | $J_P$ = *0.347* | Fig. 9-15 | | |
| Gear: | $J_G$ = *0.365* | Fig. 9-15 | | |
| *Enter:* Pitting Geometry Factor: | $I$ = *0.084* | Fig. 9-21 | | |
| **REF:** $m_G$ = | 1.25 | | | |

***Factors in Design Analysis:***

| | | | | | |
|---|---|---|---|---|---|
| Alignment Factor, $K_m = 1.0 + C_{pf} + C_{ma}$ | | If F<25 | If F>25 mm | | |
| Pinion Proportion Factor, $C_{pf}$ = | | 0.027 | 0.040 | [$0.50 < F/D_P < 2.00$] | |
| *Enter:* $C_{pf}$ = | | *0.040* | Figure 9-16 | | |
| Type of gearing: | | Open | Commer. | Precision | Ex. Prec. |
| Mesh Alignment Factor, $C_{ma}$ = | | 0.280 | 0.158 | 0.093 | 0.058 |
| *Enter:* $C_{ma}$ = | | *0.158* | Figure 9-17 | | |
| Alignment Factor: $K_m$ = | | 1.20 | [Computed] | | |
| Overload Factor: $K_o$ = | | *1.20* | Table 9-7 | | |
| Size Factor: $K_s$ = | | *1.00* | Table 9-8: Use 1.00 if $P_d$ >= 5 | | |
| Pinion Rim Thickness Factor: $K_{BP}$ = | | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | |
| Gear Rim Thickness Factor: $K_{BG}$ = | | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | |
| Dynamic Factor: $K_v$ = | | 1.26 | [Computed: See Fig. 9-19] | | |
| Service Factor: $SF$ = | | *1.00* | Use 1.00 if no unusual conditions | | |
| Reliability Factor: $K_R$ = | | *1.00* | Table 9-11 | Use 1.00 for $R$ = .99 | |
| *Enter: Design Life:* | | *100000* | hours | See Table 9-12 | |
| Pinion - Number of load cycles: $N_P$ = | | 2.7E+10 | Guidelines: $Y_N$, $Z_N$ | | |
| Gear - Number of load cycles: $N_G$ = | | 2.2E+10 | $10^7$ cycles | $>10^7$ | $<10^7$ |
| Bending Stress Cycle Factor: $Y_{NP}$ = | | *0.88* | 1.00 | 0.88 | Fig. 9-22 |
| Bending Stress Cycle Factor: $Y_{NG}$ = | | *0.89* | 1.00 | 0.89 | Fig. 9-22 |
| Pitting Stress Cycle Factor: $Z_{NP}$ = | | *0.83* | 1.00 | 0.83 | Fig. 9-23 |
| Pitting Stress Cycle Factor: $Z_{NG}$ = | | *0.84* | 1.00 | 0.84 | Fig. 9-23 |

| For $K_v$: | |
|---|---|
| $B$ | 0.397 |
| $C$ | 5.97 |

| | | | | Through-Hardened Grade 1 Steel | |
|---|---|---|---|---|---|
| ***Stress Analysis: Bending*** | | | | | |
| Pinion: Required $s_{at}$ = | 98 | MPa | See Fig. 9-11 or | HB 19 | Fig. 9-11 |
| Gear: Required $s_{at}$ = | 93 | MPa | Table 9-5 | HB 8 | Fig. 9-11 |
| ***Stress Analysis: Pitting*** | | | | | |
| Pinion: Required $s_{ac}$ = | 889 | MPa | See Fig. 9-12 or | HB 310 | Fig. 9-12 |
| Gear: Required $s_{ac}$ = | 878 | MPa | Table 9-5 | HB 305 | Fig. 9-12 |

***Specify materials, alloy and heat treatment, for most severe requirement.***

***One possible material specification:*** Steel pinion, Steel gear

Pinion requires HB 310: SAE 4340 OQT 1100; HB 321

Gear requires HB 305: SAE 4340 OQT 1100; HB 321 (Same as pinion)

## DESIGN OF SPUR GEARS

**APPLICATION:** Problem 67

*Commercial band saw driven by an electric motor*

### *Initial Input Data:*

| | | | |
|---|---|---|---|
| Overload Factor: | $K_o$ = | *1.50* | Table 9-7 |
| Transmitted Power: | $P$ = | *12* | hp |
| Design Power | $P_{des}$ = | 18 | hp |
| Diametral Pitch: | $P_d$ = | *10* | Fig. 9-24 |
| Input Speed: | $n_P$ = | *3450* | rpm |
| Number of Pinion Teeth: | $N_P$ = | *18* | |
| Desired Output Speed: | $n_G$ = | *730* | rpm |
| Computed number of gear teeth: | | 85.1 | |
| *Enter:* Chosen No. of Gear Teeth: | $N_G$ = | *85* | |

### *Computed data:*

| | | | |
|---|---|---|---|
| Actual Output Speed: | $n_G$ = | 730.6 | rpm |
| Gear Ratio: | $m_G$ = | 4.72 | |
| Pitch Diameter - Pinion: | $D_P$ = | 1.800 | in |
| Pitch Diameter - Gear: | $D_G$ = | 8.500 | in |
| Center Distance: | C = | 5.150 | in |
| Pitch Line Speed: | $v_t$ = | 1626 | ft/min |
| Transmitted Load: | $W_t$ = | 244 | lb |

### *Secondary Input Data:*

| | Min | Nom | Max |
|---|---|---|---|
| Face Width Guidelines (in): | 0.800 | 1.200 | 1.600 |
| *Enter:* Face Width: $F$ = | *1.250* | in | |
| Ratio: Face width/pinion diameter: $F/D_P$ = | 0.69 | | |
| Recommended range of ratio: | $0.50 < F/D_P < 2.00$ | | |

| | | | |
|---|---|---|---|
| *Enter:* Elastic Coefficient: | $C_p$ = *2300* | | Table 9-10 |
| *Enter:* Quality Number: | $A_v$ = *9* | | Table 9-3 |
| Dynamic Factor: | $K_v$ = 1.33 | | Table 9-9 |
| [Factors for computing $K_v$:] | $B$ = 0.630 | $C$ = 70.71 | |
| *Reference:* | $N_P$ = 18 | $N_G$ = 85 | |
| Bending Geometry Factor-Pinion: | $J_P$ = *0.310* | | Fig. 9-15 |
| Bending Geometry Factor-Gear: | $J_G$ = *0.400* | | Fig. 9-15 |
| *Reference:* | $m_G$ = 4.72 | | |
| *Enter:* Pitting Geometry Factor: | $I$ = *0.106* | | Fig. 9-21 |

### *Factors in Design Analysis:*

| Alignment Factor, $K_m=1.0+C_{pf}+C_{ma}$ | If F<1.0 | If F>1.0 | | |
|---|---|---|---|---|
| Pinion Proportion Factor, $C_{pf}$ = | 0.044 | 0.048 | $[0.50 < F/D_P < 2.00]$ | |
| *Enter:* $C_{pf}$ = | *0.048* | Figure 9-16 | | |
| Type of gearing: | Open | Commer. | Precision | Ex. Prec. |
| Mesh Alignment Factor, $C_{ma}$ = | 0.268 | 0.147 | 0.083 | 0.051 |
| *Enter:* $C_{ma}$ = | *0.147* | Figure 9-17 | | |
| Alignment Factor: $K_m$ = | 1.20 | [Computed] | | |

| | | |
|---|---|---|
| Size Factor: $K_s$ = | *1.00* | Table 9-8: Use 1.00 if $P_d >= 5$ |
| Pinion Rim Thickness Factor: $K_{BP}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank |
| Gear Rim Thickness Factor: $K_{BG}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank |
| Service Factor: $SF$ = | *1.00* | Use 1.00 if no unusual conditions |
| Reliability Factor: $K_R$ = | *1.00* | Table 9-11 Use 1.00 for $R$ = .99 |

| | | | | |
|---|---|---|---|---|
| *Enter: Design Life:* | *8000* | hours | See Table 9-12 | |
| Pinion - Number of load cycles: $N_P$ = | 1.7E+09 | Guidelines: $Y_N$, $Z_N$ | | |
| Gear - Number of load cycles: $N_G$ = | 3.5E+08 | $10^7$ cycles | $>10^7$ | $<10^7$ |
| Bending Stress Cycle Factor: $Y_{NP}$ = | *0.93* | 1.00 | 0.93 | Fig. 9-22 |
| Bending Stress Cycle Factor: $Y_{NG}$ = | *0.96* | 1.00 | 0.96 | Fig. 9-22 |
| Pitting Stress Cycle Factor: $Z_{NP}$ = | *0.89* | 1.00 | 0.89 | Fig. 9-23 |
| Pitting Stress Cycle Factor: $Z_{NG}$ = | *0.92* | 1.00 | 0.92 | Fig. 9-23 |

***Stress Analysis: Bending***

| | | | |
|---|---|---|---|
| Pinion: Required $s_{at}$ = | 16,101 | psi | See Fig. 9-11 or |
| Gear: Required $s_{at}$ = | 12,088 | psi | Table 9-5 |

***Stress Analysis: Pitting***

| | | | |
|---|---|---|---|
| Pinion: Required $s_{ac}$ = | 127,467 | psi | See Fig. 9-12 or |
| Gear: Required $s_{ac}$ = | 123,310 | psi | Table 9-5 |

| | | |
|---|---|---|
| Required hardness of pinion HB: | 305 | Equations in Fig. 9-12-Grade 1 |
| Required hardness of gear HB: | 293 | Equations in Fig. 9-12-Grade 1 |

***Specify materials, alloy and heat treatment, for most severe requirement.***

***One possible material specification:***

Pinion requires HB 305: SAE 4140 OQT 1100; HB 321, 19% Elongation

Gear requires HB 293: SAE 4140 OQT 1200; HB 293, 20% Elongation

***Comments:***

It would be reasonable to specify the same heat treatment for both the pinion and the gear because their contact stresses are very similar.

## DESIGN OF SPUR GEARS

**APPLICATION:** Problem 68

*Commercial band saw driven by an electric motor*

| | | | |
|---|---|---|---|
| ***Initial Input Data:*** | | | |
| Overload Factor: | $K_o$ = | *1.50* | Table 9-7 |
| Transmitted Power: | $P$ = | *12* | hp |
| Design Power | $P_{des}$ = | 18 | hp |
| Diametral Pitch: | $P_d$ = | *14* | Fig. 9-24 |
| Input Speed: | $n_P$ = | *3450* | rpm |
| Number of Pinion Teeth: | $N_P$ = | *18* | |
| Desired Output Speed: | $n_G$ = | *730* | rpm |
| Computed number of gear teeth: | | 85.1 | |
| *Enter:* Chosen No. of Gear Teeth: | $N_G$ = | *85* | |
| ***Computed data:*** | | | |
| Actual Output Speed: | $n_G$ = | 730.6 | rpm |
| Gear Ratio: | $m_G$ = | 4.72 | |
| Pitch Diameter - Pinion: | $D_P$ = | 1.286 | in |
| Pitch Diameter - Gear: | $D_G$ = | 6.071 | in |
| Center Distance: | $C$ = | 3.679 | in |
| Pitch Line Speed: | $v_t$ = | 1161 | ft/min |
| Transmitted Load: | $W_t$ = | 341 | lb |

| | | | |
|---|---|---|---|
| ***Secondary Input Data:*** | | | |
| | Min | Nom | Max |
| Face Width Guidelines (in): | 0.571 | 0.857 | 1.143 |
| *Enter:* Face Width: | $F$ = | *1.125* | in |
| Ratio: Face width/pinion diameter: $F/D_P$ = | 0.88 | | |
| Recommended range of ratio: | $0.50 < F/D_P < 2.00$ | | |
| *Enter:* Elastic Coefficient: | $C_p$ = *2300* | | Table 9-10 |
| *Enter:* Quality Number: | $A_v$ = *7* | | Table 9-3 |
| Dynamic Factor: | $K_v$ = 1.15 | | Table 9-9 |
| [Factors for computing $K_v$:] | $B$ = 0.397 | $C$ = 83.77 | |
| *Reference:* | $N_P$ = 18 | $N_G$ = 85 | |
| Bending Geometry Factor-Pinion: | $J_P$ = *0.310* | | Fig. 9-15 |
| Bending Geometry Factor-Gear: | $J_G$ = *0.400* | | Fig. 9-15 |
| *Reference:* | $m_G$ = 4.72 | | |
| *Enter:* Pitting Geometry Factor: | $I$ = *0.106* | | Fig. 9-21 |

***Factors in Design Analysis:***

| | | | | | |
|---|---|---|---|---|---|
| Alignment Factor, $K_m = 1.0 + C_{pf} + C_{ma}$ | | If F<1.0 | If F>1.0 | | |
| Pinion Proportion Factor, $C_{pf}$ = | | 0.063 | 0.064 | [$0.50 < F/D_P < 2.00$] | |
| *Enter:* $C_{pf}$ = | | *0.064* | Figure 9-16 | | |
| Type of gearing: | | Open | Commer. | Precision | Ex. Prec. |
| Mesh Alignment Factor, $C_{ma}$ = | | 0.266 | 0.145 | 0.082 | 0.049 |
| *Enter:* $C_{ma}$ = | | *0.145* | Figure 9-17 | | |
| Alignment Factor: $K_m$ = | | 1.21 | [Computed] | | |
| Size Factor: $K_s$ = | | *1.00* | Table 9-8: Use 1.00 if $P_d$ >= 5 | | |
| Pinion Rim Thickness Factor: $K_{BP}$ = | | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | |
| Gear Rim Thickness Factor: $K_{BG}$ = | | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | |
| Service Factor: $SF$ = | | *1.00* | Use 1.00 if no unusual conditions | | |
| Reliability Factor: $K_R$ = | | *1.00* | Table 9-11 Use 1.00 for $R$ = .99 | | |
| *Enter: Design Life:* | | *8000* | hours | See Table 9-12 | |
| Pinion - Number of load cycles: $N_P$ = | | 1.7E+09 | Guidelines: $Y_N$, $Z_N$ | | |
| Gear - Number of load cycles: $N_G$ = | | 3.5E+08 | $10^7$ cycles | $>10^7$ | $<10^7$ |
| Bending Stress Cycle Factor: $Y_{NP}$ = | | *0.93* | 1.00 | 0.93 | Fig. 9-22 |
| Bending Stress Cycle Factor: $Y_{NG}$ = | | *0.96* | 1.00 | 0.96 | Fig. 9-22 |
| Pitting Stress Cycle Factor: $Z_{NP}$ = | | *0.89* | 1.00 | 0.89 | Fig. 9-23 |
| Pitting Stress Cycle Factor: $Z_{NG}$ = | | *0.92* | 1.00 | 0.92 | Fig. 9-23 |

***Stress Analysis: Bending***

| | | | |
|---|---|---|---|
| Pinion: Required $s_{at}$ = | 30,567 | psi | See Fig. 9-11 or |
| Gear: Required $s_{at}$ = | 22,949 | psi | Table 9-5 |

***Stress Analysis: Pitting***

| | | | |
|---|---|---|---|
| Pinion: Required $s_{ac}$ = | 175,629 | psi | See Fig. 9-12 or |
| Gear: Required $s_{ac}$ = | 169,902 | psi | Table 9-5 |

| | | |
|---|---|---|
| Required hardness of pinion HB: | 455 | Equations in Fig. 9-12-Grade 1 |
| Required hardness of gear HB: | 437 | Equations in Fig. 9-12-Grade 1 |

***Specify materials, alloy and heat treatment, for most severe requirement.***

***One possible material specification:***

Pinion requires case hardening by carburizing
Gear requires case hardening by carburizing

***Specifications: Example selection***

Specify SAE 4620 DOQT 300; Case hardness HRC 62; ductile core; HB 248
For both pinion and gear

## DESIGN OF SPUR GEARS

| APPLICATION: | Problem 69 | | |
|---|---|---|---|
| *Machine tool driven by an electric motor* | | | |
| ***Initial Input Data:*** | | | |
| Overload Factor: | $K_o$ = | *1.50* | Table 9-7 |
| Transmitted Power: | $P$ = | *20* | hp |
| Design Power | $P_{des}$ = | 30 | hp |
| Diametral Pitch: | $P_d$ = | *8* | Fig. 9-24 |
| Input Speed: | $n_P$ = | *650* | rpm |
| Number of Pinion Teeth: | $N_P$ = | *22* | |
| Desired Output Speed: | $n_G$ = | *112.5* | rpm |
| Computed number of gear teeth: | | 127.1 | |
| *Enter:* Chosen No. of Gear Teeth: | $N_G$ = | *128* | |
| ***Computed data:*** | | | |
| Actual Output Speed: | $n_G$ = | 111.7 | rpm |
| Gear Ratio: | $m_G$ = | 5.82 | |
| Pitch Diameter - Pinion: | $D_P$ = | 2.750 | in |
| Pitch Diameter - Gear: | $D_G$ = | 16.000 | in |
| Center Distance: | $C$ = | 9.375 | in |
| Pitch Line Speed: | $v_t$ = | 468 | ft/min |
| Transmitted Load: | $W_t$ = | 1410 | lb |
| ***Secondary Input Data:*** | | | |
| | Min | Nom | Max |
| Face Width Guidelines (in): | 1.000 | 1.500 | 2.000 |
| *Enter:* Face Width: | $F$ = | *2.000* | in |
| Ratio: Face width/pinion diameter: | $F/D_P$ = | 0.73 | |
| Recommended range of ratio: | $0.50 < F/D_P < 2.00$ | | |
| *Enter:* Elastic Coefficient: | $C_p$ = | *2300* | Table 9-10 |
| *Enter:* Quality Number: | $A_v$ = | *7* | Table 9-3 |
| Dynamic Factor: | $K_v$ = | 1.10 | Table 9-9 |
| [Factors for computing $K_v$:] | $B$ = 0.397 | $C$ = 83.77 | |
| *Reference:* | $N_P$ = 22 | $N_G$ = 128 | |
| Bending Geometry Factor-Pinion: | $J_P$ = | *0.345* | Fig. 9-15 |
| Bending Geometry Factor-Gear: | $J_G$ = | *0.440* | Fig. 9-15 |
| *Reference:* | $m_G$ = | 5.82 | |
| *Enter:* Pitting Geometry Factor: | $I$ = | *0.106* | Fig. 9-21 |

| ***Factors in Design Analysis:*** | | | | |
|---|---|---|---|---|
| Alignment Factor, $K_m = 1.0 + C_{pf} + C_{ma}$ | If F<1.0 | If F>1.0 | | |
| Pinion Proportion Factor, $C_{pf}$ = | 0.048 | 0.060 | [$0.50 < F/D_P < 2.00$] | |
| *Enter:* $C_{pf}$ = | *0.060* | Figure 9-16 | | |
| Type of gearing: | Open | Commer. | Precision | Ex. Prec. |
| Mesh Alignment Factor, $C_{ma}$ = | 0.280 | 0.158 | 0.093 | 0.058 |
| *Enter:* $C_{ma}$ = | *0.093* | Figure 9-17 | | |
| Alignment Factor: $K_m$ = | 1.15 | [Computed] | | |
| Size Factor: $K_s$ = | *1.00* | Table 9-8: Use 1.00 if $P_d >= 5$ | | |
| Pinion Rim Thickness Factor: $K_{BP}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | |
| Gear Rim Thickness Factor: $K_{BG}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | |
| Service Factor: $SF$ = | *1.00* | Use 1.00 if no unusual conditions | | |
| Reliability Factor: $K_R$ = | *1.00* | Table 9-11 Use 1.00 for $R$ = .99 | | |
| *Enter: Design Life:* | *25000* | hours | See Table 9-12 | |
| Pinion - Number of load cycles: $N_P$ = | 9.8E+08 | Guidelines: $Y_N$, $Z_N$ | | |
| Gear - Number of load cycles: $N_G$ = | 1.7E+08 | $10^7$ cycles | $>10^7$ | $<10^7$ |
| Bending Stress Cycle Factor: $Y_{NP}$ = | *0.94* | 1.00 | 0.94 | Fig. 9-22 |
| Bending Stress Cycle Factor: $Y_{NG}$ = | *0.97* | 1.00 | 0.97 | Fig. 9-22 |
| Pitting Stress Cycle Factor: $Z_{NP}$ = | *0.90* | 1.00 | 0.90 | Fig. 9-23 |
| Pitting Stress Cycle Factor: $Z_{NG}$ = | *0.94* | 1.00 | 0.94 | Fig. 9-23 |
| ***Stress Analysis: Bending*** | | | | |
| Pinion: Required $s_{at}$ = | 32,958 | psi | See Fig. 9-11 or | |
| Gear: Required $s_{at}$ = | 25,043 | psi | Table 9-5 | |
| ***Stress Analysis: Pitting*** | | | | |
| Pinion: Required $s_{ac}$ = | 173,012 | psi | See Fig. 9-12 or | |
| Gear: Required $s_{ac}$ = | 165,650 | psi | Table 9-5 | |
| Required hardness of pinion HB: | 447 | Equations in Fig. 9-12-Grade 1 | | |
| Required hardness of gear HB: | 424 | Equations in Fig. 9-12-Grade 1 | | |
| ***Specify materials, alloy and heat treatment, for most severe requirement.*** | | | | |
| ***One possible material specification:*** | | | | |
| Pinion requires case hardening by carburizing | | | | |
| Gear requires case hardening by carburizing | | | | |
| ***Specifications: Example selection*** | | | | |
| Specify SAE 4620 DOQT 300; Case hardness HRC 62; ductile core; HB 248 | | | | |
| For both pinion and gear | | | | |

# DESIGN OF SPUR GEARS

**APPLICATION:** *Problem 70*

*Crane cable drum driven by an electric motor*

### *Initial Input Data:*

| | | | |
|---|---|---|---|
| Overload Factor: | $K_o$ = | *1.50* | Table 9-7 |
| Transmitted Power: | $P$ = | *25* | hp |
| Design Power | $P_{des}$ = | 37.5 | hp |
| Diametral Pitch: | $P_d$ = | *6* | Fig. 9-24 |
| Input Speed: | $n_P$ = | *925* | rpm |
| Number of Pinion Teeth: | $N_P$ = | *17* | |
| Desired Output Speed: | $n_G$ = | *163* | rpm |
| Computed number of gear teeth: | | 96.5 | |
| *Enter:* Chosen No. of Gear Teeth: | $N_G$ = | *96* | |

### *Computed data:*

| | | | |
|---|---|---|---|
| Actual Output Speed: | $n_G$ = | 163.8 | rpm |
| Gear Ratio: | $m_G$ = | 5.65 | |
| Pitch Diameter - Pinion: | $D_P$ = | 2.833 | in |
| Pitch Diameter - Gear: | $D_G$ = | 16.000 | in |
| Center Distance: | $C$ = | 9.417 | in |
| Pitch Line Speed: | $v_t$ = | 686 | ft/min |
| Transmitted Load: | $W_t$ = | 1202 | lb |

### *Secondary Input Data:*

| | Min | Nom | Max |
|---|---|---|---|
| Face Width Guidelines (in): | 1.333 | 2.000 | 2.667 |
| *Enter:* Face Width: $F$ = | | *2.600* | in |
| Ratio: Face width/pinion diameter: $F/D_P$ = | | 0.92 | |
| Recommended range of ratio: $0.50 < F/D_P < 2.00$ | | | |

| | | | |
|---|---|---|---|
| *Enter:* Elastic Coefficient: | $C_p$ = *2300* | | Table 9-10 |
| *Enter:* Quality Number: | $A_v$ = *7* | | Table 9-3 |
| Dynamic Factor: | $K_v$ = 1.11 | | Table 9-9 |
| [Factors for computing $K_v$:] | $B$ = 0.397 | $C$ = 83.77 | |
| *Reference:* | $N_P$ = 17 | $N_G$ = 96 | |
| Bending Geometry Factor-Pinion: | $J_P$ = *0.295* | | Fig. 9-15 |
| Bending Geometry Factor-Gear: | $J_G$ = *0.420* | | Fig. 9-15 |
| *Reference:* | $m_G$ = 5.65 | | |
| *Enter:* Pitting Geometry Factor: | $I$ = *0.109* | | Fig. 9-21 |

### *Factors in Design Analysis:*

| Alignment Factor, $K_m = 1.0 + C_{pf} + C_{ma}$ | If F<1.0 | If F>1.0 | | |
|---|---|---|---|---|
| Pinion Proportion Factor, $C_{pf}$ = | 0.067 | 0.087 | [$0.50 < F/D_P < 2.00$] | |
| *Enter:* $C_{pf}$ = | *0.087* | Figure 9-16 | | |
| Type of gearing: | Open | Commer. | Precision | Ex. Prec. |
| Mesh Alignment Factor, $C_{ma}$ = | 0.290 | 0.167 | 0.100 | 0.064 |
| *Enter:* $C_{ma}$ = | *0.167* | Figure 9-17 | | |
| Alignment Factor: $K_m$ = | 1.25 | [Computed] | | |

| | | |
|---|---|---|
| Size Factor: $K_s$ = | *1.00* | Table 9-8: Use 1.00 if $P_d >= 5$ |
| Pinion Rim Thickness Factor: $K_{BP}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank |
| Gear Rim Thickness Factor: $K_{BG}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank |
| Service Factor: $SF$ = | *1.00* | Use 1.00 if no unusual conditions |
| Reliability Factor: $K_R$ = | *1.00* | Table 9-11 Use 1.00 for $R$ = .99 |

| | | | | |
|---|---|---|---|---|
| *Enter: Design Life:* | *31200* | hours | See Table 9-12 | |
| | | Guidelines: $Y_N$, $Z_N$ | | |
| Pinion - Number of load cycles: $N_P$ = | 1.7E+09 | | | |
| Gear - Number of load cycles: $N_G$ = | 3.1E+08 | $10^7$ cycles | $>10^7$ | $<10^7$ |
| Bending Stress Cycle Factor: $Y_{NP}$ = | *0.93* | 1.00 | 0.93 | Fig. 9-22 |
| Bending Stress Cycle Factor: $Y_{NG}$ = | *0.96* | 1.00 | 0.96 | Fig. 9-22 |
| Pitting Stress Cycle Factor: $Z_{NP}$ = | *0.89* | 1.00 | 0.89 | Fig. 9-23 |
| Pitting Stress Cycle Factor: $Z_{NG}$ = | *0.92* | 1.00 | 0.92 | Fig. 9-23 |

***Stress Analysis: Bending***

| | | | |
|---|---|---|---|
| Pinion: Required $s_{at}$ = | 21,194 | psi | See Fig. 9-11 or |
| Gear: Required $s_{at}$ = | 14,421 | psi | Table 9-5 |

***Stress Analysis: Pitting***

| | | | |
|---|---|---|---|
| Pinion: Required $s_{ac}$ = | 144,763 | psi | See Fig. 9-12 or |
| Gear: Required $s_{ac}$ = | 140,043 | psi | Table 9-5 |

| | | |
|---|---|---|
| Required hardness of pinion HB: | 359 | Equations in Fig. 9-12-Grade 1 |
| Required hardness of gear HB: | 345 | Equations in Fig. 9-12-Grade 1 |

***Specify materials, alloy and heat treatment, for most severe requirement.***

***One possible material specification:***

Pinion requires HB 359; SAE 8650 OQT 1000; HB 363; 14% elongation

Pinion requires HB 345; SAE 8650 OQT 1000; HB 363; 14% elongation

***Comments:***

**SPUR GEARS**
**POWER TRANSMITTING CAPACITY**

*APPLICATION:* *Centrifugal pump driven by an electric motor*
*Chapter 9-Problem 71*

***Initial Input Data:***

| | | | |
|---|---|---|---|
| *Enter:* Face Width: | $F$ = | *1.250* | in |
| Input Speed: | $n_P$ = | *1725* | rpm |
| Diametral Pitch: | $P_d$ = | *10* | |
| Number of Pinion Teeth: | $N_P$ = | *25* | |
| Number of Gear Teeth: | $N_G$ = | *60* | |

***Computed data:***

| | | | |
|---|---|---|---|
| Actual Output Speed: | $n_G$ = | 718.8 | rpm |
| Gear Ratio: | $m_G$ = | 2.40 | |
| Pitch Diameter - Pinion: | $D_P$ = | 2.500 | in |
| Pitch Diameter - Gear: | $D_G$ = | 6.000 | in |
| Center Distance: | C = | 4.250 | in |
| Pitch Line Speed: | $v_t$ = | 1129 | ft/min |
| Transmitted Load at $P_{min}$ Capacity: | $W_t$ = | 377 | lb |

***Power Transmitting Capacity: (Using Eq. 9-32, 9-34)***

| | | |
|---|---|---|
| Pinion: Based on Bending Stress: | 25.08 | hp |
| Gear: Based on Bending Stress: | 27.27 | hp |
| Pinion: Based on Contact Stress: | 14.21 | hp |
| Gear: Based on Contact Stress: | 12.91 | hp |
| ***Power Transmitting Capacity:*** | ***12.91*** | ***hp*** |

| | | | |
|---|---|---|---|
| *Enter:* Elastic Coefficient: | $C_p$ = *2300* | | Table 9-10 |
| *Enter:* Quality Number: | $A_v$ = *9* | | Table 9-3 |
| REF: $N_P$, $N_G$ = | 25 | 60 | |
| *Enter:* Bending Geometry Factors: | Press. angle = 20 deg | | |
| Pinion: | $J_P$ = *0.363* | | Fig. 9-15 |
| Gear: | $J_G$ = *0.415* | | Fig. 9-15 |
| *Enter:* Pitting Geometry Factor: | $I$ = *0.104* | | Fig. 9-21 |
| **REF:** $m_G$ = | 2.40 | | |

***Factors in Design Analysis:***

| | | | | | |
|---|---|---|---|---|---|
| Alignment Factor, $K_m$=1.0+$C_{pf}$+$C_{ma}$ | If F<1.0 | If F>1.0 | $F/D_P$ = 0.50 | | |
| Pinion Proportion Factor, $C_{pf}$ = | 0.025 | 0.028 | [0.50 < $F/D_P$ < 2.00] | | |
| *Enter:* $C_{pf}$ = | *0.028* | Figure 9-16 | | | |
| Type of gearing: | Open | Commer. | Precision | Ex. Prec. | |
| Mesh Alignment Factor, $C_{ma}$ = | 0.268 | 0.147 | 0.083 | 0.051 | |
| *Enter:* $C_{ma}$ = | *0.147* | Figure 9-17 | | | |
| Alignment Factor: $K_m$ = | 1.18 | [Computed] | | | |
| Overload Factor: $K_o$ = | *1.50* | Table 9-7 | | | |
| Size Factor: $K_s$ = | *1.00* | Table 9-8: Use 1.00 if $P_d$ >= 5 | | | |
| Pinion Rim Thickness Factor: $K_{BP}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | | |
| Gear Rim Thickness Factor: $K_{BG}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | | |
| Dynamic Factor: $K_v$ = | 1.28 | [Computed: See Fig. 9-21] | | | |
| Service Factor: $SF$ = | *1.00* | Use 1.00 if no unusual conditions | | | |
| Reliability Factor: $K_R$ = | *1.00* | Table 9-8 | Use 1.00 for $R$ = .99 | | |
| *Enter: Design Life:* | *15000* | hours | See Table 9-7 | | |

| For $K_v$: | |
|---|---|
| *B* | 0.630 |
| *A* | 70.71 |

| | | Guidelines: $Y_N$, $Z_N$ | | |
|---|---|---|---|---|
| Pinion - Number of load cycles: $N_P$ = | 1.6E+09 | | | |
| Gear - Number of load cycles: $N_G$ = | 6.5E+08 | $10^7$ cycles | >$10^7$ | <$10^7$ |
| Bending Stress Cycle Factor: $Y_{NP}$ = | *0.93* | 1.00 | 0.93 | Fig. 9-22 |
| Bending Stress Cycle Factor: $Y_{NG}$ = | *0.94* | 1.00 | 0.94 | Fig. 9-22 |
| Pitting Stress Cycle Factor: $Z_{NP}$ = | *0.89* | 1.00 | 0.89 | Fig. 9-24 |
| Pitting Stress Cycle Factor: $Z_{NG}$ = | *0.91* | 1.00 | 0.91 | Fig. 9-24 |

***Allowable Bending Stress Numbers: (Input)***

| | | | Through-Hardened Grade 1 Steel | |
|---|---|---|---|---|
| Pinion: $s_{at}$ = | 39,100 psi | See Fig. 9-11 or | 39.1 ksi | Fig. 9-11 |
| Gear: $s_{at}$ = | 36,800 psi | Table 9-5 | 36.8 ksi | Fig. 9-11 |

***Allowable Contact Stress Numbers: (Input)***

| | | | | |
|---|---|---|---|---|
| Pinion: $s_{ac}$ = | 138,600 psi | See Fig. 9-12 or | 138.6 ksi | Fig. 9-12 |
| Gear: $s_{ac}$ = | 129,200 psi | Table 9-5 | 129.2 ksi | Fig. 9-12 |

***Material specification:*** Steel pinion; Steel gear: through hardened

| | | |
|---|---|---|
| Pinion material: | SAE 4140 OQT 1000 | 340 HB |
| Gear material: | SAE 4140 OQT 1100 | 311 HB |

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**SPUR GEARS**
**POWER TRANSMITTING CAPACITY**

*APPLICATION:* *Heavy duty conveyor driven by a gasoline engine*
*Chapter 9-Problem 72*

| *Initial Input Data:* | | | |
|---|---|---|---|
| *Enter:* Face Width: | $F$ = | *2.000* | in |
| Input Speed: | $n_P$ = | *1500* | rpm |
| Diametral Pitch: | $P_d$ = | *6* | |
| Number of Pinion Teeth: | $N_P$ = | *35* | |
| Number of Gear Teeth: | $N_G$ = | *100* | |

| *Computed data:* | | | |
|---|---|---|---|
| Actual Output Speed: | $n_G$ = | 525.0 | rpm |
| Gear Ratio: | $m_G$ = | 2.86 | |
| Pitch Diameter - Pinion: | $D_P$ = | 5.833 | in |
| Pitch Diameter - Gear: | $D_G$ = | 16.667 | in |
| Center Distance: | C = | 11.250 | in |
| Pitch Line Speed: | $v_t$ = | 2291 | ft/min |
| Transmitted Load at $P_{min}$ Capacity: | $W_t$ = | 277 | lb |

| ***Power Transmitting Capacity: (Using Eq. 9-32, 9-34)*** | | |
|---|---|---|
| Pinion: Based on Bending Stress: | 90.79 | hp |
| Gear: Based on Bending Stress: | 21.63 | hp |
| Pinion: Based on Contact Stress: | 86.50 | hp |
| Gear: Based on Contact Stress: | 19.26 | hp |
| ***Power Transmitting Capacity:*** | ***19.26*** | ***hp*** |

| | | | |
|---|---|---|---|
| *Enter:* Elastic Coefficient: | $C_p$ = *2100* | | Table 9-10 |
| *Enter:* Quality Number: | $A_v$ = *11* | | Table 9-3 |
| REF: $N_P$, $N_G$ = | 35 | 100 | |
| *Enter:* Bending Geometry Factors: | Press. angle = 20 deg | | |
| Pinion: | $J_P$ = *0.410* | | Fig. 9-15 |
| Gear: | $J_G$ = *0.450* | | Fig. 9-15 |
| *Enter:* Pitting Geometry Factor: | $I$ = *0.114* | | Fig. 9-21 |
| REF: $m_G$ = | 2.86 | | |

***Factors In Design Analysis:***

| | | | | |
|---|---|---|---|---|
| Alignment Factor, $K_m$=1.0+$C_{pf}$+$C_{ma}$ | If F<1.0 | If F>1.0 | $F/D_P$ = 0.50 | Set = 0.5 |
| Pinion Proportion Factor, $C_{pf}$ = | 0.025 | 0.038 | [0.50 < $F/D_P$ < 2.00] | |
| *Enter:* $C_{pf}$ = | *0.038* | Figure 9-16 | | |
| Type of gearing: | Open | Commer. | Precision | Ex. Prec. |
| Mesh Alignment Factor, $C_{ma}$ = | 0.280 | 0.158 | 0.093 | 0.058 |
| *Enter:* $C_{ma}$ = | *0.158* | Figure 9-17 | | |
| Alignment Factor: $K_m$ = | 1.20 | [Computed] | | |

| | | |
|---|---|---|
| Overload Factor: $K_o$ = | *2.00* | Table 9-7 |
| Size Factor: $K_s$ = | *1.00* | Table 9-8: Use 1.00 if $P_d$ >= 5 |
| Pinion Rim Thickness Factor: $K_{BP}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank |
| Gear Rim Thickness Factor: $K_{BG}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank |
| Dynamic Factor: $K_v$ = | 1.63 | [Computed: See Fig. 9-21] |
| Service Factor: $SF$ = | *1.00* | Use 1.00 if no unusual conditions |
| Reliability Factor: $K_R$ = | *1.00* | Table 9-8 Use 1.00 for $R$ = .99 |
| *Enter: Design Life:* | *15000* | hours See Table 9-7 |

| For $K_v$: | |
|---|---|
| $B$ | 0.826 |
| $A$ | 59.75 |

| | | Guidelines: $Y_N$, $Z_N$ | | |
|---|---|---|---|---|
| | | $10^7$ cycles | $>10^7$ | $<10^7$ |
| Pinion - Number of load cycles: $N_P$ = | 1.4E+09 | | | |
| Gear - Number of load cycles: $N_G$ = | 4.7E+08 | | | |
| Bending Stress Cycle Factor: $Y_{NP}$ = | *0.93* | 1.00 | 0.93 | Fig. 9-22 |
| Bending Stress Cycle Factor: $Y_{NG}$ = | *0.95* | 1.00 | 0.95 | Fig. 9-22 |
| Pitting Stress Cycle Factor: $Z_{NP}$ = | *0.89* | 1.00 | 0.89 | Fig. 9-24 |
| Pitting Stress Cycle Factor: $Z_{NG}$ = | *0.92* | 1.00 | 0.92 | Fig. 9-24 |

| ***Allowable Bending Stress Numbers: (Input)*** | | |
|---|---|---|
| Pinion: $s_{at}$ = | 40,000 psi | See Fig. 9-11 |
| Gear: $s_{at}$ = | 8,500 psi | Table 9-6 |
| ***Allowable Contact Stress Numbers: (Input)*** | | |
| Pinion: $s_{ac}$ = | 142,400 psi | See Fig. 9-12 |
| Gear: $s_{ac}$ = | 65,000 psi | Table 9-6 |

| Through-Hardened Grade 1 Steel | |
|---|---|
| 40.0 ksi | Fig. 9-11 |
| ~~12.8 ksi~~ | ~~Fig. 9-11~~ |
| 142.4 ksi | Fig. 9-12 |
| ~~29.1 ksi~~ | ~~Fig. 9-12~~ |

***Material specification:*** Steel pinion; Steel gear: through hardened
Pinion material: SAE 1040 WQT 800 352 HB
Gear: Gray cast iron, ASTM A48, Class 30

**SPUR GEARS**
**POWER TRANSMITTING CAPACITY**

*APPLICATION:* *Heavy duty conveyor driven by a gasoline engine*
*Chapter 9-Problem 73 - Redesign of system in Problem 72 to get capaciy > 25 hp*

***Initial Input Data:***

| | | | |
|---|---|---|---|
| *Enter:* Face Width: | $F$ = | *2.420* | in |
| Input Speed: | $n_P$ = | *1500* | rpm |
| Diametral Pitch: | $P_d$ = | *6* | |
| Number of Pinion Teeth: | $N_P$ = | *35* | |
| Number of Gear Teeth: | $N_G$ = | *100* | |

***Computed data:***

| | | | |
|---|---|---|---|
| Actual Output Speed: | $n_G$ = | 525.0 | rpm |
| Gear Ratio: | $m_G$ = | 2.86 | |
| Pitch Diameter - Pinion: | $D_P$ = | 5.833 | in |
| Pitch Diameter - Gear: | $D_G$ = | 16.667 | in |
| Center Distance: | $C$ = | 11.250 | in |
| Pitch Line Speed: | $v_t$ = | 2291 | ft/min |
| Transmitted Load at $P_{min}$ Capacity: | $W_t$ = | 361 | lb |

***Power Transmitting Capacity: (Using Eq. 9-32, 9-34)***

| | | |
|---|---|---|
| Pinion: Based on Bending Stress: | 118.13 | hp |
| Gear: Based on Bending Stress: | 28.14 | hp |
| Pinion: Based on Contact Stress: | 112.55 | hp |
| Gear: Based on Contact Stress: | 25.06 | hp |
| ***Power Transmitting Capacity:*** | ***25.06*** | ***hp*** |

| | | | |
|---|---|---|---|
| *Enter:* Elastic Coefficient: | $C_p$ = *2100* | | Table 9-10 |
| *Enter:* Quality Number: | $A_v$ = *10* | | Table 9-3 |
| REF: $N_P$, $N_G$ = | 35 | 100 | |
| *Enter:* Bending Geometry Factors: | Press. angle = 20 deg | | |
| Pinion: | $J_P$ = *0.410* | | Fig. 9-15 |
| Gear: | $J_G$ = *0.450* | | Fig. 9-15 |
| *Enter:* Pitting Geometry Factor: | $I$ = *0.114* | | Fig. 9-21 |
| REF: $m_G$ = | 2.86 | | |

***Note: Increased face width from 2.00 to 2.42 in.***
***Changed quality number from $A_v$ = 11 to 10 (More precise)***

***Factors in Design Analysis:***

| | | | | | |
|---|---|---|---|---|---|
| Alignment Factor, $K_m$=1.0+$C_{pf}$+$C_{ma}$ | | If F<1.0 | If F>1.0 | $F/D_P$ = 0.50 | Set = 0.5 |
| Pinion Proportion Factor, $C_{pf}$ = | | 0.025 | 0.043 | [0.50 < $F/D_P$ < 2.00] | |
| *Enter:* $C_{pf}$ = | | *0.043* | Figure 9-16 | | |
| Type of gearing: | | Open | Commer. | Precision | Ex. Prec. |
| Mesh Alignment Factor, $C_{ma}$ = | | 0.287 | 0.165 | 0.098 | 0.062 |
| *Enter:* $C_{ma}$ = | | *0.165* | Figure 9-17 | | |
| Alignment Factor: $K_m$ = | | 1.21 | [Computed] | | |
| Overload Factor: $K_o$ = | | *2.00* | Table 9-7 | | |
| Size Factor: $K_s$ = | | *1.00* | Table 9-8: Use 1.00 if $P_d$ >= 5 | | |
| Pinion Rim Thickness Factor: $K_{BP}$ = | | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | |
| Gear Rim Thickness Factor: $K_{BG}$ = | | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | |
| Dynamic Factor: $K_v$ = | | 1.50 | [Computed: See Fig. 9-21] | | |
| Service Factor: $SF$ = | | *1.00* | Use 1.00 if no unusual conditions | | |
| Reliability Factor: $K_R$ = | | *1.00* | Table 9-8 | Use 1.00 for $R$ = .99 | |

| For $K_v$: | |
|---|---|
| $B$ | 0.731 |
| $A$ | 65.04 |

| | | | | | |
|---|---|---|---|---|---|
| *Enter:* *Design Life:* | *15000* | hours | See Table 9-7 | | |
| Pinion - Number of load cycles: $N_P$ = | 1.4E+09 | Guidelines: $Y_N$, $Z_N$ | | | |
| Gear - Number of load cycles: $N_G$ = | 4.7E+08 | $10^7$ cycles | >$10^7$ | <$10^7$ | |
| Bending Stress Cycle Factor: $Y_{NP}$ = | *0.93* | 1.00 | 0.93 | Fig. 9-22 | |
| Bending Stress Cycle Factor: $Y_{NG}$ = | *0.95* | 1.00 | 0.95 | Fig. 9-22 | |
| Pitting Stress Cycle Factor: $Z_{NP}$ = | *0.89* | 1.00 | 0.89 | Fig. 9-24 | |
| Pitting Stress Cycle Factor: $Z_{NG}$ = | *0.92* | 1.00 | 0.92 | Fig. 9-24 | |

***Allowable Bending Stress Numbers: (Input)***

| | | | |
|---|---|---|---|
| Pinion: $s_{at}$ = | 40,000 | psi | See Fig. 9-11 |
| Gear: $s_{at}$ = | 8,500 | psi | Table 9-6 |

***Allowable Contact Stress Numbers: (Input)***

| | | | |
|---|---|---|---|
| Pinion: $s_{ac}$ = | 142,400 | psi | See Fig. 9-12 |
| Gear: $s_{ac}$ = | 65,000 | psi | Table 9-6 |

| Through-Hardened Grade 1 Steel | |
|---|---|
| 40.0 ksi | Fig. 9-11 |
| ~~12.8~~ ksi | ~~Fig. 9-11~~ |
| 142.4 ksi | Fig. 9-12 |
| ~~29.1~~ ksi | ~~Fig. 9-12~~ |

***Material specification:*** Steel pinion; Steel gear: through hardened
Pinion material: SAE 1040 WQT 800 352 HB
Gear: Gray cast iron, ASTM A48, Class 30

# DESIGN OF SPUR GEARS

| | | | |
|---|---|---|---|
| ***APPLICATION:*** | *Problem 74 - First pair* | | |
| *Assenbly conveyor driven by an electric motor* | | | |
| ***Double reduction - First pair - Input Data:*** | | | |
| Overload Factor: | $K_o$ = | *1.50* | Table 9-7 |
| Transmitted Power: | $P$ = | *10* | hp |
| Design Power | $P_{des}$ = | 15 | hp |
| Diametral Pitch: | $P_d$ = | *8* | Fig. 9-24 |
| Input Speed: | $n_P$ = | *1750* | rpm |
| Number of Pinion Teeth: | $N_P$ = | *18* | |
| Desired Output Speed: | $n_G$ = | *425* | rpm |
| Computed number of gear teeth: | | 74.1 | |
| *Enter:* Chosen No. of Gear Teeth: | $N_G$ = | *75* | |
| ***Computed data:*** | | | |
| Actual Output Speed: | $n_G$ = | 420.0 | rpm |
| Gear Ratio: | $m_G$ = | 4.17 | |
| Pitch Diameter - Pinion: | $D_P$ = | 2.250 | in |
| Pitch Diameter - Gear: | $D_G$ = | 9.375 | in |
| Center Distance: | C = | 5.813 | in |
| Pitch Line Speed: | $v_t$ = | 1031 | ft/min |
| Transmitted Load: | $W_t$ = | 320 | lb |

| | | | |
|---|---|---|---|
| ***Secondary Input Data:*** | | | |
| | Min | Nom | Max |
| Face Width Guidelines (in): | 1.000 | 1.500 | 2.000 |
| *Enter:* Face Width: | $F$ = | *1.500* | in |
| Ratio: Face width/pinion diameter: | $F/D_P$ = | 0.67 | |
| Recommended range of ratio: | $0.50 < F/D_P < 2.00$ | | |
| *Enter:* Elastic Coefficient: | $C_p$ = *2300* | | Table 9-10 |
| *Enter:* Quality Number: | $A_v$ = *11* | | Table 9-3 |
| Dynamic Factor: | $K_v$ = 1.43 | | Table 9-9 |
| [Factors for computing $K_v$:] | $B$ = 0.826 | $C$ = 59.75 | |
| *Reference:* | $N_P$ = 18 | $N_G$ = 75 | |
| Bending Geometry Factor-Pinion: | $J_P$ = *0.315* | | Fig. 9-15 |
| Bending Geometry Factor-Gear: | $J_G$ = *0.410* | | Fig. 9-15 |
| *Reference:* | $m_G$ = 4.17 | | |
| *Enter:* Pitting Geometry Factor: | $I$ = *0.106* | | Fig. 9-21 |

**Note: Larger part of the total reduction (4.17) in this pair**
**Higher diametral pitch - 8 compared to 6 in pair 2**

***Factors in Design Analysis:***

| | | | | | |
|---|---|---|---|---|---|
| Alignment Factor, $K_m=1.0+C_{pf}+C_{ma}$ | | If F<1.0 | If F>1.0 | | |
| Pinion Proportion Factor, $C_{pf}$ = | | 0.042 | 0.048 | [0.50 < $F/D_P$ < 2.00] | |
| *Enter:* $C_{pf}$ = | | *0.048* | Figure 9-16 | | |
| Type of gearing: | | Open | Commer. | Precision | Ex. Prec. |
| Mesh Alignment Factor, $C_{ma}$ = | | 0.272 | 0.150 | 0.086 | 0.053 |
| *Enter:* $C_{ma}$ = | | *0.150* | Figure 9-17 | | |
| Alignment Factor: $K_m$ = | | 1.20 | [Computed] | | |
| Size Factor: $K_s$ = | | *1.00* | Table 9-8: Use 1.00 if $P_d$ >= 5 | | |
| Pinion Rim Thickness Factor: $K_{BP}$ = | | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | |
| Gear Rim Thickness Factor: $K_{BG}$ = | | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | |
| Service Factor: $SF$ = | | *1.00* | Use 1.00 if no unusual conditions | | |
| Reliability Factor: $K_R$ = | | *1.00* | Table 9-11 Use 1.00 for $R$ = .99 | | |
| *Enter: Design Life:* | | *15000* | hours | See Table 9-12 | |
| Pinion - Number of load cycles: $N_P$ = | | 1.6E+09 | Guidelines: $Y_N$, $Z_N$ | | |
| Gear - Number of load cycles: $N_G$ = | | 3.8E+08 | $10^7$ cycles | $>10^7$ | $<10^7$ |
| Bending Stress Cycle Factor: $Y_{NP}$ = | | *0.93* | 1.00 | 0.93 | Fig. 9-22 |
| Bending Stress Cycle Factor: $Y_{NG}$ = | | *0.95* | 1.00 | 0.95 | Fig. 9-22 |
| Pitting Stress Cycle Factor: $Z_{NP}$ = | | *0.89* | 1.00 | 0.89 | Fig. 9-23 |
| Pitting Stress Cycle Factor: $Z_{NG}$ = | | *0.92* | 1.00 | 0.92 | Fig. 9-23 |
| ***Stress Analysis: Bending*** | | | | | |
| Pinion: Required $s_{at}$ = | | 14,940 | psi | See Fig. 9-11 or | |
| Gear: Required $s_{at}$ = | | 11,237 | psi | Table 9-5 | |
| ***Stress Analysis: Pitting*** | | | | | |
| Pinion: Required $s_{ac}$ = | | 123,772 | psi | See Fig. 9-12 or | |
| Gear: Required $s_{ac}$ = | | 119,736 | psi | Table 9-5 | |
| Required hardness of pinion HB: | | 294 | Equations in Fig. 9-12-Grade 1 | | |
| Required hardness of gear HB: | | 281 | Equations in Fig. 9-12-Grade 1 | | |

***Specify materials, alloy and heat treatment, for most severe requirement.***

***One possible material specification:***

Pinion requires HB 294; SAE 4340 OQT 1100; HB 321; 19% elongation
Gear requires HB 281; SAE 4340 OQT 1200; HB 293; 20% elongation

***Comments:***

# DESIGN OF SPUR GEARS

**APPLICATION:** *Problem 74 - Second pair*

*Assembly conveyor driven by an electric motor*

***Double reduction - Second pair - Input Data:***

| | | | |
|---|---|---|---|
| Overload Factor: | $K_o$ = | 1.50 | Table 9-7 |
| Transmitted Power: | $P$ = | 10 | hp |
| Design Power | $P_{des}$ = | 15 | hp |
| Diametral Pitch: | $P_d$ = | 6 | Fig. 9-24 |
| Input Speed: | $n_P$ = | 420 | rpm |
| Number of Pinion Teeth: | $N_P$ = | 18 | |
| Desired Output Speed: | $n_G$ = | 148 | rpm |
| Computed number of gear teeth: | | 51.1 | |
| *Enter:* Chosen No. of Gear Teeth: | $N_G$ = | 51 | |

***Computed data:***

| | | | |
|---|---|---|---|
| Actual Output Speed: | $n_G$ = | 148.2 | rpm |
| Gear Ratio: | $m_G$ = | 2.83 | |
| Pitch Diameter - Pinion: | $D_P$ = | 3.000 | in |
| Pitch Diameter - Gear: | $D_G$ = | 8.500 | in |
| Center Distance: | C = | 5.750 | in |
| Pitch Line Speed: | $v_t$ = | 330 | ft/min |
| Transmitted Load: | $W_t$ = | 1000 | lb |

***Secondary Input Data:***

| | Min | Nom | Max |
|---|---|---|---|
| Face Width Guidelines (in): | 1.333 | 2.000 | 2.667 |
| *Enter:* Face Width: | $F$ = | 2.000 | in |
| Ratio: Face width/pinion diameter: | $F/D_P$ = | 0.67 | |
| Recommended range of ratio: | $0.50 < F/D_P < 2.00$ | | |
| *Enter:* Elastic Coefficient: | $C_p$ = 2300 | | Table 9-10 |
| *Enter:* Quality Number: | $A_v$ = 11 | | Table 9-3 |
| Dynamic Factor: | $K_v$ = 1.25 | | Table 9-9 |
| [Factors for computing $K_v$:] | $B$ = 0.826 | | $C$ = 59.75 |
| *Reference:* | $N_P$ = 18 | | $N_G$ = 51 |
| Bending Geometry Factor-Pinion: | $J_P$ = 0.315 | | Fig. 9-15 |
| Bending Geometry Factor-Gear: | $J_G$ = 0.395 | | Fig. 9-15 |
| *Reference:* | $m_G$ = 2.83 | | |
| *Enter:* Pitting Geometry Factor: | $I$ = 0.108 | | Fig. 9-21 |

**Note: Smaller part of the total reduction (42.83) in this pair**
**Lower diametral pitch - 6 compared to 8 in pair 1**

***Factors in Design Analysis:***

| Alignment Factor, $K_m=1.0+C_{pf}+C_{ma}$ | If F<1.0 | If F>1.0 | | |
|---|---|---|---|---|
| Pinion Proportion Factor, $C_{pf}$ = | 0.042 | 0.054 | $[0.50 < F/D_P < 2.00]$ | |
| *Enter:* $C_{pf}$ = | 0.054 | Figure 9-16 | | |
| Type of gearing: | Open | Commer. | Precision | Ex. Prec. |
| Mesh Alignment Factor, $C_{ma}$ = | 0.280 | 0.158 | 0.093 | 0.058 |
| *Enter:* $C_{ma}$ = | 0.158 | Figure 9-17 | | |
| Alignment Factor: $K_m$ = | 1.21 | [Computed] | | |
| Size Factor: $K_s$ = | 1.00 | Table 9-8: Use 1.00 if $P_d$ >= 5 | | |
| Pinion Rim Thickness Factor: $K_{BP}$ = | 1.00 | Fig. 9-18: Use 1.00 if solid blank | | |
| Gear Rim Thickness Factor: $K_{BG}$ = | 1.00 | Fig. 9-18: Use 1.00 if solid blank | | |
| Service Factor: $SF$ = | 1.00 | Use 1.00 if no unusual conditions | | |
| Reliability Factor: $K_R$ = | 1.00 | Table 9-11 Use 1.00 for $R$ = .99 | | |
| *Enter:* Design Life: | 15000 | hours | See Table 9-12 | |
| Pinion - Number of load cycles: $N_P$ = | 3.8E+08 | Guidelines: $Y_N$, $Z_N$ | | |
| Gear - Number of load cycles: $N_G$ = | 1.3E+08 | $10^7$ cycles | $>10^7$ | $<10^7$ |
| Bending Stress Cycle Factor: $Y_{NP}$ = | 0.95 | 1.00 | 0.95 | Fig. 9-22 |
| Bending Stress Cycle Factor: $Y_{NG}$ = | 0.97 | 1.00 | 0.97 | Fig. 9-22 |
| Pitting Stress Cycle Factor: $Z_{NP}$ = | 0.92 | 1.00 | 0.92 | Fig. 9-23 |
| Pitting Stress Cycle Factor: $Z_{NG}$ = | 0.94 | 1.00 | 0.94 | Fig. 9-23 |

***Stress Analysis: Bending***

| | | | |
|---|---|---|---|
| Pinion: Required $s_{at}$ = | 22,702 | psi | See Fig. 9-11 or |
| Gear: Required $s_{at}$ = | 17,731 | psi | Table 9-5 |

***Stress Analysis: Pitting***

| | | | |
|---|---|---|---|
| Pinion: Required $s_{ac}$ = | 147,790 | psi | See Fig. 9-12 or |
| Gear: Required $s_{ac}$ = | 144,645 | psi | Table 9-5 |
| Required hardness of pinion HB: | 369 | Equations in Fig. 9-12-Grade 1 | |
| Required hardness of gear HB: | 359 | Equations in Fig. 9-12-Grade 1 | |

***Specify materials, alloy and heat treatment, for most severe requirement.***

***One possible material specification:***

Pinion requires HB 369; SAE 4340 OQT 900; HB 388; 15% elongation
Gear requires HB 359; SAE 4340 OQT 1000; HB 363; 17% elongation

***Comments:***

**Center Distances and sizes of gears are well balanced**

# DESIGN OF SPUR GEARS

**APPLICATION:** *Problem 75 - First pair*

*Food waste grinder driven by an electric motor*

***Double reduction - First pair - Input Data:***

| | | | |
|---|---|---|---|
| Overload Factor: | $K_o$ = | *1.50* | Table 9-7 |
| Transmitted Power: | $P$ = | *0.5* | hp |
| Design Power | $P_{des}$ = | 0.75 | hp |
| Diametral Pitch: | $P_d$ = | *16* | Fig. 9-24 |
| Input Speed: | $n_P$ = | *850* | rpm |
| Number of Pinion Teeth: | $N_P$ = | *18* | |
| Desired Output Speed: | $n_G$ = | *190* | rpm |
| Computed number of gear teeth: | | 80.5 | |
| *Enter:* Chosen No. of Gear Teeth: | $N_G$ = | *81* | |

***Computed data:***

| | | | |
|---|---|---|---|
| Actual Output Speed: | $n_G$ = | 188.9 | rpm |
| Gear Ratio: | $m_G$ = | 4.50 | |
| Pitch Diameter - Pinion: | $D_P$ = | 1.125 | in |
| Pitch Diameter - Gear: | $D_G$ = | 5.063 | in |
| Center Distance: | C = | 3.094 | in |
| Pitch Line Speed: | $v_t$ = | 250 | ft/min |
| Transmitted Load: | $W_t$ = | 66 | lb |

***Secondary Input Data:***

| | Min | Nom | Max |
|---|---|---|---|
| Face Width Guidelines (in): | 0.500 | 0.750 | 1.000 |
| *Enter:* Face Width: $F$ = | *0.500* | in | |
| Ratio: Face width/pinion diameter: $F/D_P$ = | 0.44 | | |
| Recommended range of ratio: | 0.50 < $F/D_P$ < 2.00 | | |

| | | |
|---|---|---|
| *Enter:* Elastic Coefficient: | $C_p$ = *2300* | Table 9-10 |
| *Enter:* Quality Number: | $A_v$ = *9* | Table 9-3 |
| Dynamic Factor: | $K_v$ = 1.14 | Table 9-9 |
| [Factors for computing $K_v$:] $B$ = 0.630 | $C$ = 70.71 | |
| *Reference:* $N_P$ = 18 | $N_G$ = 81 | |
| Bending Geometry Factor-Pinion: | $J_P$ = *0.320* | Fig. 9-15 |
| Bending Geometry Factor-Gear: | $J_G$ = *0.415* | Fig. 9-15 |
| *Reference:* | $m_G$ = 4.50 | |
| *Enter:* Pitting Geometry Factor: | $I$ = *0.106* | Fig. 9-21 |

**Note: Equal reduction ratios used for pairs 1 and 2**
**Equal diametral pitches (16) used for both pairs**

***Factors in Design Analysis:***

| | | | | |
|---|---|---|---|---|
| Alignment Factor, $K_m$=1.0+$C_{pf}$+$C_{ma}$ | If F<1.0 | If F>1.0 | | |
| Pinion Proportion Factor, $C_{pf}$ = | 0.019 | 0.013 | [0.50 < $F/D_P$ < 2.00] | |
| *Enter:* $C_{pf}$ = | *0.019* | Figure 9-16 | | |
| Type of gearing: | Open | Commer. | Precision | Ex. Prec. |
| Mesh Alignment Factor, $C_{ma}$ = | 0.255 | 0.135 | 0.074 | 0.043 |
| *Enter:* $C_{ma}$ = | *0.135* | Figure 9-17 | | |
| Alignment Factor: $K_m$ = | 1.15 | [Computed] | | |

| | | |
|---|---|---|
| Size Factor: $K_s$ = | *1.00* | Table 9-8: Use 1.00 if $P_d$ >= 5 |
| Pinion Rim Thickness Factor: $K_{BP}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank |
| Gear Rim Thickness Factor: $K_{BG}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank |
| Service Factor: $SF$ = | *1.00* | Use 1.00 if no unusual conditions |
| Reliability Factor: $K_R$ = | *1.00* | Table 9-11 Use 1.00 for $R$ = .99 |

| | | | | |
|---|---|---|---|---|
| *Enter: Design Life:* | *8000* | hours | See Table 9-12 | |
| Pinion - Number of load cycles: $N_P$ = | 4.1E+08 | Guidelines: $Y_N$, $Z_N$ | | |
| Gear - Number of load cycles: $N_G$ = | 9.1E+07 | $10^7$ cycles | $>10^7$ | $<10^7$ |
| Bending Stress Cycle Factor: $Y_{NP}$ = | *0.95* | 1.00 | 0.95 | Fig. 9-22 |
| Bending Stress Cycle Factor: $Y_{NG}$ = | *0.98* | 1.00 | 0.98 | Fig. 9-22 |
| Pitting Stress Cycle Factor: $Z_{NP}$ = | *0.92* | 1.00 | 0.92 | Fig. 9-23 |
| Pitting Stress Cycle Factor: $Z_{NG}$ = | *0.95* | 1.00 | 0.95 | Fig. 9-23 |

***Stress Analysis: Bending***

| | | | |
|---|---|---|---|
| Pinion: Required $s_{at}$ = | 13,639 | psi | See Fig. 9-11 or |
| Gear: Required $s_{at}$ = | 10,195 | psi | Table 9-5 |

***Stress Analysis: Pitting***

| | | | |
|---|---|---|---|
| Pinion: Required $s_{ac}$ = | 116,542 | psi | See Fig. 9-12 or |
| Gear: Required $s_{ac}$ = | 112,862 | psi | Table 9-5 |

| | | |
|---|---|---|
| Required hardness of pinion HB: | 272 | Equations in Fig. 9-12-Grade 1 |
| Required hardness of gear HB: | 260 | Equations in Fig. 9-12-Grade 1 |

***Specify materials, alloy and heat treatment, for most severe requirement.***

***One possible material specification:***

Pinion requires HB 272; SAE 4340 OQT 1200; HB 293; 21% elongation
Gear requires HB 260; SAE 4340 OQT 1200; HB 293; 20% elongation

***Comments:***

**Larger face width required for pair 2 (1.15 in) vs. 0.50 in for pair 1**
**Stresses higher for pair 2 than for pair 1, requiring higher hardness**

# DESIGN OF SPUR GEARS

**APPLICATION:** *Problem 75 - Second pair*

*Food waste grinder driven by an electric motor*

***Double reduction - Second pair - Input Data:***

| | | | |
|---|---|---|---|
| Overload Factor: | $K_o$ = | *1.50* | Table 9-7 |
| Transmitted Power: | $P$ = | *0.5* | hp |
| Design Power | $P_{des}$ = | 0.75 | hp |
| Diametral Pitch: | $P_d$ = | *16* | Fig. 9-24 |
| Input Speed: | $n_P$ = | *188.9* | rpm |
| Number of Pinion Teeth: | $N_P$ = | *18* | |
| Desired Output Speed: | $n_G$ = | *42* | rpm |
| Computed number of gear teeth: | | 81.0 | |
| *Enter:* Chosen No. of Gear Teeth: | $N_G$ = | *81* | |

***Computed data:***

| | | | |
|---|---|---|---|
| Actual Output Speed: | $n_G$ = | 42.0 | rpm |
| Gear Ratio: | $m_G$ = | 4.50 | |
| Pitch Diameter - Pinion: | $D_P$ = | 1.125 | in |
| Pitch Diameter - Gear: | $D_G$ = | 5.063 | in |
| Center Distance: | C = | 3.094 | in |
| Pitch Line Speed: | $v_t$ = | 56 | ft/min |
| Transmitted Load: | $W_t$ = | 297 | lb |

***Secondary Input Data:***

| | Min | Nom | Max |
|---|---|---|---|
| Face Width Guidelines (in): | 0.500 | 0.750 | 1.000 |
| *Enter:* Face Width: $F$ = | | *1.150* | in |
| Ratio: Face width/pinion diameter: $F/D_P$ = | | 1.02 | |
| Recommended range of ratio: $0.50 < F/D_P < 2.00$ | | | |

| | | |
|---|---|---|
| *Enter:* Elastic Coefficient: | $C_p$ = *2300* | Table 9-10 |
| *Enter:* Quality Number: | $A_v$ = *9* | Table 9-3 |
| Dynamic Factor: | $K_v$ = 1.07 | Table 9-9 |
| [Factors for computing $K_v$:] $B$ = 0.630 | $C$ = 70.71 | |
| *Reference:* $N_P$ = 18 | $N_G$ = 81 | |
| Bending Geometry Factor-Pinion: | $J_P$ = *0.320* | Fig. 9-15 |
| Bending Geometry Factor-Gear: | $J_G$ = *0.415* | Fig. 9-15 |
| *Reference:* | $m_G$ = 4.50 | |
| *Enter:* Pitting Geometry Factor: | $I$ = *0.106* | Fig. 9-21 |

**Note: Equal reduction ratios used for pairs 1 and 2**
**Equal diametral pitches (16) used for both pairs**

***Factors in Design Analysis:***

| | | | | |
|---|---|---|---|---|
| Alignment Factor, $K_m = 1.0 + C_{pf} + C_{ma}$ | If F<1.0 | If F>1.0 | | |
| Pinion Proportion Factor, $C_{pf}$ = | 0.077 | 0.079 | [$0.50 < F/D_P < 2.00$] | |
| *Enter:* $C_{pf}$ = | *0.079* | Figure 9-16 | | |
| Type of gearing: | Open | Commer. | Precision | Ex. Prec. |
| Mesh Alignment Factor, $C_{ma}$ = | 0.266 | 0.145 | 0.082 | 0.050 |
| *Enter:* $C_{ma}$ = | *0.145* | Figure 9-17 | | |
| Alignment Factor: $K_m$ = | 1.22 | [Computed] | | |

| | | |
|---|---|---|
| Size Factor: $K_s$ = | *1.00* | Table 9-8: Use 1.00 if $P_d >= 5$ |
| Pinion Rim Thickness Factor: $K_{BP}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank |
| Gear Rim Thickness Factor: $K_{BG}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank |
| Service Factor: $SF$ = | *1.00* | Use 1.00 if no unusual conditions |
| Reliability Factor: $K_R$ = | *1.00* | Table 9-11 Use 1.00 for $R$ = .99 |

| | | | | |
|---|---|---|---|---|
| *Enter: Design Life:* | *8000* | hours | See Table 9-12 | |
| Pinion - Number of load cycles: $N_P$ = | 9.1E+07 | Guidelines: $Y_N$, $Z_N$ | | |
| Gear - Number of load cycles: $N_G$ = | 2.0E+07 | $10^7$ cycles | $>10^7$ | $<10^7$ |
| Bending Stress Cycle Factor: $Y_{NP}$ = | *0.98* | 1.00 | 0.98 | Fig. 9-22 |
| Bending Stress Cycle Factor: $Y_{NG}$ = | *1.01* | 1.00 | 1.01 | Fig. 9-22 |
| Pitting Stress Cycle Factor: $Z_{NP}$ = | *0.95* | 1.00 | 0.95 | Fig. 9-23 |
| Pitting Stress Cycle Factor: $Z_{NG}$ = | *0.98* | 1.00 | 0.98 | Fig. 9-23 |

***Stress Analysis: Bending***

| | | | |
|---|---|---|---|
| Pinion: Required $s_{at}$ = | 25,734 | psi | See Fig. 9-11 or |
| Gear: Required $s_{at}$ = | 19,253 | psi | Table 9-5 |

***Stress Analysis: Pitting***

| | | | |
|---|---|---|---|
| Pinion: Required $s_{ac}$ = | 157,454 | psi | See Fig. 9-12 or |
| Gear: Required $s_{ac}$ = | 152,634 | psi | Table 9-5 |

| | | |
|---|---|---|
| Required hardness of pinion HB: | 399 | Equations in Fig. 9-12-Grade 1 |
| Required hardness of gear HB: | 384 | Equations in Fig. 9-12-Grade 1 |

***Specify materials, alloy and heat treatment, for most severe requirement.***

***One possible material specification:***

Pinion requires HB 399; SAE 4340 OQT 800; HB 415; 12% elongation

Gear requires HB 384; SAE 4340 OQT 800; HB 415; 12% elongation

***Comments:***

Pinion and gear made from same material and heat treatment for processing convenience.

**Larger face width required for pair 2 (1.15 in) vs. 0.50 in for pair 1**
**Stresses higher for pair 2 than for pair 1, requiring higher hardness**

# DESIGN OF SPUR GEARS

**APPLICATION:** Problem 76 - First pair

*Small hand drill driven by an electric motor*

**Double reduction - First pair - Input Data:**

| | | | |
|---|---|---|---|
| Overload Factor: | $K_o$ = | *1.50* | Table 9-7 |
| Transmitted Power: | $P$ = | *0.25* | hp |
| Design Power | $P_{des}$ = | 0.375 | hp |
| Diametral Pitch: | $P_d$ = | *24* | Fig. 9-24 |
| Input Speed: | $n_P$ = | *3000* | rpm |
| Number of Pinion Teeth: | $N_P$ = | *15* | |
| Desired Output Speed: | $n_G$ = | *1300* | rpm |
| Computed number of gear teeth: | | 34.6 | |
| *Enter:* Chosen No. of Gear Teeth: | $N_G$ = | 35 | |
| ***Computed data:*** | | | |
| Actual Output Speed: | $n_G$ = | 1285.7 | rpm |
| Gear Ratio: | $m_G$ = | 2.33 | |
| Pitch Diameter - Pinion: | $D_P$ = | 0.625 | in |
| Pitch Diameter - Gear: | $D_G$ = | 1.458 | in |
| Center Distance: | $C$ = | 1.042 | in |
| Pitch Line Speed: | $v_t$ = | 491 | ft/min |
| Transmitted Load: | $W_t$ = | 17 | lb |

**Secondary Input Data:**

| | Min | Nom | Max |
|---|---|---|---|
| Face Width Guidelines (in): | 0.333 | 0.500 | 0.667 |
| *Enter:* Face Width: $F$ = | | *0.250* | in |
| Ratio: Face width/pinion diameter: $F/D_P$ = | | 0.40 | |
| Recommended range of ratio: $0.50 < F/D_P < 2.00$ | | | |
| *Enter:* Elastic Coefficient: $C_p$ = *2300* | | | Table 9-10 |
| *Enter:* Quality Number: $A_v$ = *9* | | | Table 9-3 |
| Dynamic Factor: $K_v$ = 1.19 | | | Table 9-9 |
| [Factors for computing $K_v$:] $B$ = 0.630 | | $C$ = 70.71 | |
| *Reference:* $N_P$ = 15 | | $N_G$ = 35 | |
| Bending Geometry Factor-Pinion: $J_P$ = *0.250* | | | Fig. 9-15 |
| Bending Geometry Factor-Gear: $J_G$ = *0.355* | | | Fig. 9-15 |
| *Reference:* $m_G$ = 2.33 | | | |
| *Enter:* Pitting Geometry Factor: $I$ = *0.088* | | | Fig. 9-21 |

**Note: Equal reduction ratios used for pairs 1 and 2**
**Equal diametral pitches (24) used for both pairs**

***Factors in Design Analysis:***

| Alignment Factor, $K_m=1.0+C_{pf}+C_{ma}$ | If F<1.0 | If F>1.0 | | |
|---|---|---|---|---|
| Pinion Proportion Factor, $C_{pf}$ = | 0.015 | 0.006 | [0.50 < $F/D_P$ < 2.00] | |
| *Enter:* $C_{pf}$ = | *0.015* | Figure 9-16 | | |
| Type of gearing: | Open | Commer. | Precision | Ex. Prec. |
| Mesh Alignment Factor, $C_{ma}$ = | 0.251 | 0.131 | 0.071 | 0.041 |
| *Enter:* $C_{ma}$ = | *0.131* | Figure 9-17 | | |
| Alignment Factor: $K_m$ = | 1.15 | [Computed] | | |
| Size Factor: $K_s$ = | *1.00* | Table 9-8: Use 1.00 if $P_d$ >= 5 | | |
| Pinion Rim Thickness Factor: $K_{BP}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | |
| Gear Rim Thickness Factor: $K_{BG}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | |
| Service Factor: $SF$ = | *1.00* | Use 1.00 if no unusual conditions | | |
| Reliability Factor: $K_R$ = | *1.00* | Table 9-11 Use 1.00 for $R$ = .99 | | |
| *Enter: Design Life:* | *5000* | hours | See Table 9-12 | |
| Pinion - Number of load cycles: $N_P$ = | 9.0E+08 | Guidelines: $Y_N$, $Z_N$ | | |
| Gear - Number of load cycles: $N_G$ = | 3.9E+08 | $10^7$ cycles | $>10^7$ | $<10^7$ |
| Bending Stress Cycle Factor: $Y_{NP}$ = | *0.94* | 1.00 | 0.94 | Fig. 9-22 |
| Bending Stress Cycle Factor: $Y_{NG}$ = | *0.95* | 1.00 | 0.95 | Fig. 9-22 |
| Pitting Stress Cycle Factor: $Z_{NP}$ = | *0.90* | 1.00 | 0.90 | Fig. 9-23 |
| Pitting Stress Cycle Factor: $Z_{NG}$ = | *0.92* | 1.00 | 0.92 | Fig. 9-23 |

***Stress Analysis: Bending***

| | | | |
|---|---|---|---|
| Pinion: Required $s_{at}$ = | 14,014 | psi | See Fig. 9-11 or |
| Gear: Required $s_{at}$ = | 9,765 | psi | Table 9-5 |

***Stress Analysis: Pitting***

| | | | |
|---|---|---|---|
| Pinion: Required $s_{ac}$ = | 127,649 | psi | See Fig. 9-12 or |
| Gear: Required $s_{ac}$ = | 124,874 | psi | Table 9-5 |
| Required hardness of pinion HB: | 306 | | Equations in Fig. 9-12-Grade 1 |
| Required hardness of gear HB: | 297 | | Equations in Fig. 9-12-Grade 1 |

***Specify materials, alloy and heat treatment, for most severe requirement.***

***One possible material specification:***

Pinion requires HB 306; SAE 4340 OQT 1100; HB 321; 19% elongation
Gear requires HB 297; SAE 4340 OQT 1100; HB 321; 19% elongation

***Comments:***

**Larger face width required for pair 2 (0.56 in) vs. 0.25 in for pair 1**
**Stresses nearly equal for pair 2 and for pair 1, requiring equal hardnesses**

# DESIGN OF SPUR GEARS

**APPLICATION:** *Problem 76 - Second pair*

*Small hand drill driven by an electric motor*

***Double reduction - Second pair - Input Data:***

| | | | |
|---|---|---|---|
| Overload Factor: | $K_o$ = | *1.50* | Table 9-7 |
| Transmitted Power: | $P$ = | *0.25* | hp |
| Design Power | $P_{des}$ = | 0.375 | hp |
| Diametral Pitch: | $P_d$ = | *24* | Fig. 9-24 |
| Input Speed: | $n_P$ = | *1285.7* | rpm |
| Number of Pinion Teeth: | $N_P$ = | *15* | |
| Desired Output Speed: | $n_G$ = | *550* | rpm |
| Computed number of gear teeth: | | 35.1 | |
| *Enter:* Chosen No. of Gear Teeth: | $N_G$ = | 35 | |

***Computed data:***

| | | | |
|---|---|---|---|
| Actual Output Speed: | $n_G$ = | 551.0 | rpm |
| Gear Ratio: | $m_G$ = | 2.33 | |
| Pitch Diameter - Pinion: | $D_P$ = | 0.625 | in |
| Pitch Diameter - Gear: | $D_G$ = | 1.458 | in |
| Center Distance: | $C$ = | 1.042 | in |
| Pitch Line Speed: | $v_t$ = | 210 | ft/min |
| Transmitted Load: | $W_t$ = | 39 | lb |

***Secondary Input Data:***

| | Min | Nom | Max |
|---|---|---|---|
| Face Width Guidelines (in): | 0.333 | 0.500 | 0.667 |
| *Enter:* Face Width: $F$ = | | *0.560* | in |
| Ratio: Face width/pinion diameter: $F/D_P$ = | | 0.90 | |

Recommended range of ratio: $0.50 < F/D_P < 2.00$

| | | | |
|---|---|---|---|
| *Enter:* Elastic Coefficient: | $C_p$ = *2300* | | Table 9-10 |
| *Enter:* Quality Number: | $A_v$ = *9* | | Table 9-3 |
| Dynamic Factor: | $K_v$ = 1.12 | | Table 9-9 |
| [Factors for computing $K_v$:] | $B$ = 0.630 | $C$ = 70.71 | |
| *Reference:* | $N_P$ = 15 | $N_G$ = 35 | |
| Bending Geometry Factor-Pinion: | $J_P$ = *0.250* | | Fig. 9-15 |
| Bending Geometry Factor-Gear: | $J_G$ = *0.355* | | Fig. 9-15 |
| *Reference:* | $m_G$ = 2.33 | | |
| *Enter:* Pitting Geometry Factor: | $I$ = *0.088* | | Fig. 9-21 |

**Note: Equal reduction ratios used for pairs 1 and 2**
**Equal diametral pitches (24) used for both pairs**

***Factors in Design Analysis:***

| Alignment Factor, $K_m = 1.0 + C_{pf} + C_{ma}$ | If F<1.0 | If F>1.0 | | |
|---|---|---|---|---|
| Pinion Proportion Factor, $C_{pf}$ = | 0.065 | 0.059 | [$0.50 < F/D_P < 2.00$] | |
| *Enter:* $C_{pf}$ = | *0.065* | Figure 9-16 | | |
| Type of gearing: | Open | Commer. | Precision | Ex. Prec. |
| Mesh Alignment Factor, $C_{ma}$ = | 0.256 | 0.136 | 0.075 | 0.044 |
| *Enter:* $C_{ma}$ = | *0.136* | Figure 9-17 | | |
| Alignment Factor: $K_m$ = | 1.20 | [Computed] | | |

| | | |
|---|---|---|
| Size Factor: $K_s$ = | *1.00* | Table 9-8: Use 1.00 if $P_d$ >= 5 |
| Pinion Rim Thickness Factor: $K_{BP}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank |
| Gear Rim Thickness Factor: $K_{BG}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank |
| Service Factor: $SF$ = | *1.00* | Use 1.00 if no unusual conditions |
| Reliability Factor: $K_R$ = | *1.00* | Table 9-11 Use 1.00 for $R$ = .99 |
| *Enter:* *Design Life:* | *5000* | hours See Table 9-12 |

| | | Guidelines: $Y_N$, $Z_N$ | | |
|---|---|---|---|---|
| Pinion - Number of load cycles: $N_P$ = | 3.9E+08 | | | |
| Gear - Number of load cycles: $N_G$ = | 1.7E+08 | $10^7$ cycles | $>10^7$ | $<10^7$ |
| Bending Stress Cycle Factor: $Y_{NP}$ = | *0.95* | 1.00 | 0.95 | Fig. 9-22 |
| Bending Stress Cycle Factor: $Y_{NG}$ = | *0.97* | 1.00 | 0.97 | Fig. 9-22 |
| Pitting Stress Cycle Factor: $Z_{NP}$ = | *0.92* | 1.00 | 0.92 | Fig. 9-23 |
| Pitting Stress Cycle Factor: $Z_{NG}$ = | *0.94* | 1.00 | 0.94 | Fig. 9-23 |

***Stress Analysis: Bending***

| | | | |
|---|---|---|---|
| Pinion: Required $s_{at}$ = | 14,339 | psi | See Fig. 9-11 or |
| Gear: Required $s_{at}$ = | 9,890 | psi | Table 9-5 |

***Stress Analysis: Pitting***

| | | | |
|---|---|---|---|
| Pinion: Required $s_{ac}$ = | 126,985 | psi | See Fig. 9-12 or |
| Gear: Required $s_{ac}$ = | 124,283 | psi | Table 9-5 |
| Required hardness of pinion HB: | 304 | | Equations in Fig. 9-12-Grade 1 |
| Required hardness of gear HB: | 296 | | Equations in Fig. 9-12-Grade 1 |

***Specify materials, alloy and heat treatment, for most severe requirement.***

***One possible material specification:***

Pinion requires HB 304; SAE 4340 OQT 1100; HB 321; 19% elongation

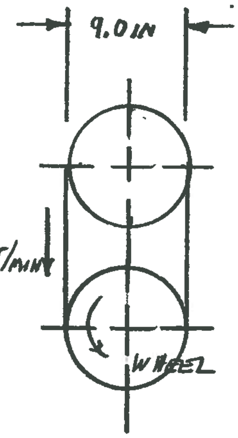
Gear requires HB 296; SAE 4340 OQT 1100; HB 321; 19% elongation

***Comments:***

**Larger face width required for pair 2 (0.56 in) vs. 0.25 in for pair 1**
**Stresses nearly equal for pair 2 and for pair 1, requiring equal hardnesses**

| DESIGN OF PLASTIC SPUR GEARS | | | |
|---|---|---|---|
| ***Application:*** | | | |
| *Band saw driven by electric motor* | | | |
| *Problem 77* | | | |
| ***Initial Input Data:*** | | | |
| Input Power: | $P$ = | 0.25 hp | |
| Input Speed: | $n_P$ = | 551 rpm | From Problem 76 |
| Diametral Pitch: | $P_d$ = | 12 | |
| Number of Pinion Teeth: | $N_P$ = | 18 | |
| Desired Output Speed: | $n_G$ = | 159.15 rpm | To produce blade |
| Computed number of gear teeth: | | 62.319 | speed of 375 ft/min |
| *Enter:* Chosen No. of Gear Teeth: | $N_G$ = | 62 | |
| ***Computed data:*** | | | |
| Actual Output Speed: | $n_G$ = | 160.0 rpm | |
| Gear Ratio: | $m_G$ = | 3.444 | |
| Pitch Diameter - Pinion: | $D_P$ = | 1.500 in | |
| Pitch Diameter - Gear: | $D_G$ = | 5.167 in | |
| Center Distance: | $C$ = | 6.667 in | |
| Pitch Line Speed: | $v_t$ = | 216.4 ft/min | |
| Transmitted Load: | $W_t$ = | 38.11 lb | |
| ***Secondary Input Data - Pinion:*** | | | |
| Tooth Form: | *20 degree full depth* | | |
| Lewis Form Factor: | $Y$ = | 0.521 | Table 9-15 |
| Safety Factor: | $SF$ = | 1.50 | Ref: Table 9-7 |
| Material: | | Unfilled Nylon | |
| Allowable Bending Stress: | $s_{at}$ = | 6000 psi | Table 9-14 or Fig. 9-28 |
| Required Face Width: | $F$ = | 0.219 in | |
| *Enter:* Specified Face Width: | $F$ = | 0.220 in | |
| Actual Bending Stress in Pinion: | $s_t$ = | 5985 psi | |
| ***Secondary Input Data - Gear:*** | | | |
| Tooth Form: | *20 degree full depth* | | Same as for pinion |
| Lewis Form Factor: | $Y$ = | 0.716 | Table 9-15 |
| Safety Factor: | $SF$ = | 1.50 | Same as for pinion |
| Material: | | Unfilled Acetal | |
| Allowable Bending Stress: | $s_{at}$ = | 5000 psi | Table 9-14 or Fig. 9-28 |
| Face Width - Gear: | $F$ = | 0.220 in | Same as for pinion |
| Actual Bending Stress in Gear: | $s_t$ = | 4355 psi | Must be < $s_{at}$ |

$v_{t\,WHEEL} = v_{BLADE} = 375\ FT/MIN$




$v_{t\,WHEEL} = \pi D_W n_W / 12$

$n_W = \dfrac{12\, v_{tW}}{\pi D_W} = \dfrac{12(375)}{\pi(9\,IN)}$

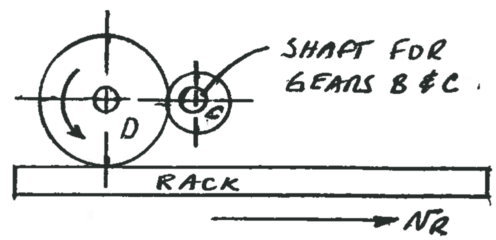
$n_W = 159.2$ RPM

CONNECT WHEEL ON OUTPUT SHAFT OF GEAR REDUCER. ONE PAIR OF GEARS.

$\boxed{78.}$ RACK DRIVES FURNACE DOOR. $v_{RACK} \geq 2.0\ FT/S = 120\ FT/MIN$

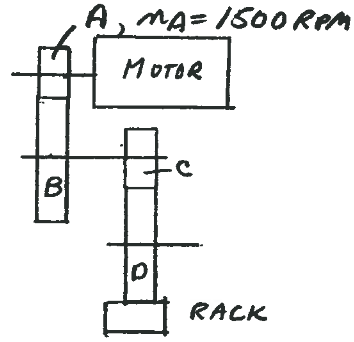
$v_R = v_{t_D} = \pi D_D\, n_D / 12$

$$n_D \geq \frac{12\, v_{tD}}{\pi\, D_D} = \frac{(12)(120)}{\pi\, D_D} = \frac{458.4}{D_D}$$




POSSIBLE VALUES FOR $D_G$:

| $D_D$ | $n_D$ | $TV = n_A/n_D$ | |
|---|---|---|---|
| 10.0 IN | 45.84 RPM | 32.72 | |
| 9.0 IN | 50.93 | 29.45 | |
| 12.167 | 37.76 | 39.72 | USED THIS |




PAIR 1: $n_A = 1500$ RPM $n_B = 172.3$ RPM
$N_A = 17$, $N_B = 148$, $VR_1 = 8.71$

PAIR 2: $n_C = n_B = 172.3$ RPM; $n_D = 37.76$
$N_C = 16$, $N_D = 73$, $VR_2 = 4.56$

SEE TWO SPREADSHEET SOLUTIONS

DESIGNED FOR OPEN GEARING

DESIGN LIFE: IN EACH CYCLE, RACK MOVES 6 FT EACH WAY
TOTAL OF 12 FT FOR FULL CYCLE. AT 2.0 FT/S, CYCLE TIME IS:

$$\frac{1\ SEC}{2.0\ FT} \times 12\ FT = 6.0\ SEC/CYCLE$$

$$\frac{6.0\ SEC}{CYCLE} \times \frac{6\ CYCLES}{hr} \times \frac{24\ hr}{DAY} \times \frac{365\ DAYS}{YR} \times \frac{15\ YRS}{} \times \frac{hr}{3600\ S} = 1314\ hr$$

USE 1500 hr DESIGN LIFE.

---

SUMMARY: POWER = 5.0 hp, $K_o = 1.50$, $P_{des} = 7.5$ hp., QUALITY = $A_v = 11$

| | $P_d$ | $N_P$ | $N_G$ | $D_P$ | $D_G$ | $F$ | $C$ | |
|---|---|---|---|---|---|---|---|---|
| PAIR 1: | 10 | 17 | 172 | 1.700 | 14.800 | 1.200 | 8.250 | WELL BALANCED IN SIZE. |
| PAIR 2: | 6 | 16 | 73 | 2.667 | 12.167 | 2.500 | 7.417 | |

ALL GEARS MADE FROM SAE 4340: PAIR 1 - OQT 1100; HB 321
PAIR 2 - OQT 900; HB 388

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## DESIGN OF SPUR GEARS

| | | | |
|---|---|---|---|
| ***APPLICATION:*** | *Problem 78 - First pair* | | |
| *Rack and pinion drive driven by a fluid power motor* | | | |
| ***Double reduction - First pair - Input Data:*** | | | |
| Overload Factor: | $K_o$ = | *1.50* | Table 9-7 |
| Transmitted Power: | $P$ = | 5 | hp |
| Design Power | $P_{des}$ = | 7.5 | hp |
| Diametral Pitch: | $P_d$ = | *10* | Fig. 9-24 |
| Input Speed: | $n_P$ = | *1500* | rpm |
| Number of Pinion Teeth: | $N_P$ = | *17* | |
| Desired Output Speed: | $n_G$ = | *172* | rpm |
| Computed number of gear teeth: | | 148.3 | |
| *Enter:* Chosen No. of Gear Teeth: | $N_G$ = | *148* | |
| ***Computed data:*** | | | |
| Actual Output Speed: | $n_G$ = | 172.30 | rpm |
| Gear Ratio: | $m_G$ = | 8.71 | |
| Pitch Diameter - Pinion: | $D_P$ = | 1.700 | in |
| Pitch Diameter - Gear: | $D_G$ = | 14.800 | in |
| Center Distance: | $C$ = | 8.250 | in |
| Pitch Line Speed: | $v_t$ = | 668 | ft/min |
| Transmitted Load: | $W_t$ = | 247 | lb |

| | | | |
|---|---|---|---|
| ***Secondary Input Data:*** | | | |
| | Min | Nom | Max |
| Face Width Guidelines (in): | 0.800 | 1.200 | 1.600 |
| *Enter:* Face Width: | $F$ = | *1.200* | in |
| Ratio: Face width/pinion diameter: $F/D_P$ = | 0.71 | | |
| Recommended range of ratio: $0.50 < F/D_P < 2.00$ | | | |
| *Enter:* Elastic Coefficient: | $C_p$ = *2300* | | Table 9-10 |
| *Enter:* Quality Number: | $A_v$ = *11* | | Table 9-3 |
| Dynamic Factor: | $K_v$ = 1.35 | | Table 9-9 |
| [Factors for computing $K_v$:] | $B$ = 0.826 | $C$ = 59.75 | |
| *Reference:* | $N_P$ = 17 | $N_G$ = 148 | |
| Bending Geometry Factor-Pinion: | $J_P$ = *0.295* | | Fig. 9-15 |
| Bending Geometry Factor-Gear: | $J_G$ = *0.425* | | Fig. 9-15 |
| *Reference:* | $m_G$ = 8.71 | | |
| *Enter:* Pitting Geometry Factor: | $I$ = *0.110* | | Fig. 9-21 |

***Factors in Design Analysis:***

| | | | | | |
|---|---|---|---|---|---|
| Alignment Factor, $K_m = 1.0 + C_{pf} + C_{ma}$ | If F<1.0 | If F>1.0 | | | |
| Pinion Proportion Factor, $C_{pf}$ = | 0.046 | 0.048 | [$0.50 < F/D_P < 2.00$] | | |
| *Enter:* $C_{pf}$ = | *0.048* | Figure 9-16 | | | |
| Type of gearing: | Open | Commer. | Precision | Ex. Prec. | |
| Mesh Alignment Factor, $C_{ma}$ = | 0.267 | 0.146 | 0.083 | 0.050 | |
| *Enter:* $C_{ma}$ = | *0.146* | Figure 9-17 | | | |
| Alignment Factor: $K_m$ = | 1.19 | [Computed] | | | |
| Size Factor: $K_s$ = | *1.00* | Table 9-8: Use 1.00 if $P_d$ >= 5 | | | |
| Pinion Rim Thickness Factor: $K_{BP}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | | |
| Gear Rim Thickness Factor: $K_{BG}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | | |
| Service Factor: $SF$ = | *1.00* | Use 1.00 if no unusual conditions | | | |
| Reliability Factor: $K_R$ = | *1.00* | Table 9-11 Use 1.00 for $R$ = .99 | | | |
| *Enter: Design Life:* | *1500* | hours | See Table 9-12 | | |
| Pinion - Number of load cycles: $N_P$ = | 1.4E+08 | Guidelines: $Y_N$, $Z_N$ | | | |
| Gear - Number of load cycles: $N_G$ = | 1.6E+07 | $10^7$ cycles | $>10^7$ | $<10^7$ | |
| Bending Stress Cycle Factor: $Y_{NP}$ = | *0.97* | 1.00 | 0.97 | Fig. 9-22 | |
| Bending Stress Cycle Factor: $Y_{NG}$ = | *1.01* | 1.00 | 1.01 | Fig. 9-22 | |
| Pitting Stress Cycle Factor: $Z_{NP}$ = | *0.94* | 1.00 | 0.94 | Fig. 9-23 | |
| Pitting Stress Cycle Factor: $Z_{NG}$ = | *0.99* | 1.00 | 0.99 | Fig. 9-23 | |

***Stress Analysis: Bending***

| | | | |
|---|---|---|---|
| Pinion: Required $s_{at}$ = | 17,347 | psi | See Fig. 9-11 or |
| Gear: Required $s_{at}$ = | 11,564 | psi | Table 9-5 |

***Stress Analysis: Pitting***

| | | | |
|---|---|---|---|
| Pinion: Required $s_{ac}$ = | 126,062 | psi | See Fig. 9-12 or |
| Gear: Required $s_{ac}$ = | 119,695 | psi | Table 9-5 |
| Required hardness of pinion HB: | 301 | | Equations in Fig. 9-12-Grade 1 |
| Required hardness of gear HB: | 281 | | Equations in Fig. 9-12-Grade 1 |

***Specify materials, alloy and heat treatment, for most severe requirement.***

***One possible material specification:***

Pinion requires HB 301; SAE 4340 OQT 1100; HB 321; 19% elongation

Gear requires HB 281; SAE 4340 OQT 1100; HB 321; 19% elongation

***Comments:***

Same material used for pinion and gear because contact stresses are close.

# DESIGN OF SPUR GEARS

| | | | |
|---|---|---|---|
| ***APPLICATION:*** | *Problem 78 - Second pair* | | |
| *Rack and pinion drive driven by a fluid power motor* | | | |
| ***Double reduction - Second pair - Input Data:*** | | | |
| Overload Factor: | $K_o$ = | *1.50* | Table 9-7 |
| Transmitted Power: | $P$ = | 5 | hp |
| Design Power | $P_{des}$ = | 7.5 | hp |
| Diametral Pitch: | $P_d$ = | *6* | Fig. 9-24 |
| Input Speed: | $n_P$ = | *172.3* | rpm |
| Number of Pinion Teeth: | $N_P$ = | *16* | |
| Desired Output Speed: | $n_G$ = | *38.2* | rpm |
| Computed number of gear teeth: | | 72.2 | |
| *Enter:* Chosen No. of Gear Teeth: | $N_G$ = | *73* | |
| ***Computed data:*** | | | |
| Actual Output Speed: | $n_G$ = | 37.76 | rpm |
| Gear Ratio: | $m_G$ = | 4.56 | |
| Pitch Diameter - Pinion: | $D_P$ = | 2.667 | in |
| Pitch Diameter - Gear: | $D_G$ = | 12.167 | in |
| Center Distance: | $C$ = | 7.417 | in |
| Pitch Line Speed: | $v_t$ = | 120 | ft/min |
| Transmitted Load: | $W_t$ = | 1372 | lb |

| | | | |
|---|---|---|---|
| ***Secondary Input Data:*** | | | |
| | Min | Nom | Max |
| Face Width Guidelines (in): | 1.333 | 2.000 | 2.667 |
| *Enter:* Face Width: | $F$ = | *2.500* | in |
| Ratio: Face width/pinion diameter: | $F/D_P$ = | 0.94 | |
| Recommended range of ratio: | $0.50 < F/D_P < 2.00$ | | |
| *Enter:* Elastic Coefficient: | $C_p$ = *2300* | | Table 9-10 |
| *Enter:* Quality Number: | $A_v$ = *11* | | Table 9-3 |
| Dynamic Factor: | $K_v$ = 1.15 | | Table 9-9 |
| [Factors for computing $K_v$:] | $B$ = 0.826 | | $C$ = 59.75 |
| *Reference:* | $N_P$ = 16 | | $N_G$ = 73 |
| Bending Geometry Factor-Pinion: | $J_P$ = *0.265* | | Fig. 9-15 |
| Bending Geometry Factor-Gear: | $J_G$ = *0.400* | | Fig. 9-15 |
| *Reference:* | $m_G$ = 4.56 | | |
| *Enter:* Pitting Geometry Factor: | $I$ = *0.102* | | Fig. 9-21 |

***Factors in Design Analysis:***

| | | | | |
|---|---|---|---|---|
| Alignment Factor, $K_m$=1.0+$C_{pf}$+$C_{ma}$ | If F<1.0 | If F>1.0 | | |
| Pinion Proportion Factor, $C_{pf}$ = | 0.069 | 0.088 | [0.50 < $F/D_P$ < 2.00] | |
| *Enter:* $C_{pf}$ = | *0.088* | Figure 9-16 | | |
| Type of gearing: | Open | Commer. | Precision | Ex. Prec. |
| Mesh Alignment Factor, $C_{ma}$ = | 0.288 | 0.166 | 0.099 | 0.063 |
| *Enter:* $C_{ma}$ = | *0.166* | Figure 9-17 | | |
| Alignment Factor: $K_m$ = | 1.25 | [Computed] | | |
| Size Factor: $K_s$ = | *1.00* | Table 9-8: Use 1.00 if $P_d$ >= 5 | | |
| Pinion Rim Thickness Factor: $K_{BP}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | |
| Gear Rim Thickness Factor: $K_{BG}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | |
| Service Factor: $SF$ = | *1.00* | Use 1.00 if no unusual conditions | | |
| Reliability Factor: $K_R$ = | *1.00* | Table 9-11 Use 1.00 for $R$ = .99 | | |
| *Enter: Design Life:* | *1500* | hours | See Table 9-12 | |
| Pinion - Number of load cycles: $N_P$ = | 1.6E+07 | Guidelines: $Y_N$, $Z_N$ | | |
| Gear - Number of load cycles: $N_G$ = | 3.4E+06 | $10^7$ cycles | $>10^7$ | $<10^7$ |
| Bending Stress Cycle Factor: $Y_{NP}$ = | *1.01* | 1.00 | 1.01 | Fig. 9-22 |
| Bending Stress Cycle Factor: $Y_{NG}$ = | *1.04* | 1.00 | 1.04 | Fig. 9-22 |
| Pitting Stress Cycle Factor: $Z_{NP}$ = | *0.99* | 1.00 | 0.99 | Fig. 9-23 |
| Pitting Stress Cycle Factor: $Z_{NG}$ = | *1.03* | 1.00 | 1.03 | Fig. 9-23 |

***Stress Analysis: Bending***

| | | | |
|---|---|---|---|
| Pinion: Required $s_{at}$ = | 26,592 | psi | See Fig. 9-11 or |
| Gear: Required $s_{at}$ = | 17,109 | psi | Table 9-5 |

***Stress Analysis: Pitting***

| | | | |
|---|---|---|---|
| Pinion: Required $s_{ac}$ = | 153,423 | psi | See Fig. 9-12 or |
| Gear: Required $s_{ac}$ = | 147,465 | psi | Table 9-5 |
| Required hardness of pinion HB: | 386 | Equations in Fig. 9-12-Grade 1 | |
| Required hardness of gear HB: | 368 | Equations in Fig. 9-12-Grade 1 | |

***Specify materials, alloy and heat treatment, for most severe requirement.***

***One possible material specification:***

Pinion requires HB 386; SAE 4340 OQT 900; HB 388; 14% elongation

Gear requires HB 368; SAE 4340 OQT 900; HB 388; 14% elongation

***Comments:***

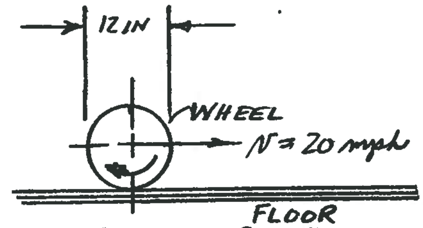
Same material and heat treatment used for pinion and gear.

Same material used for both Pair 1 and Pair2; Different tempering temperatures.

79. GEAR DRIVE FOR A LIFT TRUCK

ROLLING WHEEL IS THE INVERSE OF A PINION DRIVING A RACK.

$v$ AT CENTER OF WHEEL EQUALS SPEED OF LIFT TRUCK.




$$v = \frac{20\,mi}{hr} \times \frac{5280\,ft}{mi} \times \frac{1\,hr}{60\,MIN}$$

$v = 1760$ FT/MIN.

$v = \pi D_W n_W / 12$

$$n_W = \frac{12v}{\pi D_W} = \frac{12(1760\,FT/MIN)}{\pi(12\,IN)} = 560.23\ RPM$$

TRAIN VALUE $= 3000/560.23 = 5.355$

THIS TV COULD BE PRODUCED BY ONE PAIR OF GEARS. HOWEVER, FOR POWER REQUIRED, GEAR WOULD BE TOO LARGE TO ATTACH TO AXLE WITH 12-IN WHEEL.

USE DOUBLE REDUCTION. SEE TWO FOLLOWING PAGES.

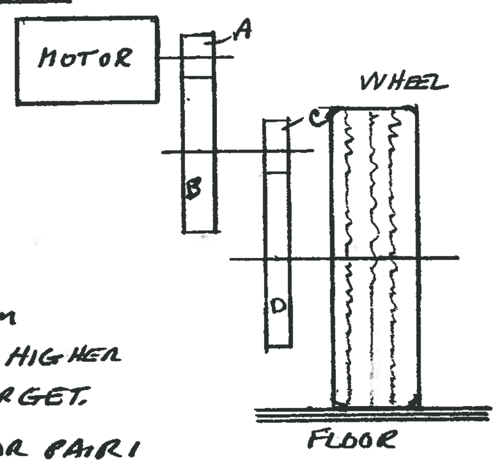
$$VR_1 = \frac{N_B}{N_A} = \frac{50}{17} = 2.94$$

$$VR_2 = \frac{N_D}{N_C} = \frac{38}{21} = 1.81$$

$$n_W = n_D = n_A \cdot \frac{N_A\ N_C}{N_B\ N_D}$$

$$n_W = 3000 \cdot \frac{17}{50} \cdot \frac{21}{38} = 563.7\ RPM$$

SLIGHTLY HIGHER THAN TARGET.




NOTE: GEARS HAVE $P_d = 6$ FOR PAIR 1 AND $P_d = 5$ FOR PAIR 2. FACE WIDTHS ARE RELATIVELY LARGE. $F_1 = 2.50$ IN, $F_2 = 3.00$ IN. REDESIGN WITH HELICAL GEARS SHOULD ALLOW A SMALLER SYSTEM.

ALSO ALL GEARS USE SAME MATERIAL AND HEAT TREATMENT.

LIFE CALCULATION:

$$\frac{16\,h}{DAY} \times \frac{6\,DAYS}{WK} \times \frac{52\,WKS}{YR} \times 20\,YR = 99\,840\,h \qquad \text{USE } L = 100\,000\,h$$

DESIGN DECISIONS: $SF = 1.00$, $K_R = 1.50$ [1 FAILURE IN 10 000; $R = 0.9999$]

CONTINUED ON NEXT TWO PAGES

## DESIGN OF SPUR GEARS

| | | | |
|---|---|---|---|
| ***APPLICATION:*** | *Problem 79 - First pair* | | |
| *Industrial lift truck drive to wheels: driven by DC motor* | | | |
| ***Double reduction - First pair - Input Data:*** | | | |
| Overload Factor: | $K_o$ = | *1.50* | Table 9-7 |
| Transmitted Power: | $P$ = | *20* | hp |
| Design Power | $P_{des}$ = | 30 | hp |
| Diametral Pitch: | $P_d$ = | *6* | Fig. 9-24 |
| Input Speed: | $n_P$ = | *3000* | rpm |
| Number of Pinion Teeth: | $N_P$ = | *17* | |
| Desired Output Speed: | $n_G$ = | *1000* | rpm |
| Computed number of gear teeth: | | 51.0 | |
| *Enter:* Chosen No. of Gear Teeth: | $N_G$ = | *50* | |
| ***Computed data:*** | | | |
| Actual Output Speed: | $n_G$ = | 1020.00 | rpm |
| Gear Ratio: | $m_G$ = | 2.94 | |
| Pitch Diameter - Pinion: | $D_P$ = | 2.833 | in |
| Pitch Diameter - Gear: | $D_G$ = | 8.333 | in |
| Center Distance: | C = | 5.583 | in |
| Pitch Line Speed: | $v_t$ = | 2225 | ft/min |
| Transmitted Load: | $W_t$ = | 297 | lb |

| ***Secondary Input Data:*** | Min | Nom | Max |
|---|---|---|---|
| Face Width Guidelines (in): | 1.333 | 2.000 | 2.667 |
| *Enter:* Face Width: | $F$ = | *2.500* | in |
| Ratio: Face width/pinion diameter: | $F/D_P$ = | 0.88 | |
| Recommended range of ratio: | $0.50 < F/D_P < 2.00$ | | |
| *Enter:* Elastic Coefficient: | $C_p$ = | *2300* | Table 9-10 |
| *Enter:* Quality Number: | $A_v$ = | *7* | Table 9-3 |
| Dynamic Factor: | $K_v$ = | 1.19 | Table 9-9 |
| [Factors for computing $K_v$:] | $B$ = 0.397 | $C$ = 83.77 | |
| *Reference:* | $N_P$ = 17 | $N_G$ = 50 | |
| Bending Geometry Factor-Pinion: | $J_P$ = | *0.293* | Fig. 9-15 |
| Bending Geometry Factor-Gear: | $J_G$ = | *0.380* | Fig. 9-15 |
| *Reference:* | $m_G$ = | 2.94 | |
| *Enter:* Pitting Geometry Factor: | $I$ = | *0.097* | Fig. 9-21 |

| ***Factors in Design Analysis:*** | | | | | |
|---|---|---|---|---|---|
| Alignment Factor, $K_m$=1.0+$C_{pf}$+$C_{ma}$ | | If F<1.0 | If F>1.0 | | |
| Pinion Proportion Factor, $C_{pf}$ = | | 0.063 | 0.082 | [0.50 < $F/D_P$ < 2.00] | |
| *Enter:* $C_{pf}$ = | | *0.082* | Figure 9-16 | | |
| Type of gearing: | | Open | Commer. | Precision | Ex. Prec. |
| Mesh Alignment Factor, $C_{ma}$ = | | 0.288 | 0.166 | 0.099 | 0.063 |
| *Enter:* $C_{ma}$ = | | *0.166* | Figure 9-17 | | |
| Alignment Factor: $K_m$ = | | 1.25 | [Computed] | | |
| Size Factor: $K_s$ = | | *1.00* | Table 9-8: Use 1.00 if $P_d$ >= 5 | | |
| Pinion Rim Thickness Factor: $K_{BP}$ = | | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | |
| Gear Rim Thickness Factor: $K_{BG}$ = | | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | |
| Service Factor: $SF$ = | | *1.00* | Use 1.00 if no unusual conditions | | |
| Reliability Factor: $K_R$ = | | *1.50* | Table 9-11 Use 1.00 for $R$ = .99 | | |
| *Enter: Design Life:* | | *100000* | hours | See Table 9-12 | |
| Pinion - Number of load cycles: $N_P$ = | | 1.8E+10 | Guidelines: $Y_N$, $Z_N$ | | |
| Gear - Number of load cycles: $N_G$ = | | 6.1E+09 | $10^7$ cycles | >$10^7$ | <$10^7$ |
| Bending Stress Cycle Factor: $Y_{NP}$ = | | *0.89* | 1.00 | 0.89 | Fig. 9-22 |
| Bending Stress Cycle Factor: $Y_{NG}$ = | | *0.91* | 1.00 | 0.91 | Fig. 9-22 |
| Pitting Stress Cycle Factor: $Z_{NP}$ = | | *0.84* | 1.00 | 0.84 | Fig. 9-23 |
| Pitting Stress Cycle Factor: $Z_{NG}$ = | | *0.86* | 1.00 | 0.86 | Fig. 9-23 |
| ***Stress Analysis: Bending*** | | | | | |
| Pinion: Required $s_{at}$ = | | 9,152 | psi | See Fig. 9-11 or | |
| Gear: Required $s_{at}$ = | | 6,901 | psi | Table 9-5 | |
| ***Stress Analysis: Pitting*** | | | | | |
| Pinion: Required $s_{ac}$ = | | 127,576 | psi | See Fig. 9-12 or | |
| Gear: Required $s_{ac}$ = | | 124,609 | psi | Table 9-5 | |
| Required hardness of pinion HB: | | 306 | Equations in Fig. 9-12-Grade 1 | | |
| Required hardness of gear HB: | | 297 | Equations in Fig. 9-12-Grade 1 | | |

***Specify materials, alloy and heat treatment, for most severe requirement.***

***One possible material specification:***

Pinion requires HB 306; SAE 4340 OQT 1100; HB 321; 19% elongation

Gear requires HB 297; SAE 4340 OQT 1100; HB 321; 19% elongation

***Comments:***

Same material used for pinion and gear because contact stresses are close.

# DESIGN OF SPUR GEARS

**APPLICATION:** *Problem 79 - Second pair*

*Industrial lift truck drive to wheels: driven by DC motor*

***Double reduction - Second pair - Input Data:***

| | | | |
|---|---|---|---|
| Overload Factor: | $K_o$ = | *1.50* | Table 9-7 |
| Transmitted Power: | $P$ = | *20* | hp |
| Design Power | $P_{des}$ = | 30 | hp |
| Diametral Pitch: | $P_d$ = | *5* | Fig. 9-24 |
| Input Speed: | $n_P$ = | *1020* | rpm |
| Number of Pinion Teeth: | $N_P$ = | *21* | |
| Desired Output Speed: | $n_G$ = | *560.23* | rpm |
| Computed number of gear teeth: | | 38.2 | |
| *Enter:* Chosen No. of Gear Teeth: | $N_G$ = | *38* | |

***Computed data:***

| | | | |
|---|---|---|---|
| Actual Output Speed: | $n_G$ = | 563.68 | rpm |
| Gear Ratio: | $m_G$ = | 1.81 | |
| Pitch Diameter - Pinion: | $D_P$ = | 4.200 | in |
| Pitch Diameter - Gear: | $D_G$ = | 7.600 | in |
| Center Distance: | C = | 5.900 | in |
| Pitch Line Speed: | $v_t$ = | 1122 | ft/min |
| Transmitted Load: | $W_t$ = | 588 | lb |

***Secondary Input Data:***

| | Min | Nom | Max |
|---|---|---|---|
| Face Width Guidelines (in): | 1.600 | 2.400 | 3.200 |
| *Enter:* Face Width: $F$ = | | *3.000* | in |
| Ratio: Face width/pinion diameter: $F/D_P$ = | | 0.71 | |
| Recommended range of ratio: $0.50 < F/D_P < 2.00$ | | | |

| | | | |
|---|---|---|---|
| *Enter:* Elastic Coefficient: | $C_p$ = | *2300* | Table 9-10 |
| *Enter:* Quality Number: | $A_v$ = | *7* | Table 9-3 |
| Dynamic Factor: | $K_v$ = | 1.14 | Table 9-9 |
| [Factors for computing $K_v$:] | $B$ = 0.397 | $C$ = 83.77 | |
| *Reference:* | $N_P$ = 21 | $N_G$ = 38 | |
| Bending Geometry Factor-Pinion: | $J_P$ = | *0.330* | Fig. 9-15 |
| Bending Geometry Factor-Gear: | $J_G$ = | *0.380* | Fig. 9-15 |
| *Reference:* | $m_G$ = | 1.81 | |
| *Enter:* Pitting Geometry Factor: | $I$ = | *0.092* | Fig. 9-21 |

***Factors in Design Analysis:***

| | | | | |
|---|---|---|---|---|
| Alignment Factor, $K_m = 1.0 + C_{pf} + C_{ma}$ | If F<1.0 | If F>1.0 | | |
| Pinion Proportion Factor, $C_{pf}$ = | 0.046 | 0.071 | [$0.50 < F/D_P < 2.00$] | |
| *Enter:* $C_{pf}$ = | *0.071* | Figure 9-16 | | |
| Type of gearing: | Open | Commer. | Precision | Ex. Prec. |
| Mesh Alignment Factor, $C_{ma}$ = | 0.296 | 0.173 | 0.105 | 0.068 |
| *Enter:* $C_{ma}$ = | *0.173* | Figure 9-17 | | |
| Alignment Factor: $K_m$ = | 1.24 | [Computed] | | |
| Size Factor: $K_s$ = | *1.00* | Table 9-8: Use 1.00 if $P_d$ >= 5 | | |
| Pinion Rim Thickness Factor: $K_{BP}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | |
| Gear Rim Thickness Factor: $K_{BG}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | |
| Service Factor: $SF$ = | *1.00* | Use 1.00 if no unusual conditions | | |
| Reliability Factor: $K_R$ = | *1.50* | Table 9-11 Use 1.00 for $R$ = .99 | | |
| *Enter:* Design Life: | *100000* | hours | See Table 9-12 | |
| Pinion - Number of load cycles: $N_P$ = | 6.1E+09 | Guidelines: $Y_N$, $Z_N$ | | |
| Gear - Number of load cycles: $N_G$ = | 3.4E+09 | $10^7$ cycles | $>10^7$ | $<10^7$ |
| Bending Stress Cycle Factor: $Y_{NP}$ = | *0.91* | 1.00 | 0.91 | Fig. 9-22 |
| Bending Stress Cycle Factor: $Y_{NG}$ = | *0.92* | 1.00 | 0.92 | Fig. 9-22 |
| Pitting Stress Cycle Factor: $Z_{NP}$ = | *0.86* | 1.00 | 0.86 | Fig. 9-23 |
| Pitting Stress Cycle Factor: $Z_{NG}$ = | *0.87* | 1.00 | 0.87 | Fig. 9-23 |

***Stress Analysis: Bending***

| | | | |
|---|---|---|---|
| Pinion: Required $s_{at}$ = | 10,447 | psi | See Fig. 9-11 or |
| Gear: Required $s_{at}$ = | 8,974 | psi | Table 9-5 |

***Stress Analysis: Pitting***

| | | | |
|---|---|---|---|
| Pinion: Required $s_{ac}$ = | 131,992 | psi | See Fig. 9-12 or |
| Gear: Required $s_{ac}$ = | 130,475 | psi | Table 9-5 |

| | | |
|---|---|---|
| Required hardness of pinion HB: | 320 | Equations in Fig. 9-12-Grade 1 |
| Required hardness of gear HB: | 315 | Equations in Fig. 9-12-Grade 1 |

***Specify materials, alloy and heat treatment, for most severe requirement.***

***One possible material specification:***

Pinion requires HB 320; SAE 4340 OQT 1100; HB 321; 19% elongation

Gear requires HB 315; SAE 4340 OQT 1100; HB 388; 19% elongation

***Comments:***

Same material and heat treatment used for pinion and gear.

Same material used for both Pair 1 and Pair2; Same tempering temperature.

| DESIGN OF PLASTIC SPUR GEARS | | | |
|---|---|---|---|
| ***Application:*** | | | |
| *Small band saw driven by an electric motor* | | | |
| *Problem 80 - One possible design* | | | |
| ***Initial Input Data:*** | | | |
| Input Power: | $P$ = | 0.5 hp | |
| Input Speed: | $n_P$ = | 860 rpm | |
| Diametral Pitch: | $P_d$ = | 12 | |
| Number of Pinion Teeth: | $N_P$ = | 18 | |
| Desired Output Speed: | $n_G$ = | 266 rpm | |
| Computed number of gear teeth: | | 58.195 | |
| *Enter:* Chosen No. of Gear Teeth: | $N_G$ = | 58 | |
| ***Computed data:*** | | | |
| Actual Output Speed: | $n_G$ = | 266.9 rpm | |
| Gear Ratio: | $m_G$ = | 3.222 | |
| Pitch Diameter - Pinion: | $D_P$ = | 1.500 in | |
| Pitch Diameter - Gear: | $D_G$ = | 4.833 in | |
| Center Distance: | $C$ = | 6.333 in | |
| Pitch Line Speed: | $v_t$ = | 337.7 ft/min | |
| Transmitted Load: | $W_t$ = | 48.84 lb | |
| ***Secondary Input Data - Pinion:*** | | | |
| Tooth Form: | | *20 degree full depth* | |
| Lewis Form Factor: | $Y$ = | 0.521 | Table 9-15 |
| Safety Factor: | $SF$ = | 1.50 | Ref. Table 9-7 |
| Material: | | Nylon | |
| Allowable Bending Stress: | $s_{at}$ = | 6000 psi | Table 9-14 or Fig. 9-28 |
| Required Face Width: | $F$ = | 0.281 in | |
| *Enter:* Specified Face Width: | $F$ = | 0.300 in | |
| Actual Bending Stress in Pinion: | $s_t$ = | 5624 psi | |
| ***Secondary Input Data - Gear:*** | | | |
| Tooth Form: | | *20 degree full depth* | Same as for pinion |
| Lewis Form Factor: | $Y$ = | 0.709 | Table 9-15 |
| Safety Factor: | $SF$ = | 1.50 | Same as for pinion |
| Material: | | Acetal | |
| Allowable Bending Stress: | $s_{at}$ = | 5000 psi | Table 9-14 or Fig. 9-28 |
| Face Width - Gear: | $F$ = | 0.300 in | Same as for pinion |
| Actual Bending Stress in Gear: | $s_t$ = | 4133 psi | Must be < $s_{at}$ |

| DESIGN OF PLASTIC SPUR GEARS | | | |
|---|---|---|---|
| ***Application:*** | | | |
| *Paper feed roll driven by an electric motor* | | | |
| *Problem 81 - One possible design* | | | |
| ***Initial Input Data:*** | | | |
| Input Power: | $P$ = | 0.06 hp | |
| Input Speed: | $n_P$ = | 88 rpm | |
| Diametral Pitch: | $P_d$ = | 20 | |
| Number of Pinion Teeth: | $N_P$ = | 16 | |
| Desired Output Speed: | $n_G$ = | 21 rpm | |
| Computed number of gear teeth: | | 67.048 | |
| *Enter:* Chosen No. of Gear Teeth: | $N_G$ = | 67 | |
| ***Computed data:*** | | | |
| Actual Output Speed: | $n_G$ = | 21.0 rpm | |
| Gear Ratio: | $m_G$ = | 4.188 | |
| Pitch Diameter - Pinion: | $D_P$ = | 0.800 in | |
| Pitch Diameter - Gear: | $D_G$ = | 3.350 in | |
| Center Distance: | $C$ = | 4.150 in | |
| Pitch Line Speed: | $v_t$ = | 18.4 ft/min | |
| Transmitted Load: | $W_t$ = | 107.4 lb | |
| ***Secondary Input Data - Pinion:*** | | | |
| Tooth Form: | | *20 degree stub* | |
| Lewis Form Factor: | $Y$ = | 0.578 | Table 9- 15 |
| Safety Factor: | $SF$ = | 1.25 | Ref. Table 9- 7 |
| Material: | | Nylon-glass filled | |
| Allowable Bending Stress: | $s_{at}$ = | 12000 psi | Table 9- 14 or Fig. 9- 28 |
| Required Face Width: | $F$ = | 0.387 in | |
| *Enter:* Specified Face Width: | $F$ = | 0.400 in | |
| Actual Bending Stress in Pinion: | $s_t$ = | 11612 psi | |
| ***Secondary Input Data - Gear:*** | | | |
| Tooth Form: | | *20 degree stub* | Same as for pinion |
| Lewis Form Factor: | $Y$ = | 0.782 | Table 9- 15 |
| Safety Factor: | $SF$ = | 1.25 | Same as for pinion |
| Material: | | Nylon-glass filled | |
| Allowable Bending Stress: | $s_{at}$ = | 12000 psi | Table 9- 14 or Fig. 9- 28 |
| Face Width - Gear: | $F$ = | 0.400 in | Same as for pinion |
| Actual Bending Stress in Gear: | $s_t$ = | 8583 psi | Must be < $s_{at}$ |

| DESIGN OF PLASTIC SPUR GEARS | | | |
|---|---|---|---|
| ***Application:*** | | | |
| *Wheels of remote control car-Electric motor drive* | | | |
| *Problem 82 - One possible design* | | | |
| ***Initial Input Data:*** | | | |
| Input Power: | $P$ = | 0.025 hp | |
| Input Speed: | $n_P$ = | 430 rpm | |
| Diametral Pitch: | $P_d$ = | 48 | |
| Number of Pinion Teeth: | $N_P$ = | 14 | |
| Desired Output Speed: | $n_G$ = | 121 rpm | |
| Computed number of gear teeth: | | 49.752 | |
| *Enter:* Chosen No. of Gear Teeth: | $N_G$ = | 50 | |
| ***Computed data:*** | | | |
| Actual Output Speed: | $n_G$ = | 120.4 rpm | |
| Gear Ratio: | $m_G$ = | 3.571 | |
| Pitch Diameter - Pinion: | $D_P$ = | 0.292 in | |
| Pitch Diameter - Gear: | $D_G$ = | 1.042 in | |
| Center Distance: | $C$ = | 1.333 in | |
| Pitch Line Speed: | $v_t$ = | 32.8 ft/min | |
| Transmitted Load: | $W_t$ = | 25.1 lb | |
| ***Secondary Input Data - Pinion:*** | | | |
| Tooth Form: | | *20 degree stub* | |
| Lewis Form Factor: | $Y$ = | 0.54 | Table 9-15 |
| Safety Factor: | $SF$ = | 1.25 | Ref: Table 9-7 |
| Material: | | Nylon-unfilled | |
| Allowable Bending Stress: | $s_{at}$ = | 6000 psi | Table 9-14 or Fig. 9-28 |
| Required Face Width: | $F$ = | 0.465 in | |
| *Enter:* Specified Face Width: | $F$ = | 0.470 in | |
| Actual Bending Stress in Pinion: | $s_t$ = | 5938 psi | |
| ***Secondary Input Data - Gear:*** | | | |
| Tooth Form: | | *20 degree stub* | Same as for pinion |
| Lewis Form Factor: | $Y$ = | 0.758 | Table 9-15 |
| Safety Factor: | $SF$ = | 1.25 | Same as for pinion |
| Material: | | Nylon-unfilled | |
| Allowable Bending Stress: | $s_{at}$ = | 6000 psi | Table 9-14 or Fig. 9-28 |
| Face Width - Gear: | $F$ = | 0.470 in | Same as for pinion |
| Actual Bending Stress in Gear: | $s_t$ = | 4230 psi | Must be < $s_{at}$ |

| DESIGN OF PLASTIC SPUR GEARS | | | |
|---|---|---|---|
| ***Application:*** | | | |
| *Food-chopping machine driven by electric motor* | | | |
| *Problem 83 - One possible design* | | | |
| ***Initial Input Data:*** | | | |
| Input Power: $P$ = | 0.65 hp | | |
| Input Speed: $n_P$ = | 1560 rpm | | |
| Diametral Pitch: $P_d$ = | 16 | | |
| Number of Pinion Teeth: $N_P$ = | 18 | | |
| Desired Output Speed: $n_G$ = | 469 rpm | | |
| Computed number of gear teeth: | 59.872 | | |
| *Enter:* Chosen No. of Gear Teeth: $N_G$ = | 60 | | |
| ***Computed data:*** | | | |
| Actual Output Speed: $n_G$ = | 468.0 rpm | | |
| Gear Ratio: $m_G$ = | 3.333 | | |
| Pitch Diameter - Pinion: $D_P$ = | 1.125 in | | |
| Pitch Diameter - Gear: $D_G$ = | 3.750 in | | |
| Center Distance: $C$ = | 4.875 in | | |
| Pitch Line Speed: $v_t$ = | 459.5 ft/min | | |
| Transmitted Load: $W_t$ = | 46.7 lb | | |
| ***Secondary Input Data - Pinion:*** | | | |
| Tooth Form: | *20 degree full depth* | | |
| Lewis Form Factor: $Y$ = | 0.521 | Table 9-15 | |
| Safety Factor: $SF$ = | 1.75 | Ref: Table 9-7 | |
| Material: | Nylon-unfilled | | |
| Allowable Bending Stress: $s_{at}$ = | 6000 psi | Table 9-14 or Fig. 9-28 | |
| Required Face Width: $F$ = | 0.418 in | | |
| *Enter:* Specified Face Width: $F$ = | 0.420 in | | |
| Actual Bending Stress in Pinion: $s_t$ = | 5971 psi | | |
| ***Secondary Input Data - Gear:*** | | | |
| Tooth Form: | *20 degree full depth* | Same as for pinion | |
| Lewis Form Factor: $Y$ = | 0.713 | Table 9-15 | |
| Safety Factor: $SF$ = | 1.75 | Same as for pinion | |
| Material: | Acetal-unfilled | | |
| Allowable Bending Stress: $s_{at}$ = | 5000 psi | Table 9-14 or Fig. 9-28 | |
| Face Width - Gear: $F$ = | 0.420 in | Same as for pinion | |
| Actual Bending Stress in Gear: $s_t$ = | 4363 psi | Must be < $s_{at}$ | |

# CHAPTER 10
## HELICAL GEARS, BEVEL GEARS, AND WORMGEARING

**1.** HELICAL GEARS: $P_d = 8$, $\phi_t = 14\frac{1}{2}°$, $N_G = 45$, $F = 2.0\,IN$, $\psi = 30°$

a. $P = 5.0\,HP$, $n_G = 1250\,RPM$: TORQUE $= T = \dfrac{63000(5.0)}{1250} = 252\,LB \cdot IN$

$W_t = \dfrac{T}{D_G/2}$ : $D_G = \dfrac{N_G}{P_d} = \dfrac{45}{8} = 5.625\,IN$

$\underline{W_t} = \dfrac{252\,LB \cdot IN}{5.625\,IN/2} = \underline{89.6\,LB}$

$\underline{W_x} = W_t \tan\psi = 89.6 \tan 30° = \underline{51.7\,LB}$

$\underline{W_r} = W_t \tan\phi_t = 89.6 \tan 14\frac{1}{2}° = \underline{23.2\,LB}$

b. $N_P = 15$; DRIVE TO A RECIPROCATING PUMP, $K_o = 1.50$

$s_{tP} = \dfrac{W_t P_d}{F J_P} \cdot K_o K_s K_m K_B K_v$; ASSUME $K_s = K_B = 1.0$

APPROXIMATE $J_P$ FROM FIG 10-5. DATA ARE FOR $\phi_n = 15°$

ACTUAL $\phi_n = \tan^{-1}[\tan\phi_t \cos\psi] = \tan^{-1}[\tan 14\frac{1}{2} \cos 30°] = 12.6°$

$J_P \approx (0.38)(0.97) = 0.369$

$D_P = N_P/P_d = 15/8 = 1.875\,IN$; $F/D_P = 1.067$; $C_{pf} = 0.075$; $C_{ma} = .155$

$K_m = 1.0 + C_{pf} + C_{ma} = 1.00 + 0.075 + 0.155 = 1.23$

$v_t = \pi D_G n_G/12 = \pi(5.625)(1250)/12 = 1841\,FT/MIN$

SPECIFY $A_v = 11$: THE $K_v = 1.55$ (FIG 9-20)

$\underline{s_{tP}} = \dfrac{(89.6)(8)}{(2.00)(0.369)}(1.50)(1.0)(1.23)(1.0)(1.55) = \underline{2778\,PSI}$

$s_c = C_p\sqrt{\dfrac{W_t K_o K_s K_m K_v}{F D_P I}} = 1960\sqrt{\dfrac{(89.6)(1.50)(1.0)(1.23)(1.55)}{(2.00)(1.875)(0.20)}}$

$\underline{s_c = 36228\,PSI}$ : I EST. FROM TABLE 10-1.

c. SPECIFIED CAST IRON BECAUSE OF LOW STRESSES

<u>GRAY CAST IRON</u>: <u>$s_{at} = 5000\,PSI$</u>, <u>$s_{ac} = 50\,000\,PSI$</u>

<u>CLASS 20</u>

$\boxed{2}$ HELICAL GEARS: $P = 2.50\ hP$, $N_P = 16$, $N_G = 48$, $P_{d_m} = 12$, $\phi_m = 20°$, $\psi = 45°$

$F = 1.50$ IN: TORQUE $= T = \dfrac{63000(2.50)}{1750} = 90.0$ LB-IN

$n_G = 1750$ RPM:

a. $W_t = \dfrac{T}{D_G/2} = \dfrac{90.0}{(5.657/2)} = \underline{31.8\ LB.}$

FROM PROBLEM 8-42: $P_d = 8.485$; $D_G = 5.657$ IN, $\phi_t = 27.2°$

$D_P = \dfrac{N_P}{P_d} = \dfrac{16}{8.485} = 1.887$ IN

$\underline{W_x} = W_t \tan\psi = 31.8\ LB \cdot \tan 45° = \underline{31.8\ LB}$

$\underline{W_r} = W_t \tan\phi_t = 31.8 \cdot \tan 27.2° = \underline{16.4\ LB}$

b. $K_o = 1.25$ (LT. SHOCK); $K_s = K_B = 1.00$; $J_P \cong 0.30$

$v_t = \pi D_G n_G/12 = \pi(5.657)(1750)/12 = 2592$ FT/MIN

CENTRIFUGAL BLOWER LET $A_v = 9$; $K_v = 1.40$

$F/D_P = 1.50/1.887 = 0.795$ $C_{pf} = 0.05$, $C_{ma} = 0.15$; $K_m = 1.20$

$\underline{S_t} = \dfrac{W_t P_d}{F J_P} K_o K_s K_m K_B K_v = \dfrac{(31.8)(8.485)(1.25)(1.0)(1.20)(1.40)}{(1.50)(0.30)} = \underline{1259\ PSI}$

PITTING: $I \approx 0.21$, $C_P = 1960$ CAST IRON

$S_c = C_P\sqrt{\dfrac{W_t K_o K_s K_m K_v}{F D_P I}} = 1960\sqrt{\dfrac{(31.8)(1.25)(1.0)(1.2)(1.40)}{(1.5)(1.887)(0.21)}}$

c. $\underline{S_c = 20{,}775\ PSI}$: SPECIFY $\underline{\text{CLASS 20 CAST IRON}}$

$S_{at} = 5000\ PSI$, $S_{ac} = 50000\ PSI$

$\boxed{3}$ HELICAL GEARS: $P = 15\ hP$, $N_P = 12$, $N_G = 36$, $P_d = 6$, $\phi_t = 14\tfrac{1}{2}°$; $\psi = 45°$, $F = 1.00$ IN

$n_G = 2200$ RPM; $T = \dfrac{63000\ P}{n} = \dfrac{63000(15)}{2200} = 430$ LB·IN

FROM PROB. 8-43: $D_G = 6.00$ IN; $D_P = \dfrac{N_P}{P_d} = \dfrac{12}{6} = 2.00$ IN

a. $\underline{W_t} = T/D_G/2 = 430/(6.00/2) = \underline{143\ LB}$

$\underline{W_x} = W_t \tan\psi = 143\ LB \cdot \tan 45° = \underline{143\ LB}$

$\underline{W_r} = W_t \tan\phi_t = 143\ LB \cdot \tan 14\tfrac{1}{2}° = \underline{37.0\ LB}$

$K_o = 2.00$ CONC. MIXER

$K_s = 1.00 = K_B$

$\dfrac{F}{D_P} = \dfrac{1.00}{2.00} = 0.50$; $C_{pf} = 0.025$

$C_{ma} = 0.15$; $K_m = 1.18$

b. $v_t = \pi D_G n_G/12 = \pi(6.00)(2200)/12 = 3456$ FT/MIN

$A_v = 9$; $K_v = 1.44$; $J_P = 0.30$ (EST.), $I = 0.190$ (EST)

$\underline{S_t} = \dfrac{W_t P_d}{F J_P} K_o K_s K_m K_B K_v = \dfrac{(143)(6)(1.0)(1.18)(1.0)(1.44)(2.0)}{(1.00)(0.30)} = \underline{9720\ PSI}$

USE NODULAR (DUCTILE) IRON $C_P = 2050$

$\underline{S_c} = C_P\sqrt{\dfrac{W_t K_o K_s K_m K_v}{F D_P I}} = 2050\sqrt{\dfrac{(143)(2.0)(1.0)(1.18)(1.44)}{(1.0)(2.00)(0.19)}} = \underline{73300\ PSI}$

c. SPECIFY: $\underline{\text{DUCTILE IRON ASTM A536 60-40-18}}$

$S_{at} = 22000\ PSI$, $S_{ac} = 77000\ PSI$

OR CI CLASS 40

$S_{at} = 13\ KSI$, $S_{ac} = 75\ KSI$

NOTE: PROB. 8-43 GIVES $P_x$ = AXIAL PITCH = 0.5236 IN

THEN $F/P_x = 1.00/0.5236 = 1.91$ - LOW - SHOULD BE 2.0 FOR FULL HELIX ACTION.

4 HELICAL GEARS: $P = 0.50$ hp, $n_G = 3450$ RPM, $P_{nd} = 24$, $\phi_n = 14\frac{1}{2}°$

$\psi = 45°$, $N_G = 72$, $N_P = 16$, $F = 0.25$ IN, WINCH-MOD. SHOCK $K_o = 1.50$

FROM PROB 8-44: $P_d = 16.97$, $D_G = 4.243$ IN $D_P = \frac{N_P}{P_d} = \frac{16}{16.97} = 0.943$ IN

$$T = \frac{63\,000(P)}{n_G} = \frac{63\,000(0.50)}{3450} = 9.13 \text{ LB·IN}$$

a. $W_t = T/(D_G/2) = (9.13)/(4.243/2) = 4.30$ LB

$W_x = W_t \tan\psi = 4.30 \text{ LB} \cdot \tan 45° = 4.30$ LB

$W_r = W_t \tan\phi_t = 4.30 \tan 20° = 1.57$ LB ; $\phi_t = 20.0°$ FROM PROB 8-44.

b. $v_t = \pi D_G n_G/12 = \pi(4.243)(3450)/12 = 3832$ FT/MIN

LET $A_v = 9$, $K_v = 1.48$, $K_s = 1.00 = K_B$. $J_P = 0.32$ (EST) $I = 0.25$ (EST)

$F/D_P = 0.25/0.943 = 0.265$; $C_{pf} \simeq 0$; $C_{ma} = 0.14$; $K_m = 1.14$

$$s_t = \frac{W_t P_d}{F J} \cdot K_o K_s K_m K_B K_v = \frac{(4.30)(16.97)(1.50)(1.0)(1.14)(1.0)(1.48)}{(0.25)(0.32)}$$

$s_t = 2308$ psi    TRY CAST IRON

$$s_c = C_p \sqrt{\frac{W_t K_o K_s K_m K_v}{F D_P I}} = 1960 \sqrt{\frac{(4.30)(1.50)(1.0)(1.14)(1.48)}{(0.25)(0.943)(0.25)}}$$

$s_c = 26\,633$ psi

SPECIFY CLASS 20 CAST IRON $s_{at} = 5000$ PSI $s_{ac} = 50\,000$ PSI

NOTE: PROBLEM 8-44 GIVES $P_x$ = AXIAL PITCH = 0.1815 IN

THEN $F/P_x = 0.25\text{IN}/0.1815\text{IN} = 1.35$ — LOW — SHOULD BE 2.0 FOR FULL HELICAL ACTION.

THE FOLLOWING PAGES GIVE SAMPLE DESIGNS FOR PROBLEMS 5-11. THE PROCEDURE IS SIMILAR TO THAT USED IN EXAMPLE PROBLEM 10-2. OTHER DESIGNS ARE POSSIBLE. READER IS ENCOURAGED TO WORK TOWARD A PARTICULAR GOAL OF MATERIAL TYPE, CENTER DISTANCE, OVERALL SIZE OR OTHER APPLICATION-SPECIFIC GOAL.

NOTE THAT TRANSVERSE DIAMETRAL PITCH MUST BE INPUT. IF NORMAL DIAMETRAL PITCH IS ORIGINALLY SPECIFIED, COMPUTE $P_d = P_{nd} \cos\psi$.

| DESIGN OF HELICAL GEARS-U.S. | | APPLICATION: | Reciprocating compressor driven by an electric motor Problem 10-5 |
|---|---|---|---|

| **Initial Input Data:** | | | |
|---|---|---|---|
| Input Power: | $P$ = | 5 | hp |
| Input Speed: | $n_P$ = | 1200 | rpm |
| Transverse Diametral Pitch, $P_d$: | $P_d$ = | 18 | |
| Number of Pinion Teeth: | $N_P$ = | 18 | |
| Desired Output Speed: | $n_G$ = | 387.5 | rpm |
| Computed number of gear teeth: | | 55.7 | |
| *Enter:* Chosen No. of Gear Teeth: | $N_G$ = | 56 | |
| **Computed data:** | | | |
| Actual Output Speed: | $n_G$ = | 385.7 | rpm |
| Gear Ratio: | $m_G$ = | 3.11 | |
| Pitch Diameter - Pinion: | $D_P$ = | 1.000 | in |
| Pitch Diameter - Gear: | $D_G$ = | 3.111 | in |
| Center Distance: | $C$ = | 2.056 | in |
| Pitch Line Speed: | $v_t$ = | 314 | ft/min |
| Transmitted Load: | $W_t$ = | 525 | lb |
| **Secondary Input Data:** | | | |
| Transverse pressure angle: | $\phi_t$ = | 20.0 | deg |
| Helix angle: | $\psi$ = | 25.0 | deg |
| Axial Pitch: | $p_x$ = | 0.3743 | in |
| Min. Face Width (2 x Axial Pitch): | $F_{min}$ = | 0.749 | in |
| *Enter:* Face Width: | $F$ = | 1.200 | in |
| *Enter:* Elastic Coefficient: | $Cp$ = | 2300 | Table 9-10 |
| *Enter:* Quality Number: | $A_v$ = | 11 | Table 9-3 |
| REF: $N_P$, $N_G$ = | 18 | 56 | |
| *Enter:* Bending Geometry Factors: | | | |
| Pinion: | $J_P$ = | 0.453 | Fig 10-5,6,7 |
| Gear: | $J_G$ = | 0.486 | Fig 10-5,6,7 |
| *Enter:* Pitting Geometry Factor: | $I$ = | 0.205 | Tab. 10-1,2 |
| REF: $m_G$ = | 3.11 | | |
| Axial Force: | $W_x$ = | 245 | lb |
| Radial Force: | $W_r$ = | 191 | lb |

| **Factors in Design Analysis:** | | | | |
|---|---|---|---|---|
| Alignment Factor, $K_m$=1.0+$C_{pf}$+$C_{ma}$ | If F<1.0 | If F>1.0 | $F/D_P$ = 1.20 | |
| Pinion Proportion Factor, $C_{pf}$ = | 0.095 | 0.098 | [0.50 < $F/D_P$ < 2.00] | |
| *Enter:* $C_{pf}$ = | 0.098 | Figure 9-16 | | |
| Type of gearing: | Open | Commer. | Precision | Ex. Prec. |
| Mesh Alignment Factor, $C_{ma}$ = | 0.267 | 0.146 | 0.083 | 0.050 |
| *Enter:* $C_{ma}$ = | 0.146 | Figure 9-17 | | |
| Alignment Factor: $K_m$ = | 1.24 | [Computed] | | |
| Overload Factor: $K_o$ = | 1.50 | Table 9-7 | | |
| Size Factor: $K_s$ = | 1.00 | Table 9-8: Use 1.00 if $P_d$ >= 5 | | |
| Pinion Rim Thickness Factor: $K_{BP}$ = | 1.00 | Fig. 9-18: Use 1.00 if solid blank | | |
| Gear Rim Thickness Factor: $K_{BG}$ = | 1.00 | Fig. 9-18: Use 1.00 if solid blank | | |
| Dynamic Factor: $K_v$ = | 1.24 | [Computed: See Fig. 9-20] | | |
| Service Factor: $SF$ = | 1.00 | Use 1.00 if no unusual conditions | | |
| Reliability Factor: $K_R$ = | 1.00 | Table 9-11 | Use 1.00 for $R$ = .99 | |
| *Enter:* Design Life: | 15000 | hours | See Table 9-12 | |
| Pinion - Number of load cycles: $N_P$ = | 1.1E+09 | Guidelines: $Y_N$, $Z_N$ | | |
| Gear - Number of load cycles: $N_G$ = | 3.5E+08 | $10^7$ cycles | >$10^7$ | <$10^7$ |
| Bending Stress Cycle Factor: $Y_{NP}$ = | 0.94 | 1.00 | 0.94 | Fig. 9-22 |
| Bending Stress Cycle Factor: $Y_{NG}$ = | 0.96 | 1.00 | 0.96 | Fig. 9-22 |
| Pitting Stress Cycle Factor: $Z_{NP}$ = | 0.90 | 1.00 | 0.90 | Fig. 9-24 |
| Pitting Stress Cycle Factor: $Z_{NG}$ = | 0.92 | 1.00 | 0.92 | Fig. 9-24 |
| ***Stress Analysis: Bending*** | | | | |
| Pinion: Required $s_{at}$ = | 42,786 | psi | See Fig. 9-11 or | |
| Gear: Required $s_{at}$ = | 39,050 | psi | Table 9-5 | |
| ***Stress Analysis: Pitting*** | | | | |
| Pinion: Required $s_{ac}$ = | 179,572 | psi | See Fig. 9-12 or | |
| Gear: Required $s_{ac}$ = | 175,668 | psi | Table 9-5 | |

***Specify materials, alloy and heat treatment, for most severe requirement.***

***One possible material specification:*** Steel pinion and gear: Carburized, Grade 1

Pinion requires HRC 58 min.: SAE 4320 SOQT 450; HRC 59; Carburized

Gear requires HRC 58 min.: SAE 4320 SOQT 450; HRC 59; Carburized

**DESIGN OF HELICAL GEARS-U.S.** *APPLICATION:* *Milling machine driven by an electric motor*
*Problem 10-6*

| | | | |
|---|---|---|---|
| ***Initial Input Data:*** | | | |
| Input Power: | $P$ = | *20* | hp |
| Input Speed: | $n_P$ = | *550* | rpm |
| Transverse Diametral Pitch, $P_d$: | $P_d$ = | *10* | |
| Number of Pinion Teeth: | $N_P$ = | *24* | |
| Desired Output Speed: | $n_G$ = | *185* | rpm |
| Computed number of gear teeth: | | 71.4 | |
| *Enter:* Chosen No. of Gear Teeth: | $N_G$ = | *72* | |
| ***Computed data:*** | | | |
| Actual Output Speed: | $n_G$ = | 183.3 | rpm |
| Gear Ratio: | $m_G$ = | 3.00 | |
| Pitch Diameter - Pinion: | $D_P$ = | 2.400 | in |
| Pitch Diameter - Gear: | $D_G$ = | 7.200 | in |
| Center Distance: | $C$ = | 4.800 | in |
| Pitch Line Speed: | $v_t$ = | 346 | ft/min |
| Transmitted Load: | $W_t$ = | 1910 | lb |
| ***Secondary Input Data:*** | | | |
| Transverse pressure angle: | $\phi_t$ = | *20.0* | deg |
| Helix angle: | $\psi$ = | *15.0* | deg |
| Axial Pitch: | $p_x$ = | 1.1725 | in |
| Min. Face Width (2 x Axial Pitch): | $F_{min}$ = | 2.345 | in |
| *Enter:* Face Width: | $F$ = | *2.500* | in |
| *Enter:* Elastic Coefficient: | $Cp$ = | *2300* | Table 9-10 |
| *Enter:* Quality Number: | $A_v$ = | *9* | Table 9-3 |
| REF: $N_P$, $N_G$ = | 24 | 72 | |
| *Enter:* Bending Geometry Factors: | | | |
| Pinion: | $J_P$ = | *0.480* | Fig 10-5,6,7 |
| Gear: | $J_G$ = | *0.526* | Fig 10-5,6,7 |
| *Enter:* Pitting Geometry Factor: | $I$ = | *0.220* | Tab. 10-1,2 |
| REF: $m_G$ = | 3.00 | | |
| Axial Force: | $W_x$ = | 512 | lb |
| Radial Force: | $W_r$ = | 695 | lb |

| | | | | |
|---|---|---|---|---|
| ***Factors in Design Analysis:*** | | | | |
| Alignment Factor, $K_m$=1.0+$C_{pf}$+$C_{ma}$ | If F<1.0 | If F>1.0 | $F/D_P$ = 1.04 | |
| Pinion Proportion Factor, $C_{pf}$ = | 0.079 | 0.098 | [0.50 < $F/D_P$ < 2.00] | |
| *Enter:* $C_{pf}$ = | *0.098* | Figure 9-16 | | |
| Type of gearing: | Open | Commer. | Precision | Ex. Prec. |
| Mesh Alignment Factor, $C_{ma}$ = | 0.288 | 0.166 | 0.099 | 0.063 |
| *Enter:* $C_{ma}$ = | *0.099* | Figure 9-17 | | |
| Alignment Factor: $K_m$ = | 1.20 | [Computed] | | |
| Overload Factor: $K_o$ = | *1.50* | Table 9-7 | | |
| Size Factor: $K_s$ = | *1.00* | Table 9-8: Use 1.00 if $P_d$ >= 5 | | |
| Pinion Rim Thickness Factor: $K_{BP}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | |
| Gear Rim Thickness Factor: $K_{BG}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | |
| Dynamic Factor: $K_v$ = | 1.16 | [Computed: See Fig. 9-20] | | |
| Service Factor: $SF$ = | *1.00* | Use 1.00 if no unusual conditions | | |
| Reliability Factor: $K_R$ = | *1.25* | Table 9-11 | Use 1.00 for $R$ = .99 | |
| *Enter:* Design Life: | *15000* | hours | See Table 9-12 | |
| Pinion - Number of load cycles: $N_P$ = | 5.0E+08 | Guidelines: $Y_N$, $Z_N$ | | |
| Gear - Number of load cycles: $N_G$ = | 1.7E+08 | $10^7$ cycles | >$10^7$ | <$10^7$ |
| Bending Stress Cycle Factor: $Y_{NP}$ = | *0.95* | 1.00 | 0.95 | Fig. 9-22 |
| Bending Stress Cycle Factor: $Y_{NG}$ = | *0.97* | 1.00 | 0.97 | Fig. 9-22 |
| Pitting Stress Cycle Factor: $Z_{NP}$ = | *0.91* | 1.00 | 0.91 | Fig. 9-24 |
| Pitting Stress Cycle Factor: $Z_{NG}$ = | *0.94* | 1.00 | 0.94 | Fig. 9-24 |
| ***Stress Analysis: Bending*** | | | | |
| Pinion: Required $s_{at}$ = | 43,560 | psi | See Fig. 9-11 or | |
| Gear: Required $s_{at}$ = | 38,931 | psi | Table 9-5 | |
| ***Stress Analysis: Pitting*** | | | | |
| Pinion: Required $s_{ac}$ = | 173,320 | psi | See Fig. 9-12 or | |
| Gear: Required $s_{ac}$ = | 167,788 | psi | Table 9-5 | |

***Specify materials, alloy and heat treatment, for most severe requirement.***

***One possible material specification:*** Steel pinion and gear: Carburized, Grade 1

Pinion requires HRC 58 min.: SAE 4320 SOQT 450; HRC 59; Carburized

Gear requires HRC 58 min.: SAE 4320 SOQT 450; HRC 59; Carburized

**DESIGN OF HELICAL GEARS-U.S.** | *APPLICATION:* *Punch press driven by an electric motor*
*Problem 10-7*

| *Initial Input Data:* | | | |
|---|---|---|---|
| Input Power: | $P$ = | *50* | hp |
| Input Speed: | $n_P$ = | *900* | rpm |
| Transverse Diametral Pitch, $P_d$: | $P_d$ = | *6* | |
| Number of Pinion Teeth: | $N_P$ = | *24* | |
| Desired Output Speed: | $n_G$ = | *227.5* | rpm |
| Computed number of gear teeth: | | 94.9 | |
| *Enter:* Chosen No. of Gear Teeth: | $N_G$ = | *95* | |
| ***Computed data:*** | | | |
| Actual Output Speed: | $n_G$ = | 227.4 | rpm |
| Gear Ratio: | $m_G$ = | 3.96 | |
| Pitch Diameter - Pinion: | $D_P$ = | 4.000 | in |
| Pitch Diameter - Gear: | $D_G$ = | 15.833 | in |
| Center Distance: | $C$ = | 9.917 | in |
| Pitch Line Speed: | $v_t$ = | 942 | ft/min |
| Transmitted Load: | $W_t$ = | 1751 | lb |
| ***Secondary Input Data:*** | | | |
| Transverse pressure angle: | $\phi_t$ = | *20.0* | deg |
| Helix angle: | $\psi$ = | *25.0* | deg |
| Axial Pitch: | $p_x$ = | 1.1229 | in |
| Min. Face Width (2 x Axial Pitch): | $F_{min}$ = | 2.246 | in |
| *Enter:* Face Width: | $F$ = | *2.500* | in |
| *Enter:* Elastic Coefficient: | $Cp$ = | *2300* | Table 9-10 |
| *Enter:* Quality Number: | $A_v$ = | *9* | Table 9-3 |
| REF: $N_P$, $N_G$ = | 24 | 95 | |
| *Enter:* Bending Geometry Factors: | | | |
| Pinion: | $J_P$ = | *0.465* | Fig 10-5,6,7 |
| Gear: | $J_G$ = | *0.496* | Fig 10-5,6,7 |
| *Enter:* Pitting Geometry Factor: | $I$ = | *0.220* | Tab. 10-1,2 |
| REF: $m_G$ = | 3.96 | | |
| Axial Force: | $W_x$ = | 816 | lb |
| Radial Force: | $W_r$ = | 637 | lb |

| ***Factors in Design Analysis:*** | | | | |
|---|---|---|---|---|
| Alignment Factor, $K_m$=1.0+$C_{pf}$+$C_{ma}$ | If F<1.0 | If F>1.0 | $F/D_P$ = 0.63 | |
| Pinion Proportion Factor, $C_{pf}$ = | 0.038 | 0.056 | [0.50 < $F/D_P$ < 2.00] | |
| *Enter:* $C_{pf}$ = | *0.056* | Figure 9-16 | | |
| Type of gearing: | Open | Commer. | Precision | Ex. Prec. |
| Mesh Alignment Factor, $C_{ma}$ = | 0.288 | 0.166 | 0.099 | 0.063 |
| *Enter:* $C_{ma}$ = | *0.166* | Figure 9-17 | | |
| Alignment Factor: $K_m$ = | 1.22 | [Computed] | | |
| Overload Factor: $K_o$ = | *1.75* | Table 9-7 | | |
| Size Factor: $K_s$ = | *1.00* | Table 9-8: Use 1.00 if $P_d$ >= 5 | | |
| Pinion Rim Thickness Factor: $K_{BP}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | |
| Gear Rim Thickness Factor: $K_{BG}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | |
| Dynamic Factor: $K_v$ = | 1.26 | [Computed: See Fig. 9-20] | | |
| Service Factor: $SF$ = | *1.00* | Use 1.00 if no unusual conditions | | |
| Reliability Factor: $K_R$ = | *1.25* | Table 9-11 | Use 1.00 for $R$ = .99 | |
| *Enter: Design Life:* | *15000* | hours | See Table 9-12 | |
| Pinion - Number of load cycles: $N_P$ = | 8.1E+08 | Guidelines: $Y_N$, $Z_N$ | | |
| Gear - Number of load cycles: $N_G$ = | 2.0E+08 | $10^7$ cycles | >$10^7$ | <$10^7$ |
| Bending Stress Cycle Factor: $Y_{NP}$ = | *0.94* | 1.00 | 0.94 | Fig. 9-22 |
| Bending Stress Cycle Factor: $Y_{NG}$ = | *0.96* | 1.00 | 0.96 | Fig. 9-22 |
| Pitting Stress Cycle Factor: $Z_{NP}$ = | *0.90* | 1.00 | 0.90 | Fig. 9-24 |
| Pitting Stress Cycle Factor: $Z_{NG}$ = | *0.93* | 1.00 | 0.93 | Fig. 9-24 |

***Stress Analysis: Bending***

| | | | |
|---|---|---|---|
| Pinion: Required $s_{at}$ = | 32,253 | psi | See Fig. 9-11 or |
| Gear: Required $s_{at}$ = | 29,607 | psi | Table 9-5 |

***Stress Analysis: Pitting***

| | | | |
|---|---|---|---|
| Pinion: Required $s_{ac}$ = | 147,637 | psi | See Fig. 9-12 or |
| Gear: Required $s_{ac}$ = | 142,875 | psi | Table 9-5 |

***Specify materials, alloy and heat treatment, for most severe requirement.***

***One possible material specification:*** Steel pinion and gear: Through hardened

Pinion requires HB 368: SAE 4140 OQT 900; HB 388

Gear requires HB 353: SAE 4140 OQT 900; HB 388

**DESIGN OF HELICAL GEARS-U.S.** | *APPLICATION:* *Small cement mixer driven by a gasoline engine*
*Problem 10-8* | *Use steel pinion with cast iron gear*

| | | | |
|---|---|---|---|
| ***Initial Input Data:*** | | | |
| Input Power: | $P$ = | 2.5 | hp |
| Input Speed: | $n_P$ = | 900 | rpm |
| Transverse Diametral Pitch, $P_d$: | $P_d$ = | 12 | |
| Number of Pinion Teeth: | $N_P$ = | 20 | |
| Desired Output Speed: | $n_G$ = | 75 | rpm |
| Computed number of gear teeth: | | 240.0 | |
| *Enter:* Chosen No. of Gear Teeth: | $N_G$ = | 240 | |
| ***Computed data:*** | | | |
| Actual Output Speed: | $n_G$ = | 75.0 | rpm |
| Gear Ratio: | $m_G$ = | 12.00 | |
| Pitch Diameter - Pinion: | $D_P$ = | 1.667 | in |
| Pitch Diameter - Gear: | $D_G$ = | 20.000 | in |
| Center Distance: | $C$ = | 10.833 | in |
| Pitch Line Speed: | $v_t$ = | 393 | ft/min |
| Transmitted Load: | $W_t$ = | 210 | lb |
| ***Secondary Input Data:*** | | | |
| Transverse pressure angle: | $\phi_t$ = | 20.0 | deg |
| Helix angle: | $\psi$ = | 25.0 | deg |
| Axial Pitch: | $p_x$ = | 0.5614 | in |
| Min. Face Width (2 x Axial Pitch): | $F_{min}$ = | 1.123 | in |
| *Enter:* Face Width: | $F$ = | 1.750 | in |
| *Enter:* Elastic Coefficient: | $Cp$ = | 2100 | Table 9-10 |
| *Enter:* Quality Number: | $A_v$ = | 12 | Table 9-3 |
| REF: $N_P$, $N_G$ = | 20 | 240 | |
| *Enter:* Bending Geometry Factors: | | | |
| Pinion: | $J_P$ = | 0.451 | Fig 10-5,6,7 |
| Gear: | $J_G$ = | 0.512 | Fig 10-5,6,7 |
| *Enter:* Pitting Geometry Factor: | $I$ = | 0.260 | Tab. 10-1,2 |
| REF: $m_G$ = | 12.00 | | |
| Axial Force: | $W_x$ = | 98 | lb |
| Radial Force: | $W_r$ = | 76 | lb |

***Factors in Design Analysis:***

| | | | | | |
|---|---|---|---|---|---|
| Alignment Factor, $K_m$=1.0+$C_{pf}$+$C_{ma}$ | | If F<1.0 | If F>1.0 | $F/D_P$ = 1.05 | |
| Pinion Proportion Factor, $C_{pf}$ = | | 0.080 | 0.089 | [0.50 < $F/D_P$ < 2.00] | |
| *Enter:* $C_{pf}$ = | | 0.089 | Figure 9-16 | | |
| Type of gearing: | | Open | Commer. | Precision | Ex. Prec. |
| Mesh Alignment Factor, $C_{ma}$ = | | 0.276 | 0.154 | 0.090 | 0.056 |
| *Enter:* $C_{ma}$ = | | 0.276 | Figure 9-17 | | |
| Alignment Factor: $K_m$ = | | 1.37 | [Computed] | | |
| Overload Factor: $K_o$ = | | 2.00 | Table 9-7 | | |
| Size Factor: $K_s$ = | | 1.00 | Table 9-8: Use 1.00 if $P_d$ >= 5 | | |
| Pinion Rim Thickness Factor: $K_{BP}$ = | | 1.00 | Fig. 9-18: Use 1.00 if solid blank | | |
| Gear Rim Thickness Factor: $K_{BG}$ = | | 1.00 | Fig. 9-18: Use 1.00 if solid blank | | |
| Dynamic Factor: $K_v$ = | | 1.33 | [Computed: See Fig. 9-20] | | |
| Service Factor: $SF$ = | | 1.00 | Use 1.00 if no unusual conditions | | |
| Reliability Factor: $K_R$ = | | 1.00 | Table 9-11 | Use 1.00 for $R$ = .99 | |
| *Enter:* *Design Life:* | | 8000 | hours | See Table 9-12 | |
| Pinion - Number of load cycles: $N_P$ = | | 4.3E+08 | Guidelines: $Y_N$, $Z_N$ | | |
| Gear - Number of load cycles: $N_G$ = | | 3.6E+07 | $10^7$ cycles | >$10^7$ | <$10^7$ |
| Bending Stress Cycle Factor: $Y_{NP}$ = | | 0.95 | 1.00 | 0.95 | Fig. 9-22 |
| Bending Stress Cycle Factor: $Y_{NG}$ = | | 0.99 | 1.00 | 0.99 | Fig. 9-22 |
| Pitting Stress Cycle Factor: $Z_{NP}$ = | | 0.92 | 1.00 | 0.92 | Fig. 9-24 |
| Pitting Stress Cycle Factor: $Z_{NG}$ = | | 0.97 | 1.00 | 0.97 | Fig. 9-24 |

***Stress Analysis: Bending***

| | | | |
|---|---|---|---|
| Pinion: Required $s_{at}$ = | 12,180 | psi | See Fig. 9-11 or |
| Gear: Required $s_{at}$ = | 10,295 | psi | Table 9-5 |

***Stress Analysis: Pitting***

| | | | |
|---|---|---|---|
| Pinion: Required $s_{ac}$ = | 72,310 | psi | See Fig. 9-12 or |
| Gear: Required $s_{ac}$ = | 68,583 | psi | Table 9-5 |

***Specify materials, alloy and heat treatment, for most severe requirement.***

***One possible material specification:*** Steel pinion and cast iron gear

Pinion requires HB 134: SAE 1020 CD; HB 160 - Or almost any steel

Gear: Grade 40 gray cast iron; HB 201; $s_{at}$ = 13 ksi; $s_{ac}$ = 75 ksi (Table 9-6)

## DESIGN OF HELICAL GEARS-U.S.

*APPLICATION:* *Wood chipper driven by a gasoline engine*

*Problem 10-9* ***Speed increaser - cells changed***

### *Initial Input Data:*

| | | | |
|---|---|---|---|
| Input Power: | $P$ = | *75* | hp |
| Input Speed: | $n_G$ = | *2200* | rpm |
| Transverse Diametral Pitch, $P_d$: | $P_d$ = | *10* | |
| Number of Pinion Teeth: | $N_P$ = | *26* | |
| Desired Output Speed: | $n_P$ = | *4550* | rpm |
| Computed number of gear teeth: | | 53.8 | |
| *Enter:* Chosen No. of Gear Teeth: | $N_G$ = | *54* | |

### *Computed data:*

| | | | |
|---|---|---|---|
| Actual Output Speed: | $n_G$ = | 4569.2 | rpm |
| Gear Ratio: | $m_G$ = | 2.08 | |
| Pitch Diameter - Pinion: | $D_P$ = | 2.600 | in |
| Pitch Diameter - Gear: | $D_G$ = | 5.400 | in |
| Center Distance: | $C$ = | 4.000 | in |
| Pitch Line Speed: | $v_t$ = | 3097 | ft/min |
| Transmitted Load: | $W_t$ = | 799 | lb |

### *Secondary Input Data:*

| | | | |
|---|---|---|---|
| Transverse pressure angle: | $\phi_t$ = | *20.0* | deg |
| Helix angle: | $\psi$ = | *25.0* | deg |
| Axial Pitch: | $p_x$ = | 0.67372 | in |
| Min. Face Width (2 x Axial Pitch): | $F_{min}$ = | 1.347 | in |
| *Enter:* Face Width: | $F$ = | *1.800* | in |
| *Enter:* Elastic Coefficient: | $Cp$ = | *2300* | Table 9-10 |
| *Enter:* Quality Number: | $A_v$ = | *9* | Table 9-3 |
| REF: $N_P$, $N_G$ = | 26 | 54 | |
| *Enter:* Bending Geometry Factors: | | | |
| Pinion: | $J_P$ = | *0.453* | Fig 10-5,6,7 |
| Gear: | $J_G$ = | *0.463* | Fig 10-5,6,7 |
| *Enter:* Pitting Geometry Factor: | $I$ = | *0.188* | Tab. 10-1,2 |
| REF: $m_G$ = | 2.08 | | |
| Axial Force: | $W_x$ = | 373 | lb |
| Radial Force: | $W_r$ = | 291 | lb |

### *Factors in Design Analysis:*

| | If F<1.0 | If F>1.0 | | |
|---|---|---|---|---|
| Alignment Factor, $K_m$=1.0+$C_{pf}$+$C_{ma}$ | | | $F/D_P$ = 0.69 | |
| Pinion Proportion Factor, $C_{pf}$ = | 0.044 | 0.054 | [0.50 < $F/D_P$ < 2.00] | |
| *Enter:* $C_{pf}$ = | *0.054* | Figure 9-16 | | |
| Type of gearing: | Open | Commer. | Precision | Ex. Prec. |
| Mesh Alignment Factor, $C_{ma}$ = | 0.277 | 0.155 | 0.090 | 0.056 |
| *Enter:* $C_{ma}$ = | *0.277* | Figure 9-17 | | |
| Alignment Factor: $K_m$ = | 1.33 | [Computed] | | |

| | | |
|---|---|---|
| Overload Factor: $K_o$ = | *2.75* | Table 9-7 |
| Size Factor: $K_s$ = | *1.00* | Table 9-8: Use 1.00 if $P_d$ >= 5 |
| Pinion Rim Thickness Factor: $K_{BP}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank |
| Gear Rim Thickness Factor: $K_{BG}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank |
| Dynamic Factor: $K_v$ = | 1.44 | [Computed: See Fig. 9-20] |
| Service Factor: $SF$ = | *1.00* | Use 1.00 if no unusual conditions |
| Reliability Factor: $K_R$ = | *1.00* | Table 9-11 Use 1.00 for $R$ = .99 |
| *Enter: Design Life:* | *8000* | hours See Table 9-12 |

| | | Guidelines: $Y_N$, $Z_N$ | | |
|---|---|---|---|---|
| Pinion - Number of load cycles: $N_P$ = | 1.1E+09 | | | |
| Gear - Number of load cycles: $N_G$ = | 5.1E+08 | $10^7$ cycles | >$10^7$ | <$10^7$ |
| Bending Stress Cycle Factor: $Y_{NP}$ = | *0.94* | 1.00 | 0.94 | Fig. 9-22 |
| Bending Stress Cycle Factor: $Y_{NG}$ = | *0.95* | 1.00 | 0.95 | Fig. 9-22 |
| Pitting Stress Cycle Factor: $Z_{NP}$ = | *0.90* | 1.00 | 0.90 | Fig. 9-24 |
| Pitting Stress Cycle Factor: $Z_{NG}$ = | *0.91* | 1.00 | 0.91 | Fig. 9-24 |

***Stress Analysis: Bending***

| | | |
|---|---|---|
| Pinion: Required $s_{at}$ = | 55,024 psi | See Fig. 9-11 or |
| Gear: Required $s_{at}$ = | 53,268 psi | Table 9-5 |

***Stress Analysis: Pitting***

| | | |
|---|---|---|
| Pinion: Required $s_{ac}$ = | 176,932 psi | See Fig. 9-12 or |
| Gear: Required $s_{ac}$ = | 174,988 psi | Table 9-5 |

***Specify materials, alloy and heat treatment, for most severe requirement.***

***One possible material specification:*** Steel pinion+gear: Carburized-Case hard.

Pinion requires HRC 55: SAE 4118 DOQT 300; HRC 62

Gear requires HRC 55: SAE 4118 DOQT 300; HRC 62

| DESIGN OF HELICAL GEARS-U.S. | | APPLICATION: | | |
|---|---|---|---|---|
| | | *Small tractor driven by a gasoline engine* | | |
| | | *Problem 10-10* | | |
| ***Initial Input Data:*** | | | | |
| Input Power: | $P$ = | *20* | hp | |
| Input Speed: | $n_P$ = | *450* | rpm | |
| Transverse Diametral Pitch, $P_d$: | $P_d$ = | *6* | | |
| Number of Pinion Teeth: | $N_P$ = | *21* | | |
| Desired Output Speed: | $n_G$ = | *77.5* | rpm | |
| Computed number of gear teeth: | | 121.9 | | |
| *Enter:* Chosen No. of Gear Teeth: | $N_G$ = | *122* | | |
| ***Computed data:*** | | | | |
| Actual Output Speed: | $n_G$ = | 77.5 | rpm | |
| Gear Ratio: | $m_G$ = | 5.81 | | |
| Pitch Diameter - Pinion: | $D_P$ = | 3.500 | in | |
| Pitch Diameter - Gear: | $D_G$ = | 20.333 | in | |
| Center Distance: | $C$ = | 11.917 | in | |
| Pitch Line Speed: | $v_t$ = | 412 | ft/min | |
| Transmitted Load: | $W_t$ = | 1601 | lb | |
| ***Secondary Input Data:*** | | | | |
| Transverse pressure angle: | $\phi_t$ = | *20.0* | deg | |
| Helix angle: | $\psi$ = | *25.0* | deg | |
| Axial Pitch: | $p_x$ = | 1.1229 | in | |
| Min. Face Width (2 x Axial Pitch): | $F_{min}$ = | 2.246 | in | |
| *Enter:* Face Width: | $F$ = | *2.250* | in | |
| *Enter:* Elastic Coefficient: | $Cp$ = | *2300* | | Table 9-10 |
| *Enter:* Quality Number: | $A_v$ = | *11* | | Table 9-3 |
| REF: $N_P$, $N_G$ = | | 21 | 122 | |
| *Enter:* Bending Geometry Factors: | | | | |
| Pinion: | $J_P$ = | *0.444* | | Fig 10-5,6,7 |
| Gear: | $J_G$ = | *0.494* | | Fig 10-5,6,7 |
| *Enter:* Pitting Geometry Factor: | $I$ = | *0.240* | | Tab. 10-1,2 |
| REF: $m_G$ = | | 5.81 | | |
| Axial Force: | $W_x$ = | 746 | lb | |
| Radial Force: | $W_r$ = | 583 | lb | |

| | | | | |
|---|---|---|---|---|
| | *Use steel pinion with steel gear* | | | |
| ***Factors in Design Analysis:*** | | | | |
| Alignment Factor, $K_m$=1.0+$C_{pf}$+$C_{ma}$ | If F<1.0 | If F>1.0 | $F/D_P$ = 0.64 | |
| Pinion Proportion Factor, $C_{pf}$ = | 0.039 | 0.055 | [0.50 < $F/D_P$ < 2.00] | |
| *Enter:* $C_{pf}$ = | *0.055* | Figure 9-16 | | |
| Type of gearing: | Open | Commer. | Precision | Ex. Prec. |
| Mesh Alignment Factor, $C_{ma}$ = | 0.284 | 0.162 | 0.096 | 0.061 |
| *Enter:* $C_{ma}$ = | *0.162* | Figure 9-17 | | |
| Alignment Factor: $K_m$ = | 1.22 | [Computed] | | |
| Overload Factor: $K_o$ = | *2.75* | Table 9-7 | | |
| Size Factor: $K_s$ = | *1.00* | Table 9-8: Use 1.00 if $P_d$ >= 5 | | |
| Pinion Rim Thickness Factor: $K_{BP}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | |
| Gear Rim Thickness Factor: $K_{BG}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | |
| Dynamic Factor: $K_v$ = | 1.27 | [Computed: See Fig. 9-20] | | |
| Service Factor: $SF$ = | *1.00* | Use 1.00 if no unusual conditions | | |
| Reliability Factor: $K_R$ = | *1.00* | Table 9-11 | Use 1.00 for $R$ = .99 | |
| *Enter: Design Life:* | *8000* | hours | See Table 9-12 | |
| Pinion - Number of load cycles: $N_P$ = | 2.2E+08 | Guidelines: $Y_N$, $Z_N$ | | |
| Gear - Number of load cycles: $N_G$ = | 3.7E+07 | $10^7$ cycles | $>10^7$ | $<10^7$ |
| Bending Stress Cycle Factor: $Y_{NP}$ = | *0.96* | 1.00 | 0.96 | Fig. 9-22 |
| Bending Stress Cycle Factor: $Y_{NG}$ = | *0.99* | 1.00 | 0.99 | Fig. 9-22 |
| Pitting Stress Cycle Factor: $Z_{NP}$ = | *0.93* | 1.00 | 0.93 | Fig. 9-24 |
| Pitting Stress Cycle Factor: $Z_{NG}$ = | *0.97* | 1.00 | 0.97 | Fig. 9-24 |
| ***Stress Analysis: Bending*** | | | | |
| Pinion: Required $s_{at}$ = | 42,676 | psi | See Fig. 9-11 or | |
| Gear: Required $s_{at}$ = | 37,194 | psi | Table 9-5 | |
| ***Stress Analysis: Pitting*** | | | | |
| Pinion: Required $s_{ac}$ = | 148,576 | psi | See Fig. 9-12 or | |
| Gear: Required $s_{ac}$ = | 142,449 | psi | Table 9-5 | |

***Specify materials, alloy and heat treatment, for most severe requirement.***

***One possible material specification:*** Steel pinion and gear: Through hardened

Pinion requires HB 371: SAE 4340 OQT 900; HB 388

Gear requires HB 352: SAE 4340 OQT 1000; HB 363

**DESIGN OF HELICAL GEARS-U.S.** | *APPLICATION:* *Electric power generator driven by a water turbine*

*Problem 10-11* *Use steel pinion with steel gear*

| | | | |
|---|---|---|---|
| ***Initial Input Data:*** | | | |
| Input Power: | $P$ = | *15* | hp |
| Input Speed: | $n_P$ = | *4500* | rpm |
| Transverse Diametral Pitch, $P_d$: | $P_d$ = | *12* | |
| Number of Pinion Teeth: | $N_P$ = | *20* | |
| Desired Output Speed: | $n_G$ = | *3600* | rpm |
| Computed number of gear teeth: | | 25.0 | |
| *Enter:* Chosen No. of Gear Teeth: | $N_G$ = | *25* | |
| ***Computed data:*** | | | |
| Actual Output Speed: | $n_G$ = | 3600.0 | rpm |
| Gear Ratio: | $m_G$ = | 1.25 | |
| Pitch Diameter - Pinion: | $D_P$ = | 1.667 | in |
| Pitch Diameter - Gear: | $D_G$ = | 2.083 | in |
| Center Distance: | $C$ = | 1.875 | in |
| Pitch Line Speed: | $v_t$ = | 1963 | ft/min |
| Transmitted Load: | $W_t$ = | 252 | lb |
| ***Secondary Input Data:*** | | | |
| Transverse pressure angle: | $\phi_t$ = | *20.0* | deg |
| Helix angle: | $\psi$ = | *25.0* | deg |
| Axial Pitch: | $p_x$ = | 0.5614 | in |
| Min. Face Width (2 x Axial Pitch): | $F_{min}$ = | 1.123 | in |
| *Enter:* Face Width: | $F$ = | *1.250* | in |
| *Enter:* Elastic Coefficient: | $Cp$ = | *2300* | Table 9-10 |
| *Enter:* Quality Number: | $A_v$ = | *9* | Table 9-3 |
| REF: $N_P$, $N_G$ = | 20 | 25 | |
| *Enter:* Bending Geometry Factors: | | | |
| Pinion: | $J_P$ = | *0.418* | Fig 10-5,6,7 |
| Gear: | $J_G$ = | *0.432* | Fig 10-5,6,7 |
| *Enter:* Pitting Geometry Factor: | $I$ = | *0.150* | Tab. 10-1,2 |
| REF: $m_G$ = | 1.25 | | |
| Axial Force: | $W_x$ = | 118 | lb |
| Radial Force: | $W_r$ = | 92 | lb |

| | | | | |
|---|---|---|---|---|
| ***Factors in Design Analysis:*** | | | | |
| Alignment Factor, $K_m=1.0+C_{pf}+C_{ma}$ | If F<1.0 | If F>1.0 | $F/D_P$ = 0.75 | |
| Pinion Proportion Factor, $C_{pf}$ = | 0.050 | 0.053 | [0.50 < $F/D_P$ < 2.00] | |
| *Enter:* $C_{pf}$ = | *0.053* | Figure 9-16 | | |
| Type of gearing: | Open | Commer. | Precision | Ex. Prec. |
| Mesh Alignment Factor, $C_{ma}$ = | 0.268 | 0.147 | 0.083 | 0.051 |
| *Enter:* $C_{ma}$ = | *0.147* | Figure 9-17 | | |
| Alignment Factor: $K_m$ = | 1.20 | [Computed] | | |
| Overload Factor: $K_o$ = | *1.20* | Table 9-7 | | |
| Size Factor: $K_s$ = | *1.00* | Table 9-8: Use 1.00 if $P_d$ >= 5 | | |
| Pinion Rim Thickness Factor: $K_{BP}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | |
| Gear Rim Thickness Factor: $K_{BG}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | |
| Dynamic Factor: $K_v$ = | 1.36 | [Computed: See Fig. 9-20] | | |
| Service Factor: $SF$ = | *1.00* | Use 1.00 if no unusual conditions | | |
| Reliability Factor: $K_R$ = | *1.25* | Table 9-11 Use 1.00 for $R$ = .99 | | |
| *Enter:* Design Life: | *100000* | hours | See Table 9-12 | |
| Pinion - Number of load cycles: $N_P$ = | 2.7E+10 | Guidelines: $Y_N$, $Z_N$ | | |
| Gear - Number of load cycles: $N_G$ = | 2.2E+10 | $10^7$ cycles | >$10^7$ | <$10^7$ |
| Bending Stress Cycle Factor: $Y_{NP}$ = | *0.88* | 1.00 | 0.88 | Fig. 9-22 |
| Bending Stress Cycle Factor: $Y_{NG}$ = | *0.89* | 1.00 | 0.89 | Fig. 9-22 |
| Pitting Stress Cycle Factor: $Z_{NP}$ = | *0.83* | 1.00 | 0.83 | Fig. 9-24 |
| Pitting Stress Cycle Factor: $Z_{NG}$ = | *0.84* | 1.00 | 0.84 | Fig. 9-24 |
| ***Stress Analysis: Bending*** | | | | |
| Pinion: Required $s_{at}$ = | 16,093 | psi | See Fig. 9-11 or | |
| Gear: Required $s_{at}$ = | 15,397 | psi | Table 9-5 | |
| ***Stress Analysis: Pitting*** | | | | |
| Pinion: Required $s_{ac}$ = | 137,623 | psi | See Fig. 9-12 or | |
| Gear: Required $s_{ac}$ = | 135,985 | psi | Table 9-5 | |

***Specify materials, alloy and heat treatment, for most severe requirement.***

***One possible material specification:*** Steel pinion and gear: Through hardened

Pinion requires HB 337: SAE 4340 OQT 1000; HB 363

Gear requires HB 332: SAE 4340 OQT 1000; HB 363

**HELICAL GEARS** | APPLICATION: *Centrifugal pump driven by an electric motor*

**POWER TRANSMISSION CAPACITY** | *Chapter 10-Problem 12* | Used: $A_v$ =12; L = 15000 h

| | | | |
|---|---|---|---|
| ***Initial Input Data:*** | | | |
| *Enter:* Face Width: | $F$ = | *2.500* | in |
| Input Speed: | $n_P$ = | *1725* | rpm |
| Diametral Pitch: | $P_d$ = | *9.659* | |
| Number of Pinion Teeth: | $N_P$ = | *20* | |
| Number of Gear Teeth: | $N_G$ = | *75* | |
| ***Computed data:*** | | | |
| Actual Output Speed: | $n_G$ = | 460.0 | rpm |
| Gear Ratio: | $m_G$ = | 3.75 | |
| Pitch Diameter - Pinion: | $D_P$ = | 2.071 | in |
| Pitch Diameter - Gear: | $D_G$ = | 7.765 | in |
| Center Distance: | $C$ = | 4.918 | in |
| Pitch Line Speed: | $v_t$ = | 935 | ft/min |
| Transmitted Load at $P_{min}$ Capacity: | $W_t$ = | 797 | lb |

| | | |
|---|---|---|
| ***Power Transmission Capacity: (Using Eq. 9-32, 9-34)*** | | |
| Pinion: Based on Bending Stress: | 41.43 | hp |
| Gear: Based on Bending Stress: | 47.41 | hp |
| Pinion: Based on Contact Stress: | 22.59 | hp |
| Gear: Based on Contact Stress: | 24.14 | hp |
| ***Power Transmission Capacity:*** | ***22.59*** | ***hp*** |

| | | | |
|---|---|---|---|
| *Enter:* Elastic Coefficient: | $Cp$ = *2300* | | Table 9-10 |
| *Enter:* Quality Number: | $A_v$ = *12* | | Table 9-3 |
| REF: $N_P$, $N_G$ = | 20 | 75 | |
| *Enter:* Bending Geometry Factors: | Press. angle = 20 deg | | |
| Pinion: | $J_P$ = *0.465* | | Fig. 10-5, 6, 7 |
| Gear: | $J_G$ = *0.521* | | Fig. 10-5, 6, 7 |
| *Enter:* Pitting Geometry Factor: | $I$ = *0.200* | | Tables 10-1,2 |
| REF: $m_G$ = | 3.75 | | |

| | | | | | |
|---|---|---|---|---|---|
| ***Factors in Design Analysis:*** | | | | | |
| Alignment Factor, $K_m$=1.0+$C_{pf}$+$C_{ma}$ | | If $F$<1.0 | If $F$>1.0 | $F/D_P$ = 1.21 | |
| Pinion Proportion Factor, $C_{pf}$ = | | 0.096 | 0.114 | [0.50 < $F/D_P$ < 2.00] | |
| *Enter:* $C_{pf}$ = | | *0.114* | Figure 9-16 | | |
| Type of gearing: | | Open | Commer. | Precision | Ex. Prec. |
| Mesh Alignment Factor, $C_{ma}$ = | | 0.288 | 0.166 | 0.099 | 0.063 |
| *Enter:* $C_{ma}$ = | | *0.166* | Figure 9-17 | | |
| Alignment Factor: $K_m$ = | | 1.28 | [Computed] | | |
| Overload Factor: $K_o$ = | | *1.25* | Table 9-7 | | |
| Size Factor: $K_s$ = | | *1.00* | Table 9-8: Use 1.00 if $P_d$ >= 5 | | |
| Pinion Rim Thickness Factor: $K_{BP}$ = | | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | |
| Gear Rim Thickness Factor: $K_{BG}$ = | | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | |
| Dynamic Factor: $K_v$ = | | 1.50 | [Computed: See Fig. 9-20] | | |
| Service Factor: $SF$ = | | *1.00* | Use 1.00 if no unusual conditions | | |
| Reliability Factor: $K_R$ = | | *1.25* | Table 9-9 | Use 1.00 for $R$ = .99 | |
| *Enter: Design Life:* | | *15000* | hours | See Table 9-7 | |
| Pinion - Number of load cycles: $N_P$ = | | 1.6E+09 | Guidelines: $Y_N$, $Z_N$ | | |
| Gear - Number of load cycles: $N_G$ = | | 4.1E+08 | $10^7$ cycles | >$10^7$ | <$10^7$ |
| Bending Stress Cycle Factor: $Y_{NP}$ = | | *0.93* | 1.00 | 0.93 | Fig. 9-20 |
| Bending Stress Cycle Factor: $Y_{NG}$ = | | *0.95* | 1.00 | 0.95 | Fig. 9-20 |
| Pitting Stress Cycle Factor: $Z_{NP}$ = | | *0.89* | 1.00 | 0.89 | Fig. 9-22 |
| Pitting Stress Cycle Factor: $Z_{NG}$ = | | *0.92* | 1.00 | 0.92 | Fig. 9-22 |
| ***Allowable Bending Stress Numbers: (Input)*** | | | | | |
| Pinion: $s_{at}$ = | | *39,200* | psi | See Fig. 9-11 or | |
| Gear: $s_{at}$ = | | *39,200* | psi | Table 9-5 | |
| ***Allowable Contact Stress Numbers: (Input)*** | | | | | |
| Pinion: $s_{ac}$ = | | *138,900* | psi | See Fig. 9-12 or | |
| Gear: $s_{ac}$ = | | *138,900* | psi | Table 9-5 | |
| ***Material specification:*** | | *Steel pinion; Steel gear, Through HT* | | | |
| *Pinion material: SAE 4140 OQT 1000* | | | *341 HB* | | |
| *Gear material: SAE 4140 OQT 1000* | | | *341 HB* | | |

| For $K_v$: | |
|---|---|
| $B$ | 0.915 |
| $C$ | 54.74 |

| Through-Hardened Grade 1 Steel | |
|---|---|
| 39.2 ksi | Fig. 9-11 |
| 39.2 ksi | Fig. 9-11 |
| 138.9 ksi | Fig. 9-12 |
| 138.9 ksi | Fig. 9-12 |

**HELICAL GEARS**
**POWER TRANSMISSION CAPACITY**

APPLICATION: *Centrifugal pump driven by an electric motor*

*Chapter 10-Problem 13* — Used: $A_v$=12; L = 15000 h

| | | | |
|---|---|---|---|
| ***Initial Input Data:*** | | | |
| *Enter:* Face Width: | $F$ = | *2.500* | in |
| Input Speed: | $n_P$ = | *1725* | rpm |
| Diametral Pitch: | $P_d$ = | *9.659* | |
| Number of Pinion Teeth: | $N_P$ = | *20* | |
| Number of Gear Teeth: | $N_G$ = | *75* | |
| ***Computed data:*** | | | |
| Actual Output Speed: | $n_G$ = | 460.0 | rpm |
| Gear Ratio: | $m_G$ = | 3.75 | |
| Pitch Diameter - Pinion: | $D_P$ = | 2.071 | in |
| Pitch Diameter - Gear: | $D_G$ = | 7.765 | in |
| Center Distance: | $C$ = | 4.918 | in |
| Pitch Line Speed: | $v_t$ = | 935 | ft/min |
| Transmitted Load at $P_{min}$ Capacity: | $W_t$ = | 1338 | lb |

***Power Transmission Capacity: (Using Eq. 9-32, 9-34)***

| | | |
|---|---|---|
| Pinion: Based on Bending Stress: | 58.12 | hp |
| Gear: Based on Bending Stress: | 66.52 | hp |
| Pinion: Based on Contact Stress: | 37.94 | hp |
| Gear: Based on Contact Stress: | 40.54 | hp |
| ***Power Transmission Capacity:*** | ***37.94*** | ***hp*** |

| | | | |
|---|---|---|---|
| *Enter:* Elastic Coefficient: | $Cp$ = *2300* | | Table 9-10 |
| *Enter:* Quality Number: | $A_v$ = *12* | | Table 9-3 |
| REF: $N_P$, $N_G$ = | 20 | 75 | |
| *Enter:* Bending Geometry Factors: | Press. angle = 20 deg | | |
| Pinion: | $J_P$ = *0.465* | | Fig. 10-5, 6, 7 |
| Gear: | $J_G$ = *0.521* | | Fig. 10-5, 6, 7 |
| *Enter:* Pitting Geometry Factor: | $I$ = *0.200* | | Tables 10-1,2 |
| REF: $m_G$ = | 3.75 | | |

***Factors in Design Analysis:***

| | | | | |
|---|---|---|---|---|
| Alignment Factor, $K_m$=1.0+$C_{pf}$+$C_{ma}$ | If $F$<1.0 | If $F$>1.0 | $F/D_P$ = 1.21 | |
| Pinion Proportion Factor, $C_{pf}$ = | 0.096 | 0.114 | [0.50 < $F/D_P$ < 2.00] | |
| *Enter:* $C_{pf}$ = | *0.114* | Figure 9-16 | | |
| Type of gearing: | Open | Commer. | Precision | Ex. Prec. |
| Mesh Alignment Factor, $C_{ma}$ = | 0.288 | 0.166 | 0.099 | 0.063 |
| *Enter:* $C_{ma}$ = | *0.166* | Figure 9-17 | | |
| Alignment Factor: $K_m$ = | 1.28 | [Computed] | | |
| Overload Factor: $K_o$ = | *1.25* | Table 9-7 | | |
| Size Factor: $K_s$ = | *1.00* | Table 9-8: Use 1.00 if $P_d$ >= 5 | | |
| Pinion Rim Thickness Factor: $K_{BP}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | |
| Gear Rim Thickness Factor: $K_{BG}$ = | *1.00* | Fig. 9-18: Use 1.00 if solid blank | | |
| Dynamic Factor: $K_v$ = | 1.50 | [Computed: See Fig. 9-20] | | |
| Service Factor: $SF$ = | *1.00* | Use 1.00 if no unusual conditions | | |
| Reliability Factor: $K_R$ = | *1.25* | Table 9-9 | Use 1.00 for $R$ = .99 | |
| *Enter: Design Life:* | *15000* | hours | See Table 9-7 | |

| For $K_v$: | |
|---|---|
| $B$ | 0.915 |
| $C$ | 54.74 |

| | | Guidelines: $Y_N$, $Z_N$ | | |
|---|---|---|---|---|
| | | $10^7$ cycles | $>10^7$ | $<10^7$ |
| Pinion - Number of load cycles: $N_P$ = | 1.6E+09 | | | |
| Gear - Number of load cycles: $N_G$ = | 4.1E+08 | | | |
| Bending Stress Cycle Factor: $Y_{NP}$ = | *0.93* | 1.00 | 0.93 | Fig. 9-20 |
| Bending Stress Cycle Factor: $Y_{NG}$ = | *0.95* | 1.00 | 0.95 | Fig. 9-20 |
| Pitting Stress Cycle Factor: $Z_{NP}$ = | *0.89* | 1.00 | 0.89 | Fig. 9-22 |
| Pitting Stress Cycle Factor: $Z_{NG}$ = | *0.92* | 1.00 | 0.92 | Fig. 9-22 |

| | | | | Through-Hardened Grade 1 Steel | |
|---|---|---|---|---|---|
| ***Allowable Bending Stress Numbers: (Input)*** | | | | | |
| Pinion: $s_{at}$ = | *55,000* | psi | See Fig. 9-11 or | 17.6 ksi | Fig. 9-11 |
| Gear: $s_{at}$ = | *55,000* | psi | Table 9-5 | 17.6 ksi | Fig. 9-11 |
| ***Allowable Contact Stress Numbers: (Input)*** | | | | | |
| Pinion: $s_{ac}$ = | *180,000* | psi | See Fig. 9-12 or | 49.1 ksi | Fig. 9-12 |
| Gear: $s_{ac}$ = | *180,000* | psi | Table 9-5 | 49.1 ksi | Fig. 9-12 |

***Material specification:*** *Steel pinion+gear, Carburized case hard.*

*Pinion material: SAE 4620 DOQT 300* — *62 HRC*
*Gear material: SAE 4620 DOQT 300* — *62 HRC*

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*Problem 10-14*

**BEVEL GEARS**

**Forces and torque for shaft and bearing load analysis:**

| | | | |
|---|---|---|---|
| **On Pinion Shaft - Torque:** | $T_P$ = | 630 | lb-in |
| Mean radius of pinion: | $r_m$ = | 1.052 | in |
| *Enter:* pressure angle: | $\phi$ = | 20 | degrees |
| On Pinion - Tangential load: | $W_{tP}$ = | 598.7 | lb |
| On Pinion - Radial load: | $W_{rP}$ = | 206.7 | lb |
| On Pinion - Axial load: | $W_{xP}$ = | 68.9 | lb |
| **On Gear Shaft - Torque:** | $T_G$ = | 1890.0 | lb-in |
| On Gear - Tangential load: | $W_{tG}$ = | 598.7 | lb |
| On Gear - Radial load: | $W_{rG}$ = | 68.9 | lb |
| On Gear - Axial load: | $W_{xG}$ = | 206.7 | lb |

**See following page for stress analysis and design details.**

| DESIGN OF BEVEL GEARS | APPLICATION: | Concrete mixer with moderate shock driven by a gasoline engine |
|---|---|---|
| | | Problem 10-14 — Neither gear straddle mounted |

**Initial Input Data:**

| | | | |
|---|---|---|---|
| Input Power: | $P$ = | 3 | hp |
| Input Speed: | $n_P$ = | 300 | rpm |
| Diametral Pitch: | $P_d$ = | 6 | |
| Number of Pinion Teeth: | $N_P$ = | 15 | |
| Desired Output Speed: | $n_G$ = | 100 | rpm |
| Computed number of gear teeth: | | 45.0 | |
| *Enter:* Chosen No. of Gear Teeth: | $N_G$ = | 45 | |

**Computed data:**

| | | | |
|---|---|---|---|
| Actual Output Speed: | $n_G$ = | 100.0 | rpm |
| Gear Ratio: | $m_G$ = | 3.00 | |
| Pitch Diameter - Pinion: | $D_P$ = | 2.500 | in |
| Pitch Diameter - Gear: | $D_G$ = | 7.500 | in |
| Pitch cone angle - Pinion: | $\gamma$ = | 18.43 | degrees |
| Pitch cone angle - Gear: | $\Gamma$ = | 71.57 | degrees |
| Outer cone distance: | $A_o$ = | 3.9528 | in |
| Pitch Line Speed: | $v_t$ = | 196 | ft/min |
| Transmitted Load: | $W_t$ = | 504 | lb |

**Secondary Input Data:**

| | Nom | Max | Max |
|---|---|---|---|
| Face Width Guidelines (in): | 1.186 | 1.318 | 1.667 |

| | | |
|---|---|---|
| *Enter:* Face Width: | $F$ = 1.250 | in |
| *Enter:* Elastic Coefficient: | $C_p$ = 2300 | Table 9-10 |
| *Enter:* Quality Number: | $A_v$ = 9 | Table 9-3 |
| *Enter:* Bending Geometry Factors: | | |
| Pinion: | $J_P$ = 0.228 | Fig. 10-15 |
| Gear: | $J_G$ = 0.190 | Fig. 10-15 |
| *Enter:* Pitting Geometry Factor: | $I$ = 0.078 | Fig. 10-19 |

**Factors in Design Analysis:**

| | | | | |
|---|---|---|---|---|
| Load distribution factor, $K_m$: | From Equation 10-16 | | | |
| Select from: | Factor, $K_{mb}$ | | | |
| Both gears straddle mounted: | 1.00 | | | |
| One gear straddle mounted: | 1.10 | | | |
| Neither gear straddle mounted: | 1.25 | | | |
| *Enter Factor,* $K_{mb}$ | 1.25 | | | |
| $K_m$ = | 1.26 | | | |
| Overload Factor: $K_o$ = | 2.00 | Table 9-7 | | |
| *Bending* Size Factor: $K_s$ = | 0.52 | Figure 10-13 (for $P_d$ < 16) | | |
| Dynamic Factor: $K_v$ = | 1.121 | Computed: Table 9-9 | | |
| *Pitting* Size Factor: $C_s$ = | 0.59 | For 0.50 < $F$ < 3.14 | | |
| For F < 0.50, $C_s$ = 0.5 | | For F > 3.14, $C_s$ = 0.83 | | |
| Enter $C_s$ = | 0.59 | | | |
| Service Factor: $SF$ = | 1.00 | Use 1.00 if no unusual conditions | | |
| Bending Reliability Factor: $K_R$ = | 1.00 | Pitting: $C_R$ = 1.00 | | |
| *Enter: Design Life:* | 1000 | hours | See Table 9-7 | |
| Pinion - Number of load cycles: $N_P$ = | 1.80E+07 | $Y_N$, $Z_N$: Pinion-Fig. 10-18; Gear-Fig. 10-20 | | |
| Gear - Number of load cycles: $N_G$ = | 6.00E+06 | $N<10^3$ | $10^3<N<10^7$ | $N>10^7$ |
| Bending Stress Cycle Factor: $K_L$ = | 1.01 | 2.70 | 0.84 | 1.01 |
| Bending Stress Cycle Factor: $K_L$ = | 0.96 | 2.70 | 0.96 | 1.03 |
| | | $N<10^4$ | $N>10^4$ | |
| Pitting Stress Cycle Factor: $C_L$ = | 1.27 | 2.00 | 1.27 | |
| Pitting Stress Cycle Factor: $C_L$ = | 1.36 | 2.00 | 1.36 | |

| For $K_v$ | |
|---|---|
| $B$ = | 0.630 |
| $C$ = | 70.7 |

| For $R$ = | $K_R$ | $C_R$ |
|---|---|---|
| 0.9 | 0.85 | 0.62 |
| 0.99 | 1.00 | 1.00 |
| 0.999 | 1.25 | 1.12 |
| 0.9999 | 1.50 | 1.22 |

| | | | | Through Hardened Grade 1 Steel | |
|---|---|---|---|---|---|
| ***Stress Analysis - Bending:*** | | | | | |
| Pinion: Required $s_{at}$ = | 15,447 | psi | See Fig. 10-17 or | HB 303 | Fig. 10-17 |
| Gear: Required $s_{at}$ = | 19,502 | psi | Table 10-4 | HB 395 | Fig. 10-17 |
| ***Stress Analysis - Pitting:*** | Assumes $C_{xc}$ = 1.5 for properly crowned teeth | | | | |
| Pinion: Required $s_{ac}$ = | 129,992 | psi | See Fig. 10-21 or | HB 312 | Fig. 10-21 |
| Gear: Required $s_{ac}$ = | 121,389 | psi | Table 10-4 | HB 287 | Fig. 10-21 |

***Specify materials, alloy and heat treatment, for most severe requirement.***

***One possible material specification:***

Pinion: HB 312 required: SAE 6150 OQT 1100; HB = 341

Gear: HB 395 required: SAE 6150 OQT 900; HB = 401

*Problem 10-15*

| | | | |
|---|---|---|---|
| **BEVEL GEARS** | | | |
| **Forces and torque for shaft and bearing load analysis:** | | | |
| **On Pinion Shaft - Torque:** | $T_P$ = | 176.4 | lb-in |
| Mean radius of pinion: | $r_m$ = | 1.093 | in |
| *Enter:* pressure angle: | $\phi$ = | 20 | degrees |
| On Pinion - Tangential load: | $W_{tP}$ = | 161.3 | lb |
| On Pinion - Radial load: | $W_{rP}$ = | 52.5 | lb |
| On Pinion - Axial load: | $W_{xP}$ = | 26.3 | lb |
| **On Gear Shaft - Torque:** | $T_G$ = | 352.8 | lb-in |
| On Gear - Tangential load: | $W_{tG}$ = | 161.3 | lb |
| On Gear - Radial load: | $W_{rG}$ = | 26.3 | lb |
| On Gear - Axial load: | $W_{xG}$ = | 52.5 | lb |

**See following page for stress analysis and design details.**

| DESIGN OF BEVEL GEARS | APPLICATION: | Conveyor with moderate shock driven by a gasoline engine |
|---|---|---|
| | | Problem 10-15 — Neither gear straddle mounted |

**Initial Input Data:**

| | | | |
|---|---|---|---|
| Input Power: | $P$ = | 3.5 | hp |
| Input Speed: | $n_P$ = | 1250 | rpm |
| Diametral Pitch: | $P_d$ = | 10 | |
| Number of Pinion Teeth: | $N_P$ = | 25 | |
| Desired Output Speed: | $n_G$ = | 625 | rpm |
| Computed number of gear teeth: | | 50.0 | |
| *Enter:* Chosen No. of Gear Teeth: | $N_G$ = | 50 | |

**Computed data:**

| | | | |
|---|---|---|---|
| Actual Output Speed: | $n_G$ = | 625.0 | rpm |
| Gear Ratio: | $m_G$ = | 2.00 | |
| Pitch Diameter - Pinion: | $D_P$ = | 2.500 | in |
| Pitch Diameter - Gear: | $D_G$ = | 5.000 | in |
| Pitch cone angle - Pinion: | $\gamma$ = | 26.57 | degrees |
| Pitch cone angle - Gear: | $\Gamma$ = | 63.43 | degrees |
| Outer cone distance: | $A_o$ = | 2.7951 | in |
| Pitch Line Speed: | $v_t$ = | 818 | ft/min |
| Transmitted Load: | $W_t$ = | 141 | lb |

**Secondary Input Data:**

| | Nom | Max | Max |
|---|---|---|---|
| Face Width Guidelines (in): | 0.839 | 0.932 | 1.000 |
| *Enter:* Face Width: | $F$ = 0.700 | in | |
| *Enter:* Elastic Coefficient: | $C_p$ = 2300 | Table 9-10 | |
| *Enter:* Quality Number: | $A_v$ = 10 | Table 9-3 | |

| *Enter:* Bending Geometry Factors: | | |
|---|---|---|
| Pinion: | $J_P$ = 0.258 | Fig. 10-15 |
| Gear: | $J_G$ = 0.220 | Fig. 10-15 |
| *Enter:* Pitting Geometry Factor: | $I$ = 0.083 | Fig. 10-19 |

**Factors In Design Analysis:**

| | | |
|---|---|---|
| Load distribution factor, $K_m$: | From Equation 10-16 | |
| Select from: | Factor, $K_{mb}$ | |
| Both gears straddle mounted: | 1.00 | |
| One gear straddle mounted: | 1.10 | |
| Neither gear straddle mounted: | 1.25 | |
| *Enter Factor,* $K_{mb}$ | 1.25 | |
| $K_m$ = | 1.25 | |
| Overload Factor: $K_o$ = | 2.00 | Table 9-7 |
| *Bending* Size Factor: $K_s$ = | 0.51 | Figure 10-13 (for $P_d$ < 16) |
| Dynamic Factor: $K_v$ = | 1.305 | Computed: Table 9-9 |
| *Pitting* Size Factor: $C_s$ = | 0.53 | For 0.50 < $F$ < 3.14 |
| For F < 0.50, $C_s$ = 0.5 | | For F > 3.14, $C_s$ = 0.83 |
| Enter $C_s$ = | 0.53 | |
| Service Factor: $SF$ = | 1.00 | Use 1.00 if no unusual conditions |
| Bending Reliability Factor: $K_R$ = | 1.00 | Pitting: $C_R$ = 1.00 |
| *Enter: Design Life:* | 15000 | hours See Table 9-7 |

| | | $Y_N$, $Z_N$: Pinion-Fig. 10-18; Gear-Fig. 10-20 | | |
|---|---|---|---|---|
| Pinion - Number of load cycles: $N_P$ = | 1.13E+09 | | | |
| Gear - Number of load cycles: $N_G$ = | 5.63E+08 | $N<10^3$ | $10^3<N<10^7$ | $N>10^7$ |
| Bending Stress Cycle Factor: $K_L$ = | 0.94 | 2.70 | 0.51 | 0.94 |
| Bending Stress Cycle Factor: $K_L$ = | 0.95 | 2.70 | 0.56 | 0.95 |
| | | $N<10^4$ | $N>10^4$ | |
| Pitting Stress Cycle Factor: $C_L$ = | 1.27 | 2.00 | 0.99 | |
| Pitting Stress Cycle Factor: $C_L$ = | 1.36 | 2.00 | 1.04 | |

| For $K_v$ |
|---|
| $B$ = 0.731 |
| $C$ = 65.0 |

| For $R$ = | $K_R$ | $C_R$ |
|---|---|---|
| 0.9 | 0.85 | 0.62 |
| 0.99 | 1.00 | 1.00 |
| 0.999 | 1.25 | 1.12 |
| 0.9999 | 1.50 | 1.22 |

**Stress Analysis - Bending:**

| | | | Through Hardened Grade 1 Steel | |
|---|---|---|---|---|
| Pinion: Required $s_{at}$ = | 13,808 psi | See Fig. 10-17 or | HB 266 | Fig. 10-17 |
| Gear: Required $s_{at}$ = | 16,023 psi | Table 10-4 | HB 316 | Fig. 10-17 |

**Stress Analysis - Pitting:** Assumes $C_{xc}$ = 1.5 for properly crowned teeth

| | | | | |
|---|---|---|---|---|
| Pinion: Required $s_{ac}$ = | 91,011 psi | See Fig. 10-21 or | HB 198 | Fig. 10-21 |
| Gear: Required $s_{ac}$ = | 84,988 psi | Table 10-4 | HB 180 | Fig. 10-21 |

***Specify materials, alloy and heat treatment, for most severe requirement.***

***One possible material specification:***

Pinion: HB 266 required: SAE 6150 OQT 1200; HB = 293

Gear: HB 316 required: SAE 6150 OQT 1100; HB = 341

Problem 10-16

**BEVEL GEARS**

**Forces and torque for shaft and bearing load analysis:**

| | | | |
|---|---|---|---|
| **On Pinion Shaft - Torque:** | $T_P$ = | 370.59 | lb-in |
| Mean radius of pinion: | $r_m$ = | 0.938 | in |
| *Enter:* pressure angle: | $\phi$ = | 20 | degrees |
| On Pinion - Tangential load: | $W_{tP}$ = | 395.2 | lb |
| On Pinion - Radial load: | $W_{rP}$ = | 135.6 | lb |
| On Pinion - Axial load: | $W_{xP}$ = | 47.9 | lb |
| **On Gear Shaft - Torque:** | $T_G$ = | 1050.0 | lb-in |
| On Gear - Tangential load: | $W_{tG}$ = | 395.2 | lb |
| On Gear - Radial load: | $W_{rG}$ = | 47.9 | lb |
| On Gear - Axial load: | $W_{xG}$ = | 135.6 | lb |

**See following page for stress analysis and design details.**

| DESIGN OF BEVEL GEARS | APPLICATION: | Conveyor with heavy shock driven by a gasoline engine |
|---|---|---|
| | | Problem 10-16 — Both gears straddle mounted |

**Initial Input Data:**

| | | | |
|---|---|---|---|
| Input Power: | $P =$ | 5 | hp |
| Input Speed: | $n_P =$ | 850 | rpm |
| Diametral Pitch: | $P_d =$ | 8 | |
| Number of Pinion Teeth: | $N_P =$ | 18 | |
| Desired Output Speed: | $n_G =$ | 300 | rpm |
| Computed number of gear teeth: | | 51.0 | |
| *Enter:* Chosen No. of Gear Teeth: | $N_G =$ | 51 | |

**Computed data:**

| | | | |
|---|---|---|---|
| Actual Output Speed: | $n_G =$ | 300.0 | rpm |
| Gear Ratio: | $m_G =$ | 2.83 | |
| Pitch Diameter - Pinion: | $D_P =$ | 2.250 | in |
| Pitch Diameter - Gear: | $D_G =$ | 6.375 | in |
| Pitch cone angle - Pinion: | $\gamma =$ | 19.44 | degrees |
| Pitch cone angle - Gear: | $\Gamma =$ | 70.56 | degrees |
| Outer cone distance: | $A_o =$ | 3.3802 | in |
| Pitch Line Speed: | $v_t =$ | 501 | ft/min |
| Transmitted Load: | $W_t =$ | 330 | lb |

**Secondary Input Data:**

| | Nom | Max | Max |
|---|---|---|---|
| Face Width Guidelines (in): | 1.014 | 1.127 | 1.250 |

| | | | |
|---|---|---|---|
| *Enter:* Face Width: | $F =$ | 1.125 | in |
| *Enter:* Elastic Coefficient: | $C_p =$ | 2300 | Table 9-10 |
| *Enter:* Quality Number: | $A_v =$ | 10 | Table 9-3 |
| *Enter:* Bending Geometry Factors: | | | |
| Pinion: | $J_P =$ | 0.240 | Fig. 10-15 |
| Gear: | $J_G =$ | 0.200 | Fig. 10-15 |
| *Enter:* Pitting Geometry Factor: | $I =$ | 0.081 | Fig. 10-19 |

**Factors In Design Analysis:**

| | | |
|---|---|---|
| Load distribution factor, $K_m$: | From Equation 10-16 | |
| Select from: | Factor, $K_{mb}$ | |
| Both gears straddle mounted: | 1.00 | |
| One gear straddle mounted: | 1.10 | |
| Neither gear straddle mounted: | 1.25 | |
| *Enter Factor,* $K_{mb}$ | 1.00 | |
| $K_m =$ | 1.00 | |
| Overload Factor: $K_o =$ | 2.00 | Table 9-7 |
| *Bending* Size Factor: $K_s =$ | 0.51 | Figure 10-13 (for $P_d < 16$) |
| Dynamic Factor: $K_v =$ | 1.241 | Computed: Table 9-9 |
| *Pitting* Size Factor: $C_s =$ | 0.58 | For $0.50 < F < 3.14$ |
| For $F < 0.50$, $C_s = 0.5$ | | For $F > 3.14$, $C_s = 0.83$ |
| Enter $C_s =$ | 0.58 | |
| Service Factor: $SF =$ | 1.00 | Use 1.00 if no unusual conditions |
| Bending Reliability Factor: $K_R =$ | 1.00 | Pitting: $C_R =$ 1.00 |
| *Enter:* Design Life: | 15000 | hours — See Table 9-7 |

| For $K_v$ |
|---|
| $B =$ 0.731 |
| $C =$ 65.0 |

| For $R =$ | $K_R$ | $C_R$ |
|---|---|---|
| 0.9 | 0.85 | 0.62 |
| 0.99 | 1.00 | 1.00 |
| 0.999 | 1.25 | 1.12 |
| 0.9999 | 1.50 | 1.22 |

| | | $Y_N$, $Z_N$: Pinion-Fig. 10-18; Gear-Fig. 10-20 | | |
|---|---|---|---|---|
| Pinion - Number of load cycles: $N_P =$ | 7.65E+08 | | | |
| Gear - Number of load cycles: $N_G =$ | 2.70E+08 | $N<10^3$ | $10^3<N<10^7$ | $N>10^7$ |
| Bending Stress Cycle Factor: $K_L =$ | 0.94 | 2.70 | 0.54 | 0.94 |
| Bending Stress Cycle Factor: $K_L =$ | 0.96 | 2.70 | 0.61 | 0.96 |
| | | $N<10^4$ | $N>10^4$ | |
| Pitting Stress Cycle Factor: $C_L =$ | 1.02 | 2.00 | 1.02 | |
| Pitting Stress Cycle Factor: $C_L =$ | 1.08 | 2.00 | 1.08 | |

**Stress Analysis - Bending:**

| | | | Through Hardened Grade 1 Steel | |
|---|---|---|---|---|
| Pinion: Required $s_{at} =$ | 13,300 psi | See Fig. 10-17 or | HB 255 | Fig. 10-17 |
| Gear: Required $s_{at} =$ | 15,628 psi | Table 10-4 | HB 307 | Fig. 10-17 |

**Stress Analysis - Pitting:** Assumes $C_{xc} = 1.5$ for properly crowned teeth

| | | | | |
|---|---|---|---|---|
| Pinion: Required $s_{ac} =$ | 133,166 psi | See Fig. 10-21 or | HB 321 | Fig. 10-21 |
| Gear: Required $s_{ac} =$ | 125,768 psi | Table 10-4 | HB 300 | Fig. 10-21 |

***Specify materials, alloy and heat treatment, for most severe requirement.***

***One possible material specification:***

Pinion: HB 321 required: SAE 6150 OQT 1100; HB = 341

Gear: HB 307 required: SAE 6150 OQT 1100; HB = 341

*Problem 10-17*

**BEVEL GEARS**

**Forces and torque for shaft and bearing load analysis:**

| | | | |
|---|---|---|---|
| **On Pinion Shaft - Torque:** | $T_P$ = | 26.25 | lb-in |
| Mean radius of pinion: | $r_m$ = | 0.386 | in |
| *Enter:* pressure angle: | $\phi$ = | 20 | degrees |
| On Pinion - Tangential load: | $W_{tP}$ = | 68.0 | lb |
| On Pinion - Radial load: | $W_{rP}$ = | 23.9 | lb |
| On Pinion - Axial load: | $W_{xP}$ = | 6.3 | lb |
| **On Gear Shaft - Torque:** | $T_G$ = | 99.2 | lb-in |
| On Gear - Tangential load: | $W_{tG}$ = | 68.0 | lb |
| On Gear - Radial load: | $W_{rG}$ = | 6.3 | lb |
| On Gear - Axial load: | $W_{xG}$ = | 23.9 | lb |

**See following page for stress analysis and design details.**

**DESIGN OF BEVEL GEARS** — APPLICATION: *Reciprocating saw driven by an electric motor*

*Problem 10-17* — *Both gears straddle mounted*

| | | | |
|---|---|---|---|
| ***Initial Input Data:*** | | | |
| Input Power: | $P$ = | *0.75* | hp |
| Input Speed: | $n_P$ = | *1800* | rpm |
| Diametral Pitch: | $P_d$ = | *20* | |
| Number of Pinion Teeth: | $N_P$ = | *18* | |
| Desired Output Speed: | $n_G$ = | *475* | rpm |
| Computed number of gear teeth: | | 68.2 | |
| *Enter:* Chosen No. of Gear Teeth: | $N_G$ = | *68* | |
| ***Computed data:*** | | | |
| Actual Output Speed: | $n_G$ = | 476.5 | rpm |
| Gear Ratio: | $m_G$ = | 3.78 | |
| Pitch Diameter - Pinion: | $D_P$ = | 0.900 | in |
| Pitch Diameter - Gear: | $D_G$ = | 3.400 | in |
| Pitch cone angle - Pinion: | $\gamma$ = | 14.83 | degrees |
| Pitch cone angle - Gear: | $\Gamma$ = | 75.17 | degrees |
| Outer cone distance: | $A_o$ = | 1.7586 | in |
| Pitch Line Speed: | $v_t$ = | 424 | ft/min |
| Transmitted Load: | $W_t$ = | 58 | lb |

***Secondary Input Data:***

| | Nom | Max | Max |
|---|---|---|---|
| Face Width Guidelines (in): | 0.528 | 0.586 | 0.500 |

| | | | |
|---|---|---|---|
| *Enter:* Face Width: | $F$ = | *0.500* | in |
| *Enter:* Elastic Coefficient: | $C_p$ = | *2300* | Table 9-10 |
| *Enter:* Quality Number: | $A_v$ = | *11* | Table 9-3 |
| *Enter:* Bending Geometry Factors: | | | |
| Pinion: | $J_P$ = | *0.250* | Fig. 10-15 |
| Gear: | $J_G$ = | *0.210* | Fig. 10-15 |
| *Enter:* Pitting Geometry Factor: | $I$ = | *0.085* | Fig. 10-19 |

***Factors in Design Analysis:***

Load distribution factor, $K_m$: From Equation 10-16

| Select from: | Factor, $K_{mb}$ |
|---|---|
| Both gears straddle mounted: | 1.00 |
| One gear straddle mounted: | 1.10 |
| Neither gear straddle mounted: | 1.25 |
| *Enter Factor,* $K_{mb}$ | *1.00* |
| $K_m$ = | 1.00 |

| | | |
|---|---|---|
| Overload Factor: $K_o$ = | *1.75* | Table 9-7 |
| *Bending* Size Factor: $K_s$ = | *0.50* | Figure 10-13 (for $P_d$ < 16) |
| Dynamic Factor: $K_v$ = | 1.277 | Computed: Table 9-9 |
| *Pitting* Size Factor: $C_s$ = | 0.50 | For 0.50 < $F$ < 3.14 |
| For F < 0.50, $C_s$ = *0.5* | | For F > 3.14, $C_s$ = *0.83* |
| Enter $C_s$ = | 0.5 | |
| Service Factor: $SF$ = | *1.00* | Use 1.00 if no unusual conditions |
| Bending Reliability Factor: $K_R$ = | *1.00* | Pitting: $C_R$ = 1.00 |
| *Enter: Design Life:* | *15000* | hours See Table 9-7 |

| For $K_v$ |
|---|
| $B$ = 0.826 |
| $C$ = 59.7 |

| | | $Y_N$, $Z_N$: Pinion-Fig. 10-18; Gear-Fig. 10-20 | | |
|---|---|---|---|---|
| | | $N<10^3$ | $10^3<N<10^7$ | $N>10^7$ |
| Pinion - Number of load cycles: $N_P$ = | 1.62E+09 | | | |
| Gear - Number of load cycles: $N_G$ = | 4.29E+08 | | | |
| Bending Stress Cycle Factor: $K_L$ = | *0.93* | 2.70 | 0.49 | 0.93 |
| Bending Stress Cycle Factor: $K_L$ = | *0.95* | 2.70 | 0.58 | 0.95 |
| | | $N<10^4$ | $N>10^4$ | |
| Pitting Stress Cycle Factor: $C_L$ = | *0.97* | 2.00 | 0.97 | |
| Pitting Stress Cycle Factor: $C_L$ = | *1.05* | 2.00 | 1.05 | |

| For $R$ = | $K_R$ | $C_R$ |
|---|---|---|
| 0.9 | 0.85 | 0.62 |
| 0.99 | 1.00 | 1.00 |
| 0.999 | 1.25 | 1.12 |
| 0.9999 | 1.50 | 1.22 |

| | | | |
|---|---|---|---|
| ***Stress Analysis - Bending:*** | | | |
| Pinion: Required $s_{at}$ = | 11,171 | psi | See Fig. 10-17 or |
| Gear: Required $s_{at}$ = | 13,018 | psi | Table 10-4 |
| ***Stress Analysis - Pitting:*** | Assumes $C_{xc}$ = *1.5* for properly crowned teeth | | |
| Pinion: Required $s_{ac}$ = | 119,964 | psi | See Fig. 10-21 or |
| Gear: Required $s_{ac}$ = | 110,824 | psi | Table 10-4 |

| Through Hardened Grade 1 Steel | |
|---|---|
| HB 206 | Fig. 10-17 |
| HB 248 | Fig. 10-17 |
| | |
| HB 283 | Fig. 10-21 |
| HB 256 | Fig. 10-21 |

***Specify materials, alloy and heat treatment, for most severe requirement.***

***One possible material specification:***

Pinion: HB 283 required: SAE 6150 OQT 1200; HB = 293

Gear: HB 256 required: SAE 6150 OQT 1200; HB = 293

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WORMGEARING DATA FROM PROB. 8-52: $T_o = 924$ LB·IN., $n_G = 30$ rpm

FORCES: $W_{tG} = W_{xW} = T_o/(D_G/2) = 924\text{ LB·IN.}/2.00\text{IN} = 462\text{ LB}$

PITCH LINE SPEED OF GEAR $= v_{tG} = \pi D_G n_G/12 = \pi(4.00)(30)/12 = 31.4$ FT/MIN.

SLIDING VELOCITY $= v_s = v_{tG}/\sin\lambda = 31.4/\sin(4.57) = 394$ FT/MIN.

FROM FIG. 10-25; $\mu = 0.0323$ [COMPUTED FROM EQ. 10-27]

$$\underset{\text{(EQ. 10-30)}}{W_{xG} = W_{tW}} = 462 \times \frac{\cos(14.5)\sin(4.57) + 0.0323\cos(4.57)}{\cos(14.5)\cos(4.57) - 0.0323\cdot\sin(4.57)} = 53\text{ LB}$$

$$\underset{\text{(EQ. 10-31)}}{W_{rG} = W_{rW}} = \frac{462\cdot\sin(14.5)}{\cos(14.5)\cos(4.57) - 0.0323\cdot\sin(4.57)} = 120\text{ LB}$$

$$\underset{\text{(EQ. 10-32)}}{\text{FRICTION FORCE}} = W_f = \frac{(0.0323)(462)}{\cos(4.57)\cos(14.5) - 0.0323\sin(4.57)} = 15.5\text{ LB}$$

$$\text{FRICTION POWER LOSS} = P_L = \frac{v_s W_f}{33000} = \frac{(394)(15.5)}{33000} = 0.185\text{ hp}$$

$$\text{INPUT POWER} = P_{IN} = P_o + P_L = \frac{T_o n_G}{63000} + 0.185 = 0.44 + 0.185 = 0.625\text{ hp}$$

EFFICIENCY $= P_o/P_{IN} \times 100\% = 70.4\%$ ; INPUT SPEED $= n_G \cdot VR = (30)(40) = 1200$ rpm

STRESS

$$\sigma_G = \frac{W_d}{y F p_m} = \frac{W_{tG}}{K_v\, y\, F\, \pi \cos\lambda} = \frac{(462)(10)}{(0.974)(0.100)(0.625)(\pi)(\cos 4.57)} = 24235\text{ psi}$$

$$K_v = 1200/(1200 + v_{tG}) = 1200/(1200 + 31.4) = 0.974$$

PITTING

$$W_{tR} = C_s D_G^{0.8} F_e C_m \underset{\text{(FIG 12-24)}}{C_v} = (1000)(4.00)^{0.8}(0.625)(0.814)(0.427) = 659\text{ LB}$$

BECAUSE $W_{tR} > W_{tG}$ – OK FOR PITTING.

$\sigma_G = 24235$ psi SLIGHTLY HIGHER THAN $S_{at} = 24000$ psi FOR PHOSPHOR BRONZE

(SEE SPREADSHEET SOLUTION ON FOLLOWING PAGE.)

**Wormgearing - Design** | Problem: *10-18*

| *Input Data:* | | |
|---|---|---|
| Desired output torque: | $T_o$ = | *924* lb-in |
| Output speed: | $n_G$ = | *30* rpm |
| Velocity Ratio: | *VR* = | *40* |

| *Design Decisions:* | | |
|---|---|---|
| Diametral pitch: | $P_d$ = | *10* |
| No. of worm threads: | $N_W$ = | *1* |
| Required No. of gear teeth: | $N_G$ = | 40 |
| *Specify* No. of gear teeth: | $N_G$ = | *40* |
| Normal pressure angle: | $\phi_n$ = | *14.5* degrees |

| *Computed Results and Additional Inputs:* | | |
|---|---|---|
| Actual input speed: | $n_W$ = | *1200* rpm |
| Actual velocity ratio: | VR = | 40 |
| Gear pitch diameter: | $D_G$ = | 4 in |
| *Specify* worm diameter: | $D_W$ = | 1.25 in |
| Actual center distance: | $C$ = | 2.625 in |
| | $C^{0.875}/D_W$ = 1.86 | |
| | Should be >1.6 and <3.0 | |
| Circular pitch of gear: | $p_G$ = | 0.314 in |
| Axial pitch of worm: | $p_{xW}$ = | 0.314 in |
| Lead of the worm: | $L$ = | 0.314 in |
| Lead angle: | $\lambda$ = | 4.574 deg |
| Addendum: | $a$ = | 0.100 in |
| Dedendum: | $b$ = | 0.116 in |
| Worm outside diameter: | $D_{oW}$ = | 1.450 in |
| Worm root diameter: | $D_{RW}$ = | 1.019 in |
| Nominal worm face length: | $F_{Wnom}$ = | 1.789 in |
| Gear throat diameter: | $D_{tG}$ = | 4.200 in |
| Nominal gear face width: | $F_{eG}$ = | 0.735 in |
| Max effective gear face width: | $0.67{*}D_W$ = | 0.8375 in |
| Effective gear face width: | $F_e$ = | 0.625 in |
| | | *(Used given face width)* |

Notes: 1. Bending stress on gear slightly high
2. Suggest using larger face width; Say $F = F_e$ = *0.75 in.*

***Additional Computed Results:***

| | | |
|---|---|---|
| Pitch line speed - Gear: | 31.42 ft/min | |
| Sliding velocity $v_s$ = | 394 ft/min | |
| Coefficient of friction: | 0.032 | If $v_s$ > 10 ft/min |

| *Forces: (lb)* | Gear | Worm |
|---|---|---|
| Tangential: | 462 | 53 |
| Radial: | 120 | 120 |
| Axial: | 53 | 462 |
| **Friction force, $W_f$ =** | **15.6 lb** | |

| *Power:* | |
|---|---|
| Power output from gear: | 0.440 hp |
| Power loss - friction: | 0.186 hp |
| **Power Input:** | **0.626 hp** |
| ***Efficiency:*** | **70.3 %** |

***Stresses:***

| Normal pressure angle, $\phi_n$ | | | |
|---|---|---|---|
| 14.5 | 20 | 25 | 30 |
| Lewis form factor, $y$ | | | |
| 0.100 | 0.125 | 0.150 | 0.175 |

***Bending Stress on Gear:***

| | |
|---|---|
| *Enter:* Lewis form factor: $y$ = | 0.100 --------> |
| Normal circular pitch: | 0.313 in |
| Dynamic factor: $K_v$ = | 0.974 |
| ***Bending stress on gear:*** | ***24223 psi*** [Using effective gear face width] |

*Allowable stresses-Bronze: Manganese = 17000 psi; Phosphor = 24000 psi*

***Surface Durability: [Hardened steel worm; bronze gear]***

| *Type of bronze:/$D_G$ --->* | *>2.5 in* | *<2.5 in* | *>8 in* | *<8 in* | *>25 in* | *<25 in* |
|---|---|---|---|---|---|---|
| Sand cast: $C_s$ = | 903 | 1000 | | | | |
| Chill cast or forged: $C_s$ = | | | 1137 | 1000 | | |
| Centrifugally cast: $C_s$ = | | | | | 1143 | 1000 |
| *Enter:* Materials factor: $C_s$ = | 1000 | | | | | |

| | | | | |
|---|---|---|---|---|
| Gear Ratio: $m_G$ = | 6 to 20 | 20 to 76 | >76 | Actual $m_G$ = 40 |
| Ratio correction factor: $C_m$ = | #NUM! | 0.814 | 0.885 | |
| *Enter:* $C_m$ = | 0.814 | | | |
| Sliding velocity: | <700 | 700-3000 | >3000 | Actual $v_s$ = 394 |
| Velocity factor: $C_v$ = | 0.427 | 0.439 | 0.642 | |
| *Enter:* $C_v$ = | 0.427 | | | |
| ***Rated tangential load:*** $W_{tR}$ = | 659 lb | | | |
| ***Must be >*** $W_t$ = | 462 lb | | | |

3. Equation for $C_m$ produces invalid result for large gear ratio as VR = 40

| Wormgearing - Design | Problem: *10-18A* | |
|---|---|---|
| | Adjusted $F_e$ = *0.750 in* | |

| *Input Data:* | | |
|---|---|---|
| Desired output torque: | $T_o$ = | *924* lb-in |
| Output speed: | $n_G$ = | *30* rpm |
| Velocity Ratio: | $VR$ = | *40* |
| ***Design Decisions:*** | | |
| Diametral pitch: | $P_d$ = | *10* |
| No. of worm threads: | $N_W$ = | *1* |
| Required No. of gear teeth: | $N_G$ = | 40 |
| *Specify* No. of gear teeth: | $N_G$ = | *40* |
| Normal pressure angle: | $\phi_n$ = | *14.5* degrees |
| ***Computed Results and Additional Inputs:*** | | |
| Actual input speed: | $n_W$ = | *1200* rpm |
| Actual velocity ratio: | VR = | 40 |
| Gear pitch diameter: | $D_G$ = | 4 in |
| *Specify* worm diameter: | $D_W$ = | 1.25 in |
| Actual center distance: | $C$ = | 2.625 in |
| | $C^{0.875}/D_W$ = 1.86 | |
| | Should be >1.6 and <3.0 | |
| Circular pitch of gear: | $p_G$ = | 0.314 in |
| Axial pitch of worm: | $p_{xW}$ = | 0.314 in |
| Lead of the worm: | $L$ = | 0.314 in |
| Lead angle: | $\lambda$ = | 4.574 deg |
| Addendum: | $a$ = | 0.100 in |
| Dedendum: | $b$ = | 0.116 in |
| Worm outside diameter: | $D_{oW}$ = | 1.450 in |
| Worm root diameter: | $D_{RW}$ = | 1.019 in |
| Nominal worm face length: | $F_{Wnom}$ = | 1.789 in |
| Gear throat diameter: | $D_{tG}$ = | 4.200 in |
| Nominal gear face width: | $F_{eG}$ = | 0.735 in |
| Max effective gear face width: | $0.67^*D_W$ = | 0.8375 in |
| Effective gear face width: | $F_e$ = | 0.750 in |
| | ***(Used given face width)*** | |

Notes: 1. Bending stress on gear OK for Phosphor Bronze
2. Using $F = F_e$ = *0.75 in.*

| ***Additional Computed Results:*** | | | | | | |
|---|---|---|---|---|---|---|
| Pitch line speed - Gear: | 31.42 ft/min | | | | | |
| Sliding velocity $v_s$ = | 394 ft/min | | | | | |
| Coefficient of friction: | 0.032 | If $v_s$ > 10 ft/min | | | | |
| ***Forces: (lb)*** | Gear | Worm | | | | |
| Tangential: | 462 | 53 | | | | |
| Radial: | 120 | 120 | | | | |
| Axial: | 53 | 462 | | | | |
| **Friction force, $W_f$ =** | **15.6 lb** | | | | | |
| ***Power:*** | | | | | | |
| Power output from gear: | 0.440 hp | | | | | |
| Power loss - friction: | 0.186 hp | | | | | |
| **Power input:** | **0.626 hp** | | | | | |
| ***Efficiency:*** | **70.3 %** | | Normal pressure angle, $\phi_n$ | | | |
| ***Stresses:*** | | | 14.5 | 20 | 25 | 30 |
| ***Bending Stress on Gear:*** | | | Lewis form factor, $y$ | | | |
| *Enter:* Lewis form factor: $y$ = | 0.100 | --------> | 0.100 | 0.125 | 0.150 | 0.175 |
| Normal circular pitch: | 0.313 in | | | | | |
| Dynamic factor: $K_v$ = | 0.974 | | | | | |
| ***Bending stress on gear:*** | ***20186 psi*** | [Using effective gear face width] | | | | |
| *Allowable stresses-Bronze: Manganese = 17000 psi; Phosphor = 24000 psi* | | | | | | |

| ***Surface Durability: [Hardened steel worm; bronze gear]*** | | | | | | |
|---|---|---|---|---|---|---|
| *Type of bronze:/$D_G$ --->* | *>2.5 in* | *<2.5 in* | *>8 in* | *<8 in* | *>25 in* | *<25 in* |
| Sand cast: $C_s$ = | 903 | 1000 | | | | |
| Chill cast or forged: $C_s$ = | | | 1137 | 1000 | | |
| Centrifugally cast: $C_s$ = | | | | | 1143 | 1000 |
| *Enter:* Materials factor: $C_s$ = | 1000 | | | | | |
| Gear Ratio: $m_G$ = | 6 to 20 | 20 to 76 | >76 | Actual $m_G$ = 40 | | |
| Ratio correction factor: $C_m$ = | #NUM! | 0.814 | 0.885 | | | |
| *Enter:* $C_m$ = | 0.814 | | | | | |
| Sliding velocity: | <700 | 700-3000 | >3000 | Actual $v_s$ = 394 | | |
| Velocity factor: $C_v$ = | 0.427 | 0.439 | 0.642 | | | |
| *Enter:* $C_v$ = | 0.427 | | | | | |
| ***Rated tangential load: $W_{tR}$ =*** | 790 lb | | | | | |
| ***Must be > $W_t$ =*** | 462 lb | | | | | |

3. Equation for $C_m$ produces invalid result for large gear ratio as VR = 40

**Problems 10-19 and 10-20** — **COMPARISON OF THREE PROPOSED DESIGNS**

***See details on following three spreadsheets***

***Given data:***

| | | |
|---|---|---|
| Diametral pitch, $P_d$ = | 12 | |
| Velocity ratio, *VR* = | 20 | |
| Output speed (Gear) = | 90 rpm | |
| Worm pitch diameter, $D_W$ = | 1.000 in | ***See comment*** |
| Gear face width, *F* = | 0.500 in | ***See comment*** |
| Normal pressure angle, $\phi_n$ = | 14.5 degrees | |

***Assumed gear is made from chilled cast phosphor bronze***

***Allowable bending stress = 24,000 psi***

***Results:***

| | DESIGN | | | |
|---|---|---|---|---|
| | A | B | C | |
| ***Number of threads in worm:*** | 1 | 2 | 4 | |
| Output torque (lb-in), $T_o$ = | 202 | 484 | 878 | |
| Output power (hp), *Po* = | 0.289 | 0.691 | 1.254 | |
| Gear bending stress (psi) | 19190 | ***23963*** | ***23987*** | Limits in ***Bold*** |
| Allowable bending stress (psi) | 24000 | 24000 | 24000 | |
| Rated load for surface durability (lb) | ***242*** | 418 | 714 | Limits in ***Bold*** |
| Gear transmitted load (lb) | 242 | 290 | 263 | |
| Efficiency (%) [Problem 20] | 72.9 | 84.1 | 90.8 | |
| Power input (hp) | 0.396 | 0.822 | 1.381 | |
| Lead angle (degrees) | 4.76 | 9.46 | 18.4 | |
| Gear pitch diameter (in) | 1.667 | 3.333 | 6.667 | |
| Center distance (in) | 1.333 | 2.167 | 3.833 | |

***Comments on results:***

The given face width is small. Could use *F* > 0.601 in to maximize effective face width.
Worm diameter is too large for Design A. See Equations 10-46 and 10-47
Worm diameter is too small for Design C. See Equations 10-46 and 10-47
Design A is limited by surface durability
Designs B and C are limited by bending stress in gear teeth.

***As number of threads in worm increases:***

Lead angle increases
Efficiency increases
Torque and power capacity increase
**BUT**: Gear size and center distance increase

## Wormgearing - Design

Problem: 10-19a and 20a

**Input Data:**

| | | |
|---|---|---|
| Desired output torque: | $T_o$ = | 202 lb-in |
| Output speed: | $n_G$ = | 90 rpm |
| Velocity Ratio: | $VR$ = | 20 |

**Design Decisions:**

| | | |
|---|---|---|
| Diametral pitch: | $P_d$ = | 12 |
| No. of worm threads: | $N_W$ = | 1 |
| Required No. of gear teeth: | $N_G$ = | 20 |
| Specify No. of gear teeth: | $N_G$ = | 20 |
| Normal pressure angle: | $\phi_n$ = | 14.5 degrees |

**Computed Results and Additional Inputs:**

| | | |
|---|---|---|
| Actual input speed: | $n_W$ = | 1800 rpm |
| Actual velocity ratio: | VR = | 20 |
| Gear pitch diameter: | $D_G$ = | 1.66667 in |
| Specify worm diameter: | $D_W$ = | 1.000 in |
| Actual center distance: | $C$ = | 1.333 in |
| | $C^{0.875}/D_W$ = 1.29 | ***LOW*** |
| ***Use smaller worm diameter*** | Should be >1.6 and <3.0 | |
| Circular pitch of gear: | $p_G$ = | 0.262 in |
| Axial pitch of worm: | $p_{xW}$ = | 0.262 in |
| Lead of the worm: | $L$ = | 0.262 in |
| Lead angle: | $\lambda$ = | 4.764 deg |
| Addendum: | $a$ = | 0.083 in |
| Dedendum: | $b$ = | 0.096 in |
| Worm outside diameter: | $D_{oW}$ = | 1.167 in |
| Worm root diameter: | $D_{RW}$ = | 0.807 in |
| Nominal worm face length: | $F_{Wnom}$ = | 1.054 in |
| Gear throat diameter: | $D_{tG}$ = | 1.833 in |
| Nominal gear face width: | $F_{eG}$ = | 0.601 in |
| Max effective gear face width: | $0.67^*D_W$ = | 0.670 in |
| Effective gear face width: | $F_e$ = | 0.500 in |
| | ***[Used given face width]*** | |
| Given face width is small; Could use $F$ > | | 0.601 |

**Additional Computed Results:**

| | | |
|---|---|---|
| Pitch line speed - Gear: | 39.27 ft/min | |
| Sliding velocity $v_s$ = | 473 ft/min | |
| Coefficient of friction: | 0.030 | if $v_s$ > 10 ft/min |

***Forces: (lb)***

| | Gear | Worm |
|---|---|---|
| Tangential: | 242 | 28 |
| Radial: | 63 | 63 |
| Axial: | 28 | 242 |
| **Friction force, $W_f$ =** | **7.5 lb** | |

***Power:***

| | |
|---|---|
| Power output from gear: | 0.289 hp |
| Power loss - friction: | 0.107 hp |
| **Power input:** | **0.396 hp** |
| ***Efficiency:*** | **72.9 %** |

***Stresses:***

| Normal pressure angle, $\phi_n$ | | | |
|---|---|---|---|
| 14.5 | 20 | 25 | 30 |
| Lewis form factor, $y$ | | | |
| 0.100 | 0.125 | 0.150 | 0.175 |

***Bending Stress on Gear:***

| | |
|---|---|
| Enter: Lewis form factor: $y$ = | 0.100 ------> |
| Normal circular pitch: | 0.261 in |
| Dynamic factor: $K_v$ = | 0.968 |
| ***Bending stress on gear:*** | ***19190 psi*** [Using effective gear face width] |

*Allowable stresses-Bronze: Manganese = 17000 psi; Phosphor = 24000 psi*

***Surface Durability: [Hardened steel worm; bronze gear]***

| *Type of bronze:/$D_G$ --->* | *>2.5 in* | *<2.5 in* | *>8 in* | *<8 in* | *>25 in* | *<25 in* |
|---|---|---|---|---|---|---|
| Sand cast: $C_s$ = | 1084 | 1000 | | | | |
| Chill cast or forged: $C_s$ = | | | 1311 | 1000 | | |
| Centrifugally cast: $C_s$ = | | | | | 1211 | 1000 |
| Enter: Materials factor: $C_s$ = | 1000 | | | | | |

| | | | | |
|---|---|---|---|---|
| Gear Ratio: $m_G$ = | 6 to 20 | 20 to 76 | >76 | Actual $m_G$ = 20 |
| Ratio correction factor: $C_m$ = | 0.820 | 0.819 | 1.017 | |
| Enter: $C_m$ = | 0.819 | | | |
| Sliding velocity: | <700 | 700-3000 | >3000 | Actual $v_s$ = 473 |
| Velocity factor: $C_v$ = | 0.392 | 0.395 | 0.557 | |
| Enter: $C_v$ = | 0.392 | | | |
| ***Rated tangential load: $W_{tR}$ =*** | 242 lb | | | |
| ***Must be > $W_t$ =*** | 242 lb | | | |

Adjusted output torque until limits reached on either bending or surface durability
Surface durability controls this design

| Wormgearing - Design | | Problem: 10-19b and 20b | |
|---|---|---|---|
| **Input Data:** | | | |
| Desired output torque: | $T_o$ = | 484 | lb-in |
| Output speed: | $n_G$ = | 90 | rpm |
| Velocity Ratio: | $VR$ = | 20 | |
| **Design Decisions:** | | | |
| Diametral pitch: | $P_d$ = | 12 | |
| No. of worm threads: | $N_W$ = | 2 | |
| Required No. of gear teeth: | $N_G$ = | 40 | |
| Specify No. of gear teeth: | $N_G$ = | 40 | |
| Normal pressure angle: | $\phi_n$ = | 14.5 | degrees |
| **Computed Results and Additional Inputs:** | | | |
| Actual input speed: | $n_W$ = | 1800 | rpm |
| Actual velocity ratio: | VR = | 20 | |
| Gear pitch diameter: | $D_G$ = | 3.33333 | in |
| Specify worm diameter: | $D_W$ = | 1.000 | in |
| Actual center distance: | $C$ = | 2.167 | in |
| | $C^{0.875}/D_W$ = | 1.97 | |
| | Should be >1.6 and <3.0 | | |
| Circular pitch of gear: | $p_G$ = | 0.262 | in |
| Axial pitch of worm: | $p_{xW}$ = | 0.262 | in |
| Lead of the worm: | $L$ = | 0.524 | in |
| Lead angle: | $\lambda$ = | 9.462 | deg |
| Addendum: | $a$ = | 0.083 | in |
| Dedendum: | $b$ = | 0.096 | in |
| Worm outside diameter: | $D_{oW}$ = | 1.167 | in |
| Worm root diameter: | $D_{RW}$ = | 0.807 | in |
| Nominal worm face length: | $F_{Wnom}$ = | 1.491 | in |
| Gear throat diameter: | $D_{tG}$ = | 3.500 | in |
| Nominal gear face width: | $F_{eG}$ = | 0.601 | in |
| Max effective gear face width: | $0.67*D_W$ = | 0.670 | in |
| Effective gear face width: | $F_e$ = | 0.500 | in |
| | *[Used given face width]* | | |
| Given face width is small; Could use $F$ > | | 0.601 | in |

***Additional Computed Results:***

| | | |
|---|---|---|
| Pitch line speed - Gear: | 78.54 ft/min | |
| Sliding velocity $v_s$ = | 478 ft/min | |
| Coefficient of friction: | 0.030 | If $v_s$ > 10 ft/min |

| ***Forces: (lb)*** | Gear | Worm |
|---|---|---|
| Tangential: | 290 | 58 |
| Radial: | 77 | 77 |
| Axial: | 58 | 290 |
| **Friction force, $W_f$ =** | **9.1 lb** | |

***Power:***

| | |
|---|---|
| Power output from gear: | 0.691 hp |
| Power loss - friction: | 0.131 hp |
| **Power input:** | **0.822 hp** |

***Efficiency:*** **84.1 %**

***Stresses:***

***Bending Stress on Gear:***

| Normal pressure angle, $\phi_n$ | | | |
|---|---|---|---|
| 14.5 | 20 | 25 | 30 |
| Lewis form factor, $y$ | | | |
| 0.100 | 0.125 | 0.150 | 0.175 |

| | |
|---|---|
| *Enter:* Lewis form factor: $y$ = | 0.100 ———> |
| Normal circular pitch: | 0.258 in |
| Dynamic factor: $K_v$ = | 0.939 |
| ***Bending stress on gear:*** | ***23963 psi*** [Using effective gear face width] |

*Allowable stresses-Bronze: Manganese = 17000 psi; Phosphor = 24000 psi*

***Surface Durability: [Hardened steel worm; bronze gear]***

| *Type of bronze:/$D_G$ —>* | *>2.5 in* | *<2.5 in* | *>8 in* | *<8 in* | *>25 in* | *<25 in* |
|---|---|---|---|---|---|---|
| Sand cast: $C_s$ = | 940 | 1000 | | | | |
| Chill cast or forged: $C_s$ = | | | 1173 | 1000 | | |
| Centrifugally cast: $C_s$ = | | | | | 1157 | 1000 |

*Enter:* Materials factor: $C_s$ = 1000 Chilled Cast - Phosphor bronze

| | | | | |
|---|---|---|---|---|
| Gear Ratio: $m_G$ = | 6 to 20 | 20 to 76 | >76 | Actual $m_G$ = 20 |
| Ratio correction factor: $C_m$ = | 0.820 | 0.819 | 1.017 | |
| *Enter:* $C_m$ = | 0.819 | | | |
| Sliding velocity: | <700 | 700-3000 | >3000 | Actual $v_s$ = 478 |
| Velocity factor: $C_v$ = | 0.390 | 0.393 | 0.553 | |
| *Enter:* $C_v$ = | 0.390 | | | |
| ***Rated tangential load: $W_{tR}$ =*** | 418 lb | | | |
| ***Must be > $W_t$ =*** | 290 lb | | | |

Adjusted output torque until limits reached on either bending or surface durability
Bending stress controls this design

| Wormgearing - Design | | Problem: 10-19c and 20c | |
|---|---|---|---|
| **Input Data:** | | | |
| Desired output torque: | $T_o$ = | 878 | lb-in |
| Output speed: | $n_G$ = | 90 | rpm |
| Velocity Ratio: | VR = | 20 | |
| **Design Decisions:** | | | |
| Diametral pitch: | $P_d$ = | 12 | |
| No. of worm threads: | $N_W$ = | 4 | |
| Required No. of gear teeth: | $N_G$ = | 80 | |
| Specify No. of gear teeth: | $N_G$ = | 80 | |
| Normal pressure angle: | $\phi_n$ = | 14.5 | degrees |
| **Computed Results and Additional Inputs:** | | | |
| Actual input speed: | $n_W$ = | 1800 | rpm |
| Actual velocity ratio: | VR = | 20 | |
| Gear pitch diameter: | $D_G$ = | 6.66667 | in |
| Specify worm diameter: | $D_W$ = | 1.000 | in |
| Actual center distance: | $C$ = | 3.833 | in |
| | $C^{0.875}/D_W$ = 3.24 | | HIGH |
| *Use larger worm diameter* | Should be >1.6 and <3.0 | | |
| Circular pitch of gear: | $p_G$ = | 0.262 | in |
| Axial pitch of worm: | $p_{xW}$ = | 0.262 | in |
| Lead of the worm: | $L$ = | 1.047 | in |
| Lead angle: | $\lambda$ = | 18.435 | deg |
| Addendum: | $a$ = | 0.083 | in |
| Dedendum: | $b$ = | 0.096 | in |
| Worm outside diameter: | $D_{oW}$ = | 1.167 | in |
| Worm root diameter: | $D_{RW}$ = | 0.807 | in |
| Nominal worm face length: | $F_{Wnom}$ = | 2.108 | in |
| Gear throat diameter: | $D_{tG}$ = | 6.833 | in |
| Nominal gear face width: | $F_{eG}$ = | 0.601 | in |
| Max effective gear face width: | $0.67^*D_W$ = | 0.670 | in |
| Effective gear face width: | $F_e$ = | 0.500 | in |
| | *[Used given face width]* | | |
| Given face width is small; Could use $F$ > | | 0.601 | in |

**Additional Computed Results:**

| | | | |
|---|---|---|---|
| Pitch line speed - Gear: | 157.08 ft/min | | |
| Sliding velocity $v_s$ = | 497 ft/min | | |
| Coefficient of friction: | 0.029 | If $v_s$ > 10 ft/min | |

| *Forces: (lb)* | Gear | Worm |
|---|---|---|
| Tangential: | 263 | 97 |
| Radial: | 73 | 73 |
| Axial: | 97 | 263 |
| Friction force, $W_f$ = | 8.4 lb | |

| *Power:* | |
|---|---|
| Power output from gear: | 1.254 hp |
| Power loss - friction: | 0.127 hp |
| Power input: | 1.381 hp |
| *Efficiency:* | 90.8 % |
| *Stresses:* | |

| Normal pressure angle, $\phi_n$ | | | |
|---|---|---|---|
| 14.5 | 20 | 25 | 30 |
| Lewis form factor, $y$ | | | |
| 0.100 | 0.125 | 0.150 | 0.175 |

*Bending Stress on Gear:*

| | | |
|---|---|---|
| Enter: Lewis form factor: $y$ = | 0.100 | ---> |
| Normal circular pitch: | 0.248 in | |
| Dynamic factor: $K_v$ = | 0.884 | |
| *Bending stress on gear:* | *23987 psi* | [Using effective gear face width] |

*Allowable stresses-Bronze: Manganese = 17000 psi; Phosphor = 24000 psi*

*Surface Durability: [Hardened steel worm; bronze gear]*

| *Type of bronze:/$D_G$ --->* | >2.5 in | <2.5 in | >8 in | <8 in | >25 in | <25 in |
|---|---|---|---|---|---|---|
| Sand cast: $C_s$ = | 797 | 1000 | | | | |
| Chill cast or forged: $C_s$ = | | | 1036 | 1000 | | |
| Centrifugally cast: $C_s$ = | | | | | 1103 | 1000 |

| | | | | | |
|---|---|---|---|---|---|
| Enter: Materials factor: $C_s$ = | 1000 | Chilled Cast - Phosphor bronze | | | |
| Gear Ratio: $m_G$ = | 6 to 20 | 20 to 76 | >76 | Actual $m_G$ = | 20 |
| Ratio correction factor: $C_m$ = | 0.820 | 0.819 | 1.017 | | |
| Enter: $C_m$ = | 0.819 | | | | |
| Sliding velocity: | <700 | 700-3000 | >3000 | Actual $v_s$ = | 497 |
| Velocity factor: $C_v$ = | 0.382 | 0.384 | 0.537 | | |
| Enter: $C_v$ = | 0.382 | | | | |
| *Rated tangential load:* $W_{tR}$ = | 714 lb | | | | |
| *Must be >* $W_t$ = | 263 lb | | | | |

Adjusted output torque until limits reached on either bending or surface durability
Bending stress controls this design

## Wormgearing - Design

Problem: 10-21

### Input Data:

| | | |
|---|---|---|
| Desired output torque: | $T_o$ = | 984 lb-in |
| Output speed: | $n_G$ = | 80 rpm |
| Velocity Ratio: | $VR$ = | 7.5 |

### Design Decisions:

| | | |
|---|---|---|
| Diametral pitch: | $P_d$ = | 8 |
| No. of worm threads: | $N_W$ = | 4 |
| Required No. of gear teeth: | $N_G$ = | 30 |
| Specify No. of gear teeth: | $N_G$ = | 30 |
| Normal pressure angle: | $\phi_n$ = | 14.5 degrees |

### Computed Results and Additional Inputs:

| | | |
|---|---|---|
| Actual input speed: | $n_W$ = | 600 rpm |
| Actual velocity ratio: | VR = | 7.5 |
| Gear pitch diameter: | $D_G$ = | 3.75 in |
| Specify worm diameter: | $D_W$ = | 1.375 in |
| Actual center distance: | $C$ = | 2.563 in |
| | $C^{0.875}/D_W$ = | 1.66 |
| | Should be >1.6 and <3.0 | |
| Circular pitch of gear: | $p_G$ = | 0.393 in |
| Axial pitch of worm: | $p_{xW}$ = | 0.393 in |
| Lead of the worm: | $L$ = | 1.571 in |
| Lead angle: | $\lambda$ = | 19.983 deg |
| Addendum: | $a$ = | 0.125 in |
| Dedendum: | $b$ = | 0.145 in |
| Worm outside diameter: | $D_{oW}$ = | 1.625 in |
| Worm root diameter: | $D_{RW}$ = | 1.086 in |
| Nominal worm face length: | $F_{Wnom}$ = | 1.936 in |
| Gear throat diameter: | $D_{tG}$ = | 4.000 in |
| Nominal gear face width: | $F_{eG}$ = | 0.866 in |
| Max effective gear face width: | $0.67 \cdot D_W$ = | 0.92125 in |
| Effective gear face width: | $F_e$ = | 0.866 in |

### Additional Computed Results:

| | | |
|---|---|---|
| Pitch line speed - Gear: | 78.54 | ft/min |
| Sliding velocity $v_s$ = | 230 | ft/min |
| Coefficient of friction: | 0.041 | If $v_s$ > 10 ft/min |

| Forces: (lb) | Gear | Worm |
|---|---|---|
| Tangential: | 525 | 216 |
| Radial: | 147 | 147 |
| Axial: | 216 | 525 |
| **Friction force, $W_f$ =** | **24.0 lb** | |

**Power:**

| | |
|---|---|
| Power output from gear: | 1.250 hp |
| Power loss - friction: | 0.167 hp |
| **Power input:** | **1.416 hp** |

**Efficiency:** **88.2 %**

**Stresses:**

| Normal pressure angle, $\phi_n$ | | | |
|---|---|---|---|
| 14.5 | 20 | 25 | 30 |
| Lewis form factor, $y$ | | | |
| 0.100 | 0.125 | 0.150 | 0.175 |

**Bending Stress on Gear:**

Enter: Lewis form factor: $y$ = 0.100 ——>

| | |
|---|---|
| Normal circular pitch: | 0.369 in |
| Dynamic factor: $K_v$ = | 0.939 |

**Bending stress on gear: 17495 psi** [Using effective gear face width]

*Allowable stresses-Bronze: Manganese = 17000 psi; Phosphor = 24000 psi*

**Surface Durability: [Hardened steel worm; bronze gear]**

| *Type of bronze:/$D_G$ —>* | *>2.5 in* | *<2.5 in* | *>8 in* | *<8 in* | *>25 in* | *<25 in* |
|---|---|---|---|---|---|---|
| Sand cast: $C_s$ = | 916 | 1000 | | | | |
| Chill cast or forged: $C_s$ = | | | 1150 | 1000 | | |
| Centrifugally cast: $C_s$ = | | | | | 1148 | 1000 |

Enter: Materials factor: $C_s$ = 1000

| | | | | |
|---|---|---|---|---|
| Gear Ratio: $m_G$ = | 6 to 20 | 20 to 76 | >76 | Actual $m_G$ = 7.5 |
| Ratio correction factor: $C_m$ = | 0.719 | 0.794 | 1.099 | |

Enter: $C_m$ = 0.719

| | | | | |
|---|---|---|---|---|
| Sliding velocity: | <700 | 700-3000 | >3000 | Actual $v_s$ = 230 |
| Velocity factor: $C_v$ = | 0.512 | 0.597 | 0.974 | |

Enter: $C_v$ = 0.512

***Rated tangential load: $W_{tR}$ =*** 918 lb

***Must be > $W_t$ =*** 525 lb

## Wormgearing - Design

Problem: 10-22

### Input Data:

| | | | |
|---|---|---|---|
| Desired output torque: | $T_o$ = | 52.5 | lb-in |
| Output speed: | $n_G$ = | 600 | rpm |
| Velocity Ratio: | $VR$ = | 3 | |

### Design Decisions:

| | | | |
|---|---|---|---|
| Diametral pitch: | $P_d$ = | 12 | |
| No. of worm threads: | $N_W$ = | 6 | |
| Required No. of gear teeth: | $N_G$ = | 18 | |
| Specify No. of gear teeth: | $N_G$ = | 18 | |
| Normal pressure angle: | $\phi_n$ = | 25 | degrees |

### Computed Results and Additional Inputs:

| | | | |
|---|---|---|---|
| Actual input speed: | $n_W$ = | 1800 | rpm |
| Actual velocity ratio: | VR = | 3 | |
| Gear pitch diameter: | $D_G$ = | 1.5 | in |
| Specify worm diameter: | $D_W$ = | 0.5 | in |
| Actual center distance: | $C$ = | 1.000 | in |
| | $C^{0.875}/D_W$ = | 2.00 | |
| | Should be >1.6 and <3.0 | | |
| Circular pitch of gear: | $p_G$ = | 0.262 | in |
| Axial pitch of worm: | $p_{xW}$ = | 0.262 | in |
| Lead of the worm: | $L$ = | 1.571 | in |
| Lead angle: | $\lambda$ = | 45.000 | deg |
| Addendum: | $a$ = | 0.083 | in |
| Dedendum: | $b$ = | 0.096 | in |
| Worm outside diameter: | $D_{oW}$ = | 0.667 | in |
| Worm root diameter: | $D_{RW}$ = | 0.307 | in |
| Nominal worm face length: | $F_{Wnom}$ = | 1.000 | in |
| Gear throat diameter: | $D_{tG}$ = | 1.667 | in |
| Nominal gear face width: | $F_{eG}$ = | 0.441 | in |
| Max effective gear face width: | $0.67^*D_W$ = | 0.335 | in |
| Effective gear face width: | $F_e$ = | 0.335 | in |

### Additional Computed Results:

| | | | |
|---|---|---|---|
| Pitch line speed - Gear: | 235.62 | ft/min | |
| Sliding velocity $v_s$ = | 333 | ft/min | |
| Coefficient of friction: | 0.035 | If $v_s$ > 10 ft/min | |

| **Forces: (lb)** | Gear | Worm |
|---|---|---|
| Tangential: | 70 | 76 |
| Radial: | 48 | 48 |
| Axial: | 76 | 70 |
| **Friction force, $W_f$ =** | **4.0** | **lb** |

**Power:**

| | | |
|---|---|---|
| Power output from gear: | 0.500 | hp |
| Power loss - friction: | 0.040 | hp |
| **Power input:** | **0.540** | **hp** |

**Efficiency:** **92.6 %**

**Stresses:**

| Normal pressure angle, $\phi_n$ | | | |
|---|---|---|---|
| 14.5 | 20 | 25 | 30 |
| Lewis form factor, $y$ | | | |
| 0.100 | 0.125 | 0.150 | 0.175 |

**Bending Stress on Gear:**

| | | |
|---|---|---|
| Enter: Lewis form factor: $y$ = | 0.150 | ------> |
| Normal circular pitch: | 0.185 | in |
| Dynamic factor: $K_v$ = | 0.836 | |
| **Bending stress on gear:** | **9003** | **psi** [Using effective gear face width] |

*Allowable stresses-Bronze: Manganese = 17000 psi; Phosphor = 24000 psi*

**Surface Durability: [Hardened steel worm; bronze gear]**

| *Type of bronze:/$D_G$ --->* | *>2.5 in* | *<2.5 in* | *>8 in* | *<8 in* | *>25 in* | *<25 in* |
|---|---|---|---|---|---|---|
| Sand cast: $C_s$ = | 1106 | 1000 | | | | |
| Chill cast or forged: $C_s$ = | | | 1331 | 1000 | | |
| Centrifugally cast: $C_s$ = | | | | | 1220 | 1000 |
| Enter: Materials factor: $C_s$ = | 1000 | | | | | |

| | | | | |
|---|---|---|---|---|
| Gear Ratio: $m_G$ = | 6 to 20 | 20 to 76 | >76 | Actual $m_G$ = 3 |
| Ratio correction factor: $C_m$ = | 0.578 | 0.779 | 1.129 | |
| Enter: $C_m$ = | 0.578 | | | |
| Sliding velocity: | <700 | 700-3000 | >3000 | Actual $v_s$ = 333 |
| Velocity factor: $C_v$ = | 0.457 | 0.483 | 0.731 | |
| Enter: $C_v$ = | 0.457 | | | |

| | | |
|---|---|---|
| ***Rated tangential load: $W_{tR}$ =*** | 122 | lb |
| ***Must be > $W_t$ =*** | 70 | lb |

## Wormgearing - Design

Problem: 10-23

**Input Data:**

| | | |
|---|---|---|
| Desired output torque: | $T_o$ = | 4200 lb-in |
| Output speed: | $n_G$ = | 45 rpm |
| Velocity Ratio: | $VR$ = | 40 |

**Design Decisions:**

| | | |
|---|---|---|
| Diametral pitch: | $P_d$ = | 8 |
| No. of worm threads: | $N_W$ = | 2 |
| Required No. of gear teeth: | $N_G$ = | 80 |
| Specify No. of gear teeth: | $N_G$ = | 80 |
| Normal pressure angle: | $\phi_n$ = | 14.5 degrees |

**Computed Results and Additional Inputs:**

| | | |
|---|---|---|
| Actual input speed: | $n_W$ = | 1800 rpm |
| Actual velocity ratio: | VR = | 40 |
| Gear pitch diameter: | $D_G$ = | 10 in |
| Specify worm diameter: | $D_W$ = | 2.25 in |
| Actual center distance: | $C$ = | 6.125 in |
| | $C^{0.875}/D_W$ = | 2.17 |
| | Should be >1.6 and <3.0 | |
| Circular pitch of gear: | $p_G$ = | 0.393 in |
| Axial pitch of worm: | $p_{xW}$ = | 0.393 in |
| Lead of the worm: | $L$ = | 0.785 in |
| Lead angle: | $\lambda$ = | 6.340 deg |
| Addendum: | $a$ = | 0.125 in |
| Dedendum: | $b$ = | 0.145 in |
| Worm outside diameter: | $D_{oW}$ = | 2.500 in |
| Worm root diameter: | $D_{RW}$ = | 1.961 in |
| Nominal worm face length: | $F_{Wnom}$ = | 3.162 in |
| Gear throat diameter: | $D_{tG}$ = | 10.250 in |
| Nominal gear face width: | $F_{eG}$ = | 1.090 in |
| Max effective gear face width: | $0.67 \cdot D_W$ = | 1.5075 in |
| Effective gear face width: | $F_e$ = | 1.090 in |

**Additional Computed Results:**

| | | |
|---|---|---|
| Pitch line speed - Gear: | 117.81 | ft/min |
| Sliding velocity $v_s$ = | 1067 | ft/min |
| Coefficient of friction: | 0.020 | If $v_s$ > 10 ft/min |

| **Forces: (lb)** | Gear | Worm |
|---|---|---|
| Tangential: | 840 | 111 |
| Radial: | 219 | 219 |
| Axial: | 111 | 840 |
| **Friction force, $W_f$ =** | **17.6 lb** | |

**Power:**

| | |
|---|---|
| Power output from gear: | 3.000 hp |
| Power loss - friction: | 0.570 hp |
| **Power input:** | **3.570 hp** |

***Efficiency:*** **84.0 %**

***Stresses:***

| Normal pressure angle, $\phi_n$ | | | |
|---|---|---|---|
| 14.5 | 20 | 25 | 30 |
| Lewis form factor, $y$ | | | |
| 0.100 | 0.125 | 0.150 | 0.175 |

***Bending Stress on Gear:***

| | |
|---|---|
| Enter: Lewis form factor, $y$ = | 0.100 ——> |
| Normal circular pitch: | 0.390 in |
| Dynamic factor: $K_v$ = | 0.911 |
| ***Bending stress on gear:*** | ***21689 psi*** [Using effective gear face width] |

*Allowable stresses-Bronze: Manganese = 17000 psi; Phosphor = 24000 psi*

***Surface Durability: [Hardened steel worm; bronze gear]***

| *Type of bronze:/$D_G$ —>* | *>2.5 in* | *<2.5 in* | *>8 in* | *<8 in* | *>25 in* | *<25 in* |
|---|---|---|---|---|---|---|
| Sand cast: $C_s$ = | 713 | 1000 | | | | |
| Chill cast or forged: $C_s$ = | | | 956 | 1000 | | |
| Centrifugally cast: $C_s$ = | | | | | 1072 | 1000 |

Enter: Materials factor $C_s$ = 713 Sand Cast

| | | | | |
|---|---|---|---|---|
| Gear Ratio: $m_G$ = | 6 to 20 | 20 to 76 | >76 | Actual $m_G$ = 40 |
| Ratio correction factor: $C_m$ = | #NUM! | 0.814 | 0.885 | |
| Enter: $C_m$ = | 0.814 | | | |
| Sliding velocity: | <700 | 700-3000 | >3000 | Actual $v_s$ = 1067 |
| Velocity factor: $C_v$ = | 0.204 | 0.248 | 0.297 | |
| Enter: $C_v$ = | 0.248 | | | |

***Rated tangential load: $W_{tR}$ =*** 990 lb

***Must be > $W_t$ =*** 840 lb

**PROBLEM 10-24** . COMPARISON OF DESIGNS A and B.
***Note: Also includes a revised Design A with a smaller worm diameter and larger gear face width***

| | Design | | | |
|---|---|---|---|---|
| ***Given data:*** | ***A*** | ***B*** | ***A Revised*** | |
| Diametral pitch: | 6 | 10 | 6 | |
| Threads in worm: | 1 | 2 | 1 | |
| Teeth in gear: | 30 | 60 | 30 | |
| Worm diameter (in): | 2.000 | 1.250 | 1.750 | Design A Rev. - Smaller diameter |
| Face width - Gear (in): | 1.000 | 0.625 | 1.130 | Design A Rev. - Larger face width |
| Pressure angle (deg): | 14.5 | 14.5 | 14.5 | |
| ***Results:*** | | | | |
| Forces - Gear (lb): | | | | |
| Tangential: | 480 | 400 | 480 | |
| Radial: | 125 | 106 | 125 | |
| Axial: | 58 | 82 | 65 | |
| Forces - Worm (lb): | | | | |
| Tangential: | 58 | 82 | 65 | |
| Radial: | 125 | 106 | 125 | |
| Axial: | 480 | 400 | 480 | |
| Lead angle (degrees): | 4.76 | 9.09 | 5.44 | Design B - Higher lead angle |
| Efficiency (%): | 69.1 | 77.6 | 70.6 | Design B - Higher efficiency |
| Power output (hp): | 0.381 | 0.381 | 0.381 | |
| Power input (hp): | 0.552 | 0.491 | 0.54 | Design B OK for 0.50 hp motor |
| Gear pitch diameter (in): | 5.000 | 6.000 | 5.000 | Design A smaller |
| Center distance (in): | 3.500 | 3.625 | 3.375 | Design A smaller |
| Stress - Wormgear (psi): | 9400 | 21171 | 8324 | Design A - Lower bending stress |
| Rated load - Durability (lb): | 1193 | 937 | 1406 | |
| | | | | Design A - OK for sand cast Manganese Bronze |
| | | | | Design B - OK for sand cast Phosphor bronze |

| Wormgearing - Design | Problem: 10-24A | |
|---|---|---|
| **_Input Data:_** | | |
| Desired output torque: | $T_o$ = | 1200 lb-in |
| Output speed: | $n_G$ = | 20 rpm |
| Velocity Ratio: | $VR$ = | 30 |
| **_Design Decisions:_** | | |
| Diametral pitch: | $P_d$ = | 6 |
| No. of worm threads: | $N_W$ = | 1 |
| Required No. of gear teeth: | $N_G$ = | 30 |
| Specify No. of gear teeth: | $N_G$ = | 30 |
| Normal pressure angle: | $\phi_n$ = | 14.5 degrees |
| **_Computed Results and Additional Inputs:_** | | |
| Actual input speed: | $n_W$ = | 600 rpm |
| Actual velocity ratio: | VR = | 30 |
| Gear pitch diameter: | $D_G$ = | 5 in |
| Specify worm diameter: | $D_W$ = | 2.000 in |
| Actual center distance: | $C$ = | 3.500 in |
| | $C^{0.875}/D_W$ = 1.50 | ***LOW*** |
| ***Use smaller worm diameter*** | Should be >1.6 and <3.0 | |
| Circular pitch of gear: | $p_G$ = | 0.524 in |
| Axial pitch of worm: | $p_{xW}$ = | 0.524 in |
| Lead of the worm: | $L$ = | 0.524 in |
| Lead angle: | $\lambda$ = | 4.764 deg |
| Addendum: | $a$ = | 0.167 in |
| Dedendum: | $b$ = | 0.193 in |
| Worm outside diameter: | $D_{oW}$ = | 2.333 in |
| Worm root diameter: | $D_{RW}$ = | 1.614 in |
| Nominal worm face length: | $F_{Wnom}$ = | 2.582 in |
| Gear throat diameter: | $D_{tG}$ = | 5.333 in |
| Nominal gear face width: | $F_{eG}$ = | 1.202 in |
| Max effective gear face width: | $0.67^*D_W$ = | 1.340 in |
| Effective gear face width: | $F_e$ = | 1.000 in |
| | ***[Used given face width]*** | |
| Given face width is small; Could use $F$ > | | 1.202 |

| ***Additional Computed Results:*** | | |
|---|---|---|
| Pitch line speed - Gear: | 26.18 ft/min | |
| Sliding velocity $v_s$ = | 315 ft/min | |
| Coefficient of friction: | 0.036 If $v_s$ > 10 ft/min | |
| ***Forces: (lb)*** | Gear | Worm |
| Tangential: | 480 | 58 |
| Radial: | 125 | 125 |
| Axial: | 58 | 480 |
| **Friction force, $W_f$ =** | **17.9 lb** | |
| ***Power:*** | | |
| Power output from gear: | 0.381 hp | |
| Power loss - friction: | 0.171 hp | |
| **Power input:** | **0.552 hp** | |
| ***Efficiency:*** | **69.1 %** | |
| ***Stresses:*** | | |

| Normal pressure angle, $\phi_n$ | | | |
|---|---|---|---|
| 14.5 | 20 | 25 | 30 |
| Lewis form factor, $y$ | | | |
| 0.100 | 0.125 | 0.150 | 0.175 |

***Bending Stress on Gear:***

Enter: Lewis form factor: $y$ = 0.100 ——> 

Normal circular pitch: 0.522 in

Dynamic factor: $K_v$ = 0.979

***Bending stress on gear:*** ***9400 psi*** [Using effective gear face width]

*Allowable stresses-Bronze: Manganese = 17000 psi; Phosphor = 24000 psi*

***Surface Durability: [Hardened steel worm; bronze gear]***

| *Type of bronze:/$D_G$ —>* | *>2.5 in* | *<2.5 in* | *>8 in* | *<8 in* | *>25 in* | *<25 in* |
|---|---|---|---|---|---|---|
| Sand cast: $C_s$ = | 857 | 1000 | | | | |
| Chill cast or forged: $C_s$ = | | | 1093 | 1000 | | |
| Centrifugally cast: $C_s$ = | | | | | 1126 | 1000 |

Enter: Materials factor: $C_s$ = 857 Sand cast

| Gear Ratio: $m_G$ = | 6 to 20 | 20 to 76 | >76 | Actual $m_G$ = 30 |
|---|---|---|---|---|
| Ratio correction factor: $C_m$ = | 0.759 | 0.824 | 0.951 | |
| Enter: $C_m$ = | 0.824 | | | |
| Sliding velocity: | <700 | 700-3000 | >3000 | Actual $v_s$ = 315 |
| Velocity factor: $C_v$ = | 0.466 | 0.498 | 0.763 | |
| Enter: $C_v$ = | 0.466 | | | |

***Rated tangential load: $W_{tR}$ =*** 1193 lb OK For sand cast bronze

***Must be > $W_t$ =*** 480 lb

Can use Manganese bronze based on bending stress in gear.

***Worm diameter is too large.***

## Wormgearing - Design

Problem: 10-24B

### Input Data:

| | | |
|---|---|---|
| Desired output torque: | $T_o$ = | 1200 lb-in |
| Output speed: | $n_G$ = | 20 rpm |
| Velocity Ratio: | VR = | 30 |

### Design Decisions:

| | | |
|---|---|---|
| Diametral pitch: | $P_d$ = | 10 |
| No. of worm threads: | $N_W$ = | 2 |
| Required No. of gear teeth: | $N_G$ = | 60 |
| Specify No. of gear teeth: | $N_G$ = | 60 |
| Normal pressure angle: | $\phi_n$ = | 14.5 degrees |

### Computed Results and Additional Inputs:

| | | |
|---|---|---|
| Actual input speed: | $n_W$ = | 600 rpm |
| Actual velocity ratio: | VR = | 30 |
| Gear pitch diameter: | $D_G$ = | 6 in |
| Specify worm diameter: | $D_W$ = | 1.250 in |
| Actual center distance: | $C$ = | 3.625 in |
| | $C^{0.875}/D_W$ = | 2.47 OK |
| | Should be >1.6 and <3.0 | |
| Circular pitch of gear: | $p_G$ = | 0.314 in |
| Axial pitch of worm: | $p_{xW}$ = | 0.314 in |
| Lead of the worm: | $L$ = | 0.628 in |
| Lead angle: | $\lambda$ = | 9.090 deg |
| Addendum: | $a$ = | 0.100 in |
| Dedendum: | $b$ = | 0.116 in |
| Worm outside diameter: | $D_{oW}$ = | 1.450 in |
| Worm root diameter: | $D_{RW}$ = | 1.019 in |
| Nominal worm face length: | $F_{Wnom}$ = | 2.191 in |
| Gear throat diameter: | $D_{tG}$ = | 6.200 in |
| Nominal gear face width: | $F_{eG}$ = | 0.735 in |
| Max effective gear face width: | $0.67{*}D_W$ = | 0.838 in |
| Effective gear face width: | $F_e$ = | 0.625 in |
| | *[Used given face width]* | |
| Given face width is small; Could use $F$ > | | 0.735 |

### Additional Computed Results:

| | |
|---|---|
| Pitch line speed - Gear: | 31.42 ft/min |
| Sliding velocity $v_s$ = | 199 ft/min |
| Coefficient of friction: | 0.043 If $v_s$ > 10 ft/min |

| **Forces: (lb)** | Gear | Worm |
|---|---|---|
| Tangential: | 400 | 82 |
| Radial: | 106 | 106 |
| Axial: | 82 | 400 |
| **Friction force, $W_f$ =** | **18.3 lb** | |

**Power:**

| | |
|---|---|
| Power output from gear: | 0.381 hp |
| Power loss - friction: | 0.110 hp |
| **Power input:** | **0.491 hp** |

**Efficiency:** **77.6 %**

**Stresses:**

| Normal pressure angle, $\phi_n$ | | | |
|---|---|---|---|
| 14.5 | 20 | 25 | 30 |
| Lewis form factor, $y$ | | | |
| 0.100 | 0.125 | 0.150 | 0.175 |

**Bending Stress on Gear:**

| | |
|---|---|
| Enter: Lewis form factor: $y$ = | 0.100 ———> |
| Normal circular pitch: | 0.310 in |
| Dynamic factor: $K_v$ = | 0.974 |
| ***Bending stress on gear:*** | ***21171 psi*** [Using effective gear face width] |

*Allowable stresses-Bronze: Manganese = 17000 psi; Phosphor = 24000 psi*

**Surface Durability: [Hardened steel worm; bronze gear]**

| *Type of bronze:/$D_G$ —>* | *>2.5 in* | *<2.5 in* | *>8 in* | *<8 in* | *>25 in* | *<25 in* |
|---|---|---|---|---|---|---|
| Sand cast: $C_s$ = | 819 | 1000 | | | | |
| Chill cast or forged: $C_s$ = | | | 1057 | 1000 | | |
| Centrifugally cast: $C_s$ = | | | | | 1111 | 1000 |
| Enter: Materials factor: $C_s$ = | 819 | Sand cast | | | | |

| | | | | |
|---|---|---|---|---|
| Gear Ratio: $m_G$ = | 6 to 20 | 20 to 76 | >76 | Actual $m_G$ = 30 |
| Ratio correction factor: $C_m$ = | 0.759 | 0.824 | 0.951 | |
| Enter: $C_m$ = | 0.824 | | | |
| Sliding velocity: | <700 | 700-3000 | >3000 | Actual $v_s$ = 199 |
| Velocity factor: $C_v$ = | 0.530 | 0.648 | 1.090 | |
| Enter: $C_v$ = | 0.530 | | | |
| ***Rated tangential load: $W_{tR}$ =*** | 937 lb | | OK For sand cast bronze | |
| ***Must be > $W_t$ =*** | 400 lb | | | |

Can use Phosphor bronze based on bending stress in gear.

## Wormgearing - Design

Problem: 10-24A - Revised

| | | |
|---|---|---|
| ***Input Data:*** | | |
| Desired output torque: | $T_o$ = | 1200 lb-in |
| Output speed: | $n_G$ = | 20 rpm |
| Velocity Ratio: | VR = | 30 |
| ***Design Decisions:*** | | |
| Diametral pitch: | $P_d$ = | 6 |
| No. of worm threads: | $N_W$ = | 1 |
| Required No. of gear teeth: | $N_G$ = | 30 |
| Specify: No. of gear teeth: | $N_G$ = | 30 |
| Normal pressure angle: | $\phi_n$ = | 14.5 degrees |
| ***Computed Results and Additional Inputs:*** | | |
| Actual input speed: | $n_W$ = | *600* rpm |
| Actual velocity ratio: | VR = | 30 |
| Gear pitch diameter: | $D_G$ = | 5 in |
| Specify: worm diameter: | $D_W$ = | 1.750 in |
| Actual center distance: | $C$ = | 3.375 in |
| | $C^{0.875}/D_W$ = 1.66 | ***OK*** |
| | Should be >1.6 and <3.0 | |
| Circular pitch of gear: | $p_G$ = | 0.524 in |
| Axial pitch of worm: | $p_{xW}$ = | 0.524 in |
| Lead of the worm: | $L$ = | 0.524 in |
| Lead angle: | $\lambda$ = | 5.440 deg |
| Addendum: | $a$ = | 0.167 in |
| Dedendum: | $b$ = | 0.193 in |
| Worm outside diameter: | $D_{oW}$ = | 2.083 in |
| Worm root diameter: | $D_{RW}$ = | 1.364 in |
| Nominal worm face length: | $F_{Wnom}$ = | 2.582 in |
| Gear throat diameter: | $D_{tG}$ = | 5.333 in |
| Nominal gear face width: | $F_{eG}$ = | 1.130 in |
| Max effective gear face width: | $0.67{*}D_W$ = | 1.173 in |
| Effective gear face width: | $F_e$ = | 1.130 in |
| | | ***[Used nominal face width]*** |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| ***Additional Computed Results:*** | | | | | | | |
| Pitch line speed - Gear: | 26.18 ft/min | | | | | | |
| Sliding velocity $v_s$ = | 276 ft/min | | | | | | |
| Coefficient of friction: | 0.038 | If $v_s$ > 10 ft/min | | | | | |
| ***Forces: (lb)*** | Gear | Worm | | | | | |
| Tangential: | 480 | 65 | | | | | |
| Radial: | 125 | 125 | | | | | |
| Axial: | 65 | 480 | | | | | |
| **Friction force, $W_f$ =** | **19.0 lb** | | | | | | |
| ***Power:*** | | | | | | | |
| Power output from gear: | 0.381 hp | | | | | | |
| Power loss - friction: | 0.159 hp | | | | | | |
| **Power Input:** | **0.540 hp** | | | | | | |
| ***Efficiency:*** | **70.6 %** | | Normal pressure angle, $\phi_n$ | | | | |
| ***Stresses:*** | | | 14.5 | 20 | 25 | 30 | |
| ***Bending Stress on Gear:*** | | | Lewis form factor, $y$ | | | | |
| Enter: Lewis form factor $y$ = | 0.100 | ------> | 0.100 | 0.125 | 0.150 | 0.175 | |
| Normal circular pitch: | 0.521 in | | | | | | |
| Dynamic factor: $K_v$ = | 0.979 | | | | | | |
| ***Bending stress on gear:*** | ***8324 psi*** | [Using effective gear face width] | | | | | |
| *Allowable stresses-Bronze: Manganese = 17000 psi; Phosphor = 24000 psi* | | | | | | | |
| ***Surface Durability: [Hardened steel worm; bronze gear]*** | | | | | | | |
| *Type of bronze:/$D_G$ --->* | *>2.5 in* | *<2.5 in* | *>8 in* | *<8 in* | *>25 in* | *<25 in* | |
| Sand cast: $C_s$ = | 857 | 1000 | | | | | |
| Chill cast or forged: $C_s$ = | | | 1093 | 1000 | | | |
| Centrifugally cast: $C_s$ = | | | | | 1126 | 1000 | |
| Enter: Materials factor $C_s$ = | 857 | Sand cast | | | | | |
| Gear Ratio: $m_G$ = | 6 to 20 | 20 to 76 | >76 | Actual $m_G$ = 30 | | | |
| Ratio correction factor: $C_m$ = | 0.759 | 0.824 | 0.951 | | | | |
| Enter: $C_m$ = | 0.824 | | | | | | |
| Sliding velocity: | <700 | 700-3000 | >3000 | Actual $v_s$ = 276 | | | |
| Velocity factor: $C_v$ = | 0.486 | 0.537 | 0.845 | | | | |
| Enter: $C_v$ = | 0.486 | | | | | | |
| ***Rated tangential load: $W_{tR}$ =*** | 1406 lb | | OK For sand cast bronze | | | | |
| ***Must be > $W_t$ =*** | 480 lb | | | | | | |

Can use Manganese bronze based on bending stress in gear.
***Changed worm diameter to 1.75 in; Changed face width to 1.13 in***

# CHAPTER 11
# KEYS, COUPLINGS, AND SEALS

GENERAL NOTES FOR KEY DESIGN PROBLEMS 1-15:

FOR SAE 1018 CD STEEL: $S_y = 54000$ psi. IF KEY MATERIAL IS WEAKEST -

$$\tau_d = \frac{0.5 S_y}{N} = \frac{0.5(54000)}{3} = 9000 \text{ psi}$$

$$\sigma_d = \frac{S_y}{N} = \frac{54000}{3} = 18000 \text{ psi}$$

$\boxed{1.}$ $D_{SHAFT} = 2.00$ IN; USE 1/2 IN SQ. KEY, SAE 1018 CD

$$L = \frac{2T}{\tau_d D W} = \frac{2(21000 \text{ LB·IN})}{(9000 \text{ LB/IN}^2)(2.00 \text{ IN})(0.5 \text{ IN})} = 4.667 \text{ IN}$$

BUT HUB LENGTH = 4.00 IN. USE SAE 1045 CD; $S_y = 77000$ psi

$\tau_d = 0.5 S_y / N = 0.5(77)/3 = 12.83$ ksi

$$L = \frac{2(21000)}{(12830)(2.00)0.5} = 3.27 \text{ IN}; \text{ USE } L = 3\tfrac{3}{4} = 3.75 \text{ IN.}$$

$\boxed{2.}$ $D_s = 3.60$ IN; USE 7/8 IN SQUARE KEY; SAE 1018 CD

$$L = \frac{2(21000)}{(9000)(3.60)(0.875)} = 1.48 \text{ IN}$$

USE $L = 3.50$ IN TO MORE NEARLY MATCH HUB LENGTH

$\boxed{3.}$ $D_s = 1.75$ IN; USE 3/8 IN. SQ. KEY; SAE 1018 CD; $\tau_d = 9000$ psi

FOR HUB; CLASS 20 CI; $S_u = 20\,000$ psi

FOR BEARING $\sigma_d = \frac{20000}{3} = 6667$ psi (CONSERVATIVE)

BECAUSE COMPRESSIVE STRENGTH OF CI IS MUCH GREATER THAN TENSILE STRENGTH.

SHEAR OF KEY: $L = \frac{2T}{\tau_d D W} = \frac{2(1112)}{(9000)(1.75)(.375)} = 0.377$ IN

BEARING ON HUB: $L = \frac{4T}{\sigma_d D H} = \frac{4(1112)}{(6667)(1.75)(.375)} = 1.02$ IN

USE $L = 1.50$ IN TO MATCH HUB LENGTH.

$\boxed{4.}$ $T = 63000(110)/1700 = 4076$ LB·IN; $D_s = 2.50$ IN; USE 5/8 SQ. KEY

$$L = \frac{2T}{\tau_d D W} = \frac{2(4076)}{(9000)(2.50)(0.625)} = 0.580 \text{ IN}$$

USE $L = 2.50$ IN TO MORE NEARLY MATCH HUB LENGTH.

$\boxed{5.}$ EXPRESS DATA FROM TABLE 11-6 AS $T = KD^2L$
REQ'D $K = T/D^2L$ ($L$ = HUB LENGTH) USE B-FIT

a) PROB. 1 DATA; $T = 21000$; $D = 2.00$ IN, $L = 4.00$ IN.

$$K = \frac{21000}{(2.00)^2(4.0)} = 1313$$ TOO HIGH FOR ANY SPLINE IN TABLE 11-6

b) PROB. 2 DATA: $T = 21\,000$ LB·IN; $D = 3.60$ IN; $L = 4.00$ IN.

$$K = \frac{21000}{(3.60)^2(4.0)} = 405$$ ; USE 16 SPLINES; $K = 521$

c) PROB. 3 DATA: $T = 1112$ LB-IN; $D = 1.75$ IN; $L = 1.75$ IN

$$K = \frac{1112}{(1.75)^2(1.75)} = 208$$ USE 6 SPLINES

d) PROB. 4 DATA: $T = 4076$ LB-IN; $D = 2.50$ IN; $L = 3.25$ IN

$$K = \frac{4076}{(2.50)^2(3.25)} = 201$$ USE 6 SPLINES

$\boxed{6.}$ AT 220 hp: $T_2 = 63000(220)/1700 = 8153$ LB·IN
AT 110 hp: $T_1 = 63000(110)/1700 = 4076$ LB·IN

PIN SHOULD SHEAR AT $T_2$; $\tau_2 = S_{us} = S_u(0.75)$ (SECT. 2-2)

PIN SHOULD NOT YIELD AT $T_1$; $\tau_1 \le S_{ys} = S_y(0.5)$

FOR A GIVEN PIN $d$ AND SHAFT $D$ IN EQ. 11-18

$$.5S_y = \tau_1 = \frac{4T_1}{D\pi d^2} \;;\; \tau_2 = \frac{4T_2}{D\pi d^2} = 0.75 S_u$$

RATIO $$\frac{0.5S_y}{0.75S_u} = \frac{\frac{4T_1}{D\pi d^2}}{\frac{4T_2}{D\pi d^2}} = \frac{T_1}{T_2} \;:\; S_y = \frac{0.75 T_1 S_u}{0.5 T_2}$$

MIN. $S_y = \dfrac{0.75(4076)S_u}{0.5(8153)} = 0.75 S_u$ MOST COLD DRAWN STEELS HAVE $S_y \ge 0.75 S_u$

FOR SAE 1018 CD.
$S_y = 54$ ksi; $S_u = 64$ ksi

$$d = \sqrt{\frac{4T_2}{D\pi\,\tau_2}} = \sqrt{\frac{4(8153)}{(2.50)(\pi)(.75)(64000)}} = 0.294 \text{ in.}$$

AT $T_1 = 4076$ LB·IN.; $\tau = \dfrac{4(4076)}{(2.50)(\pi)(0.294)^2} = 24\,007$ psi $< \dfrac{S_y}{2} = 54000\text{ psi}/2 = 27000$ PSI

**7.** FROM EX.PROB. 12-3: $T = 4168$ LB·IN; $D_Z = 2\frac{1}{4}$ IN; $D_S = 1\frac{1}{2}$ IN. (SPROCKET, WORMGEAR)

SPROCKET: $\frac{1}{2}$ IN SQUARE KEY; SAE 1018 CD

$$L = \frac{2T}{\tau_d DW} = \frac{2(4168)}{(9000)(2.25)(0.5)} = \underline{0.823\ IN} \quad \underline{USE\ L = 1.00\ IN}$$

WORMGEAR: 3/8 SQ. KEY

$$L = \frac{2(4168)}{(9000)(1.50)(0.375)} = \underline{1.647\ IN.} \quad \underline{USE\ L = 1.75\ IN.}$$

**8.**

WOODRUFF KEY 204: NOMINAL $W = \frac{2}{32} = \frac{1}{16}$ IN: NOMINAL $B = \frac{4}{8} = \frac{1}{2}$ IN

ACTUAL DIMS. IN TABLE 11-3

**9.** WOODRUFF KEY 1628: NOM. $W = \frac{16}{32} = \frac{1}{2}$ IN; NOM. $B = \frac{28}{8} = 3\frac{1}{2}$ IN

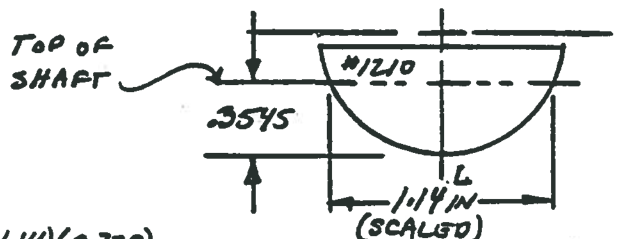
10, 11, 12 ARE DRAWINGS

**13.** $T = FD/2$ : $F = 2T/D$

KEY $W = 3/8$

$A_s = LW$; $\tau = \frac{F}{A_s} = \frac{2T}{DLW}$

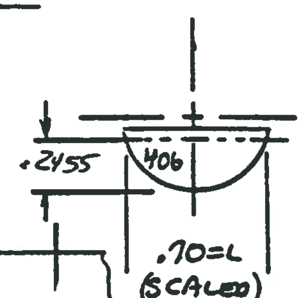
$$\underline{T} = \frac{\tau_d DLW}{2} = \frac{(9000)(1.500)(1.14)(0.375)}{2} = \underline{2886\ LB\cdot IN}$$




**14.**

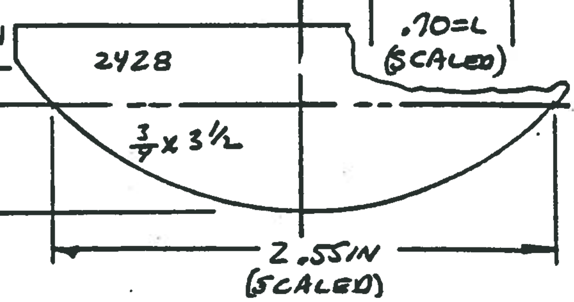
$$\underline{T} = \frac{\tau_d DLW}{2} = \frac{(9000)(.500)(.70)(.125)}{2} = \underline{197\ LB\cdot IN}$$

KEY $W = 1/8$ IN




**15.**

$$\underline{T} = \frac{(9000)(3.25)(2.55)(0.75)}{2} = \underline{27,970\ LB\cdot IN}$$




**19.** DATA FROM (PROB 16) 0.558

a) $T = 139D^2 = 139(1.50)^2$
$= \underline{313\ LB\cdot IN / IN\ OF\ HUB\ L.}$

b) $T = 326D^2 = 326(3.50)^2 =$
$= \underline{3994\ LB\cdot IN / IN\ OF\ HUB\ L.}$ (PROB 17)

c.) $T = 688D^2 = 688(2.500)^2 = \underline{4300\ LB\cdot IN / IN.\ OF\ HUB\ L}$ (PROB 18)

**NOTE: Problems 20-46 call for narrative answers for which the proper information can be found in the text. Guidance is provided below for sections in which additional information can be found.**

20. Section 11-6 includes discussion of applying set screws to transmit torque. A table of approximate holding force capacity vs. set screw size is provided.
21. Press fit is described briefly in Section 11-6. More discussion follows in Chapter 13.
22. Section 11-7 describes both rigid and flexible couplings and compares their performance. Examples of commercially available couplings are shown and described.
23. Section 11-8 contains general information about universal joints.
24. Section 11-9 contains general information about retaining rings and other means of locating machine elements axially on shafts and in other devices. Included are collars, shoulders, spacers, and locknuts.
25. to 38. Section 11-10 contains general information about seals.
39. to 46. Section 11-11 contains general information about seal materials, including elastomers.

    40. to 45. A list of 14 elastomers in included in Section 11-11. Following the list of 14, elastomers, their general performance capabilities are described.

    46. The required conditions for shafts on which elastomeric seals operate are discussed in the last part of Section 11-11. Examples are:

    - Steels, hardened to HRC 30 with tolerances of less than ±0.005 in (0.13 mm) are typically used for shafts on which seals operate. The surface must be free of burrs with a surface finish of 10 to 20 microinches is recommended. Lubrication is recommended.

# CHAPTER 12

# SHAFT DESIGN

## GENERAL NOTES CONCERNING SOLUTIONS TO SHAFT DESIGN PROBLEMS

- Design values for stress concentrations as given in Section 12-4 are used for the initial calculations. <u>These values must be checked once final design details are specified for diameters, fillet radii, and other features.</u>
- Estimates are originally used for the size factors used in calculations because they depend on the shaft sizes that are unknown at the start of a design problem. <u>These values must be checked once final design decisions have been specified.</u>
- The choice of the reliability factor, $C_R$, is a design decision. <u>Other values may be preferred.</u>
- In most cases, the proposed final values for diameters are expected to be safe because trial values are typically conservative and because final specified diameters are typically made to the next larger preferred size according to Appendix Table A-2.
- Final specifications for diameters where bearings are to be mounted must await the selections for suitable bearings that can accommodate the radial and thrust loads applied to them. This process is described in Chapter 14 and the MDESIGN – MOTT software is an excellent tool for making those decisions. The computed 'minimum required diameter' from the shaft design process should be used as to limit the bearing selection to only feasible sizes.

**Shafts with Only Radial Loads Applied to Them:**

Problems P1 through P30 relate to one of the Figures P12-1 through P12-17 showing shafts carrying a variety of combinations of gears, belt sheaves, chain sprockets, and a few other items such as a flywheel and a propeller-type fan. All of these elements apply only radial loads to the shafts on which they are mounted.

- **Problems 1-11 include only forces and torques exerted by gears on shafts. No separate solutions for these problems are included here.**
- **Problems 12-21 include only forces and torques exerted by belt drives and chain drives on shafts. No separate solutions are included here.**
- **Problems 22-30 are comprehensive design problems that use the same shaft assemblies that are used for Problems 1-21. The solutions to these problems include the analyses of forces and torques and should be used as the solutions for problems 1-21.**
- **The parts of the solutions for torques and forces give discrete, single-answers.**
- **The remaining parts of the comprehensive shaft designs include many design decisions and multiple solutions are possible. The given solutions should be considered examples only.**

There are multiple ways in which the problems P1 through P30 may be assigned. The following table may help instructors decide how to assign the problems for student solution and may help students

comprehend how the sets of problems lead to the more general shaft design. Any combination of problems may be chosen.

| Torques and Forces Acting Radial to Shaft | Comprehensive |
|---|---|
| **Figure P12-1:** P1 – Gear *B*; P14 – Sheave *D* | P22 |
| **Figure P12-2:** P2 – Gear *C*; P12 – Sprocket *D*; P13 – Pulley *A* | P23 |
| **Figure P12-3:** P3 – Gear *B*; P15 – Sprocket *C*; P16 – Sheaves *D, E* | P24 |
| **Figure P12-4:** P4 – Gear *A*; P19 – Sprockets *C, D* | P25 |
| **Figure P12-5:** P5 – Gear *D*; P20 – Sheave *A*; P21 – Sprocket *E* | P26 |
| **Figure P12-6:** P6 – Gear *E*; (No separate analysis of Sheave *A*) | P27 (Includes Sheave *A*) |
| **Figure P12-7:** P7 – Gear *C*; P8 – Gear *A* | P28 (Includes Sheaves *D, E*) |
| **Figure P12-9:** P9 – Gear *C*; P10 – Gear *D*; P11 – Gear *F* | P29 (Includes Sheave *B*) |
| **Figure P12-17:** P17 – Sheave *C*; P18 – Pulley *D* | P30 (Includes Fan *A*) |

**Shafts with both Radial and Axial Loads Applied to Them:**

Problems P31 to P34 deal with shafts carrying helical gears and wormgears that produce forces directed axially in addition to radial forces. Solutions are only shown for the comprehensive problems (12-32 and 12-34) in which the details of the analyses of torques and forces are included.

| Torques and Forces Acting Radial and Axial to Shaft | Comprehensive |
|---|---|
| **Figure P12-31:** P31 – Helical Gear *B* | P32 |
| **Figure P12-33:** P33 – Wormgear *C* | P34 (Includes Sheave *A*) |

**Other Comprehensive Design Problems**

Problems 35 to 41 contain a variety of loading situations for which the general solution procedure must be adapted. Some of the problems involve more than one shaft, considering shafts for mating gears and multiple reductions.

**Figure P12-35:** P35 – Double reduction helical drive

**Figure P10-8 in Chapter 10:** P36 – Bevel gear drive

**Figure P12-37:** P37 – Bevel gear drive with two chain sprockets

**Figure P12-38:** P38 – Double reduction spur gear drive; design three shafts.

**Figure P12-39:** P39 – Drive system consisting of an electric motor, a V-belt drive, a double reduction spur gear type reducer, and a chain drive.

**Figure P12-40:** P40 – Shaft with three spur gears

**Figure P12-41:** P41 – Shaft for windshield wiper mechanism with two levers

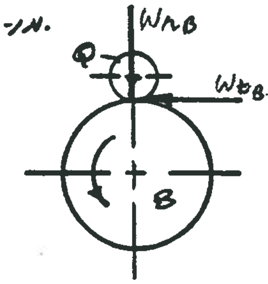
FIGURE P12-1

**22** TORQUE ON GEAR B: $T_B = 63000(30)/550 = 3436$ LB-IN

SAME TORQUE ON SHEAVE D: $T_D = 3436$ LB-IN

TORQUE IN SHAFT: $T_{A-B} = 0$ ; $T_{B-D} = 3436$ LB-IN.

FORCES ON GEAR B:




$$W_{t_B} = \frac{T_B}{R_B} = \frac{3436 \text{ LB-IN}}{8 \text{ IN}} = 430 \text{ LB} \leftarrow = F_{Bx}$$

$$W_{n_B} = W_{tB} \tan 20° = 156 \text{ LB} \downarrow = F_{By}$$

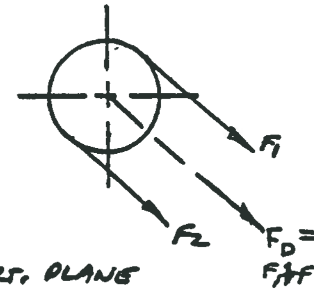
FORCES ON SHEAVE D

$$F_1 - F_2 = \frac{T_D}{R_D} = \frac{3436 \text{ LB-IN}}{5.0 \text{ IN}} = 687 \text{ LB} \quad 40°$$

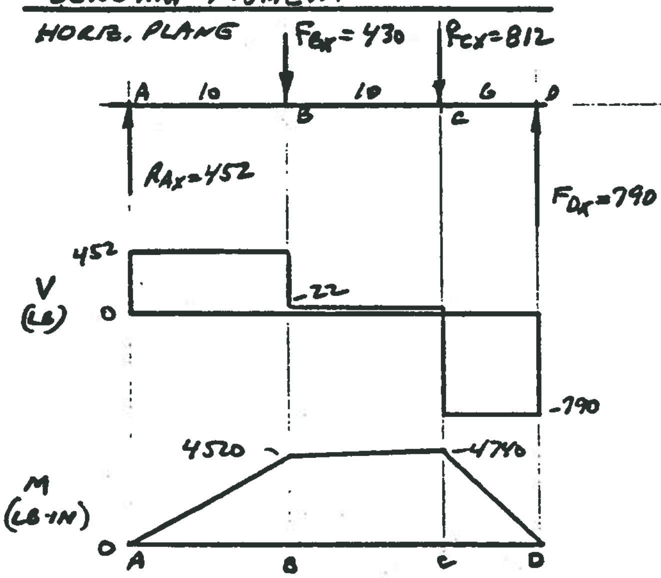
$$F_1 + F_2 = 1.5(F_1 - F_2) = 1.5(687) = 1031 \text{ LB} = F_D$$

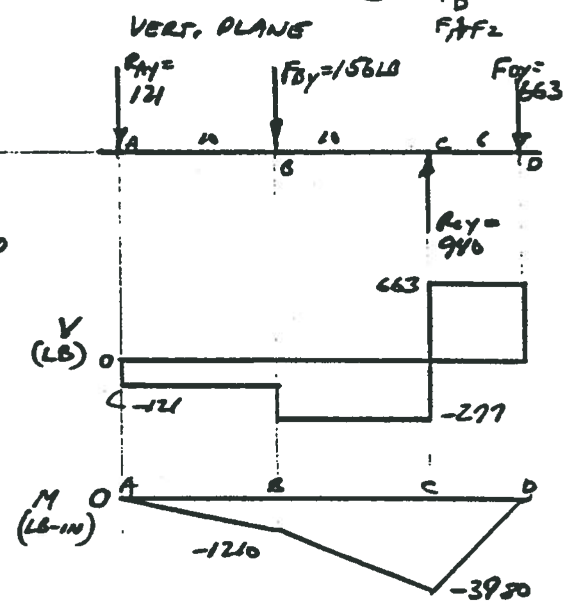
$$F_{Dx} = F_D \cos 40° = 790 \text{ LB} \rightarrow$$

$$F_{Dy} = F_D \sin 40° = 663 \text{ LB} \downarrow$$




BENDING MOMENT DIAGRAMS




$$M_B = \sqrt{4520^2 + 1210^2} = 4679 \text{ LB-IN}$$

$$M_C = \sqrt{4740^2 + 3980^2} = 6189 \text{ LB-IN}$$

$T_C = 3436$ LB-IN

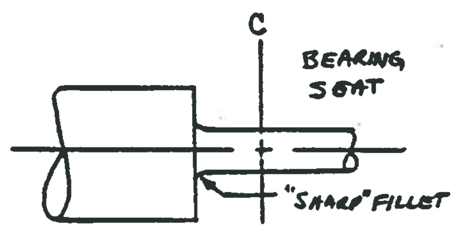
ASSUME $K_t = 2.5$ AT LEFT OF C

SAE 1040 CD: $S_u = 80$ ksi; $S_y = 71$ ksi

$S_n = 30$ ksi (FIG. 5-8 MACHINED) $C_R = 0.81$, $C_s = 0.80$

$S_n' = C_s C_R S_n = (0.80)(0.81)(30) = 19\,440$ psi

USE N = 3




DESIGN GEOMETRY AT C

$$D_C = \left[\frac{32(3)}{\pi}\sqrt{\left[\frac{2.5(6189)}{19\,440}\right]^2 + \frac{3}{4}\left[\frac{3436}{71000}\right]^2}\right]^{1/3} = 2.90 \text{ IN.}$$

SPECIFY D = 3.00 IN.
$C_s = 0.78$ OK

23

FIGURE P12-2

TORQUE ON PULLEY A: $T_A = 63000(10)/200 = 3150$ LB-IN

" ON GEAR C: $T_C = 63000(6)/200 = 1890$ LB-IN

" ON SPROCKET D: $T_D = 63000(4)/200 = 1260$ LB-IN

TORQUE DISTRIBUTION IN SHAFT: $T_{A-C} = 3150$ LB-IN; $T_{C-D} = 1260$ LB-IN; $T_{D-E} = 0$

FORCES ON PULLEY A:

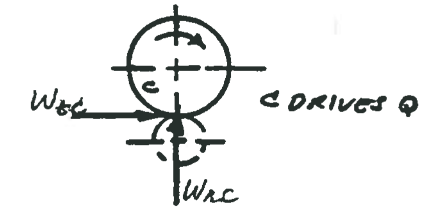
$F_1 - F_2 = T_A/R_A = 3150/10 = 315$ LB

$F_1 + F_2 = 2.0(F_1 - F_2) = 2.0(315) = 630$ LB $\downarrow = F_{Ay}$; $F_{Ax} = 0$

FORCES ON GEAR C:

$W_{tC} = T_C/R_C = \dfrac{1890 \text{ LB-IN}}{5.0 \text{ IN}} = 378$ LB $\rightarrow = F_{Cx}$

$W_{NC} = W_{tC} \tan 20° = 138$ LB $\uparrow = F_{Cy}$

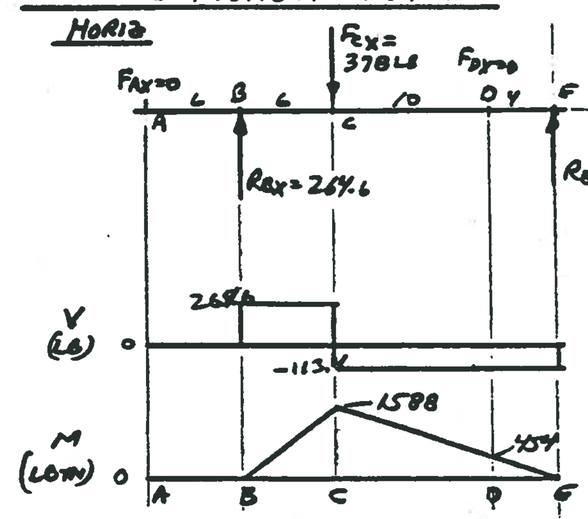


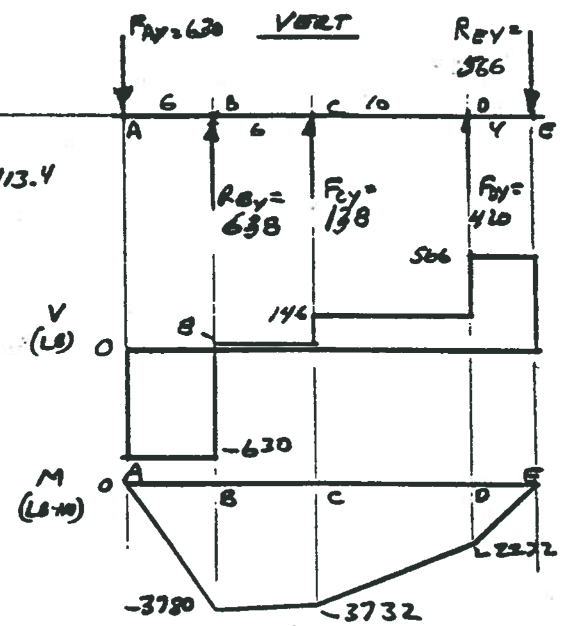

FORCES ON SPROCKET D:

$F_1 = F_D = F_{Dy} = \dfrac{T_D}{R_D} = \dfrac{1260}{3} = 420$ LB $\uparrow$

$F_{Dx} = 0$

BENDING MOMENT DIAGRAMS




$M_B = 3780$ LB-IN

$M_C = \sqrt{1588^2 + 3732^2} = 4056$ LB-IN

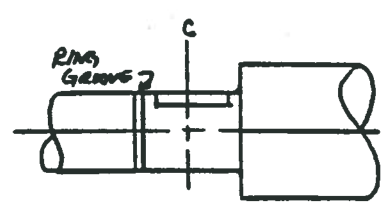
$M_D = \sqrt{454^2 + 2272^2} = 2317$ LB-IN

$T_{C_L} = 3150$ LB-IN TO LEFT; $K_t = 3.0$ RING GR.

$T_{C_R} = 1260$ LB-IN TO RIGHT; $K_t = 2.0$ KEY

SAE 1117 CD: $S_u = 69$ KSI; $S_y = 57$ KSI

$S_n = 26$ KSI; $S_n' = C_s C_R S_n = (0.85)(0.81)(26) = 17.9$ KSI




$$D_{C_L} = \left[\frac{32(3)}{\pi}\sqrt{\left(\frac{3.0(4056)}{17900}\right)^2 + \frac{3}{4}\left(\frac{3150}{57000}\right)^2}\right]^{1/3} = 2.75 \text{ IN.}$$

INCREASE BY 6% AT GROOVE $D \approx 1.06(2.75) = 2.92$ IN.

DESIGN AT C

SPECIFY $D_C = 3.00$ IN

$C_s = 0.78$ OK

FIGURE P12-3

24 TORQUE ON GEAR B: $T_B = 63000(5)/480 = 656$ LB·IN

" ON SHEAVES D AND E: $T_D = T_E = 63000(3)/480 = 394$ LB·IN

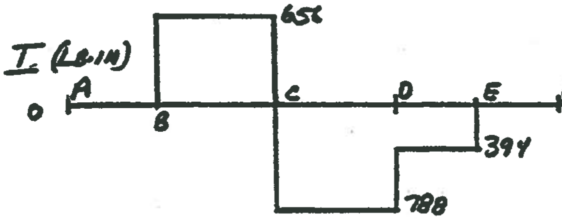
" ON SPROCKET C: $T_C = 63000(11)/480 = 1444$ LB·IN

TORQUE DISTRIBUTION IN SHAFT:

$T_{AB} = 0$ ; $T_{B-C} = 656$ LB·IN

$T_{CD} = 788$ LB·IN

$T_{DE} = 394$ LB·IN ; $T_{EF} = 0$




FORCES ON GEAR B:

$W_{tB} = T_B/R_B = 656/1.5 = 437$ LB $\longrightarrow = F_{Bx}$

$W_{nB} = W_{tB} \tan 20° = 159$ LB $\uparrow = F_{By}$

FORCES ON SPROCKET C: $F_C = T_C/R_C = 1444/5.0 = 289$ LB 15°

$F_{Cx} = F_C \sin 15° = 289 \sin 15° = 75$ LB $\longleftarrow$

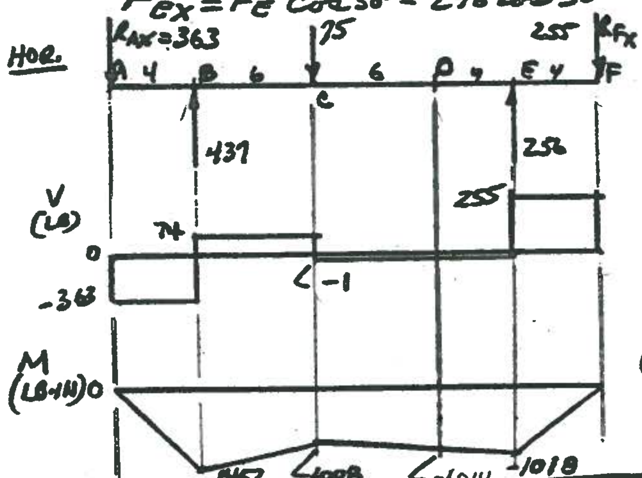
$F_{Cy} = F_C \cos 15° = 289 \cos 15° = 279$ LB $\downarrow$

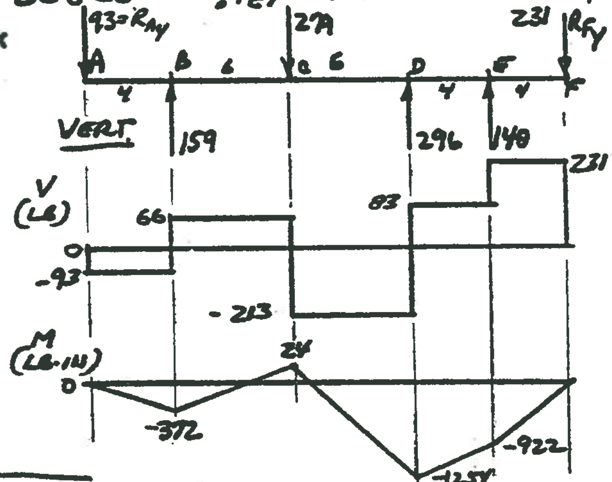
FORCES ON SHEAVE D: $F_1 - F_2 = T_D/R_D = 394/2.0 = 197$ LB

$F_D = F_1 + F_2 = 1.5(F_1 - F_2) = 1.5(197) = 296$ LB $\uparrow = F_{Dy}$ : $F_{Dx} = 0$

FORCES ON SHEAVE E: $F_E = F_D = 296$ LB 30

$F_{Ex} = F_E \cos 30° = 296 \cos 30° = 256$ LB $\longrightarrow$ ; $F_{Ey} = 296 \sin 30° = 148$ LB $\uparrow$



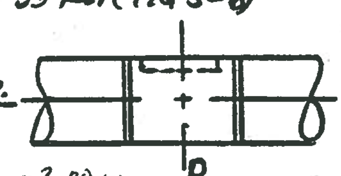

WORST CASE AT D: $M_D = \sqrt{1014^2 + 1254^2} = 1613$ LB·IN

$T_D = 788$ LB-IN (AT D TO LEFT) ; $K_t = 3.0$ RET. RING

SAE 1137 OQT 1300; $S_u = 87$ ksi ; $S_y = 60$ ksi; $S_n = 33$ ksi (FIG 5-8)

$S_n' = (0.9)(0.81)(33) = 24.1$ ksi ($C_S = 0.80$ EST.)

$$D_D = \left[\frac{32(3)}{\pi}\sqrt{\left(\frac{3.0(1613)}{24100}\right)^2 + \frac{3}{4}\left(\frac{788}{60000}\right)^2}\right]^{1/3} = 1.83 \text{ IN.}$$




INCREASE BY 6% ; $D_D = 1.06(1.83) = 1.94$ IN.; SPECIFY $D_D = 2.00$ IN

CHECK: $C_S = 0.81$; $S_n' = 21\,700$ psi; $D_{D\,REQ} = 2.01$ IN; ACCEPTABLE TO USE $D_D = 2.00$ IN

25 FIGURE P12-4

TORQUE ON GEAR A: $T_A = 63000(40)/120 = 21000$ LB·IN

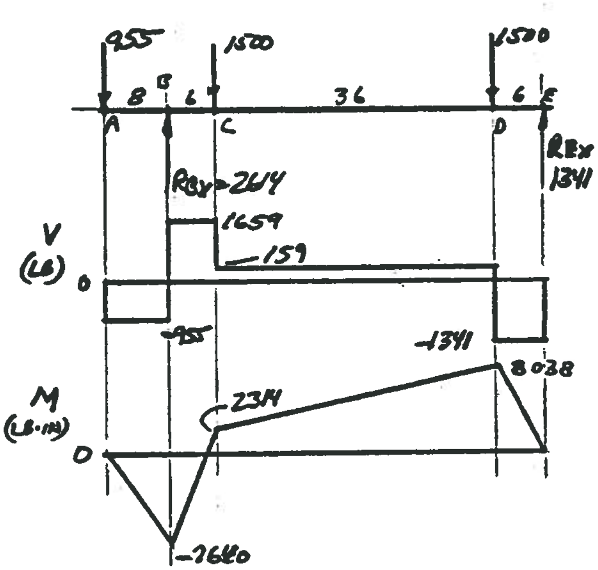
" " ON SPROCKETS C & D: $T_C = T_D = 63000(20)/120 = 10500$ LB·IN

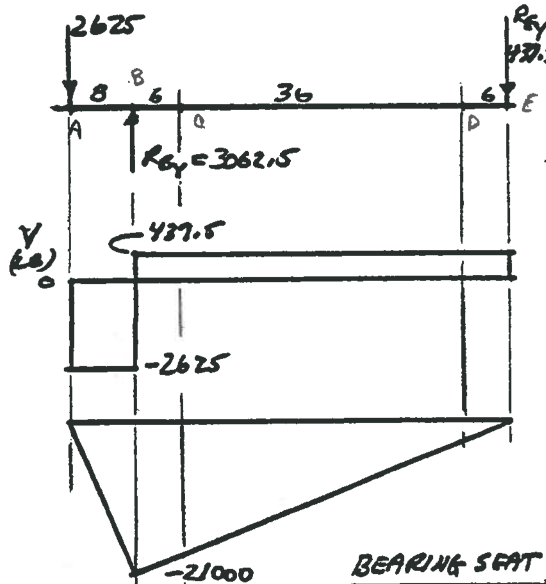
TORQUE IN SHAFT: $T_{A-C} = 21000$ LB·IN; $T_{CD} = 10500$ LB·IN; $T_{D-E} = 0$

FORCES ON GEAR A: $W_{tA} = T_A/R_A = 21000/8 = 2625$ LB $\downarrow = F_{AY}$

$W_{rA} = W_{tA} \tan 20° = 955$ LB $\leftarrow = F_{AX}$

FORCES ON SPROCKETS C & D: $F_{Cx} = F_{Dx} = T/R = 10500/7 = 1500$ LB $\leftarrow$




AT B: $T = 21000$ LB·IN

$M = \sqrt{7640^2 + 21000^2} = 22347$ LB·IN

TO RIGHT OF B: $K_t \cong 2.5$ - SMALL FILLET RAD.

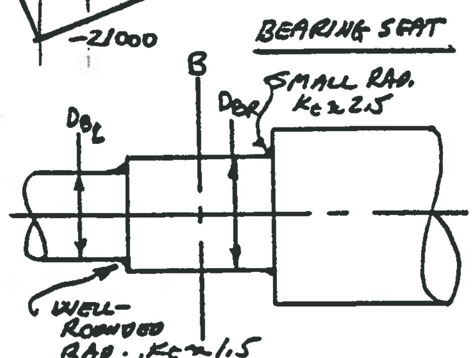
$$D_{BR} = \left[\frac{32(3)}{\pi}\sqrt{\left(\frac{2.5(22347)}{13400}\right)^2 + \frac{3}{4}\left(\frac{21000}{51000}\right)^2}\right]^{1/3}$$

$D_{BR} = 5.05$ IN. SPECIFY 5.20 IN $C_s = 0.73$ OK

TO LEFT OF B: $K_t \cong 1.5$ - WELL RND. FILLET

$$D_{BL} = \left[\frac{32(3)}{\pi}\sqrt{\left(\frac{1.5(22347)}{13400}\right)^2 + \frac{3}{4}\left(\frac{21000}{51000}\right)^2}\right]^{1/3}$$

$D_{BL} = 4.27$ IN. SPECIFY 4.50 IN $C_s = 0.74$ OK




SAE 1020 CD: $S_y = 51000$ PSI

$S_u = 61000$ PSI;

$S_n = 22000$ PSI (FIG 5-8)

USE $C_s = 0.75$ - LARGE DIA.

$S_n' = (0.81)(0.75)(22000) = 13400$ PSI
$\quad\quad\quad C_R \quad C_s$

FIGURE P12-5

26 TORQUE ON SHEAVE A: $T_A = 63000(10)/240 = 2625$ LB·IN $= T_{AD}$ IN SHAFT

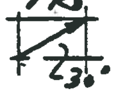
TORQUE ON GEAR D: $T_D = 63000(15)/240 = 3938$ LB·IN

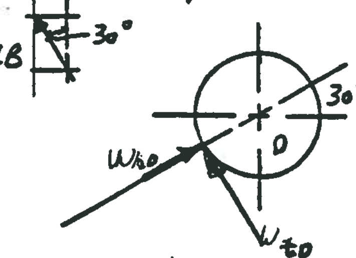
TORQUE ON SPROCKET E: $T_E = 63000(5)/240 = 1313$ LB·IN $= T_{DE}$ IN SHAFT

FORCES ON SHEAVE A: $F_1 - F_2 = T_A/R_A = 2625/6 = 438$ LB ; $F_{Ax} = 0$

$F_A = F_{Ay} = F_1 + F_2 = 1.5(F_1 - F_2) = 1.5(438) = 657$ LB ↓

FORCES ON GEAR D: $W_{tD} = T_D/R_D = 3938/4 = 985$ LB
$W_{RD} = W_{tD} \tan 20° = 358$ LB

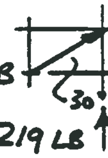




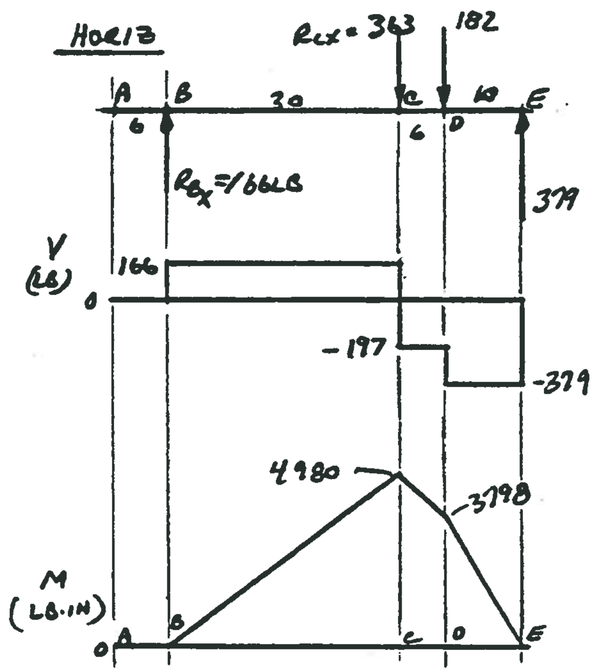
$F_{Dx} = 985 \sin 30° - 358 \cos 30° = 182$ LB ←

$F_{Dy} = 985 \cos 30° + 358 \sin 30° = 1032$ LB ↑

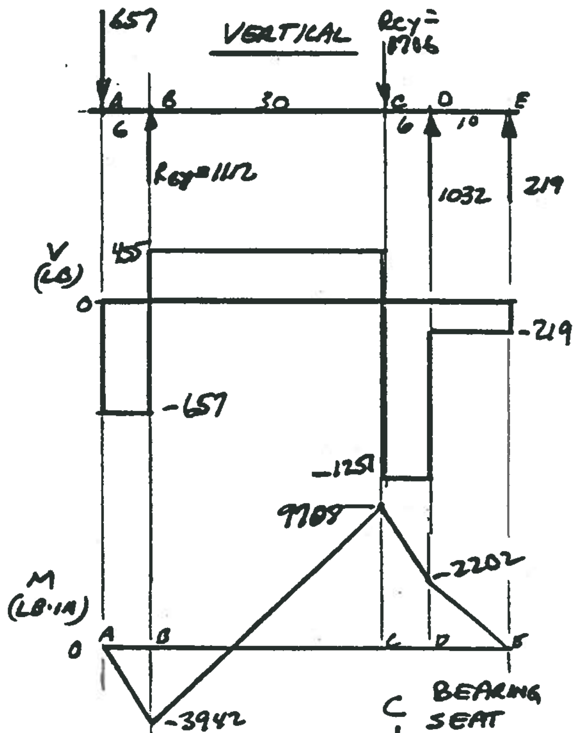
FORCES ON SPROCKET E: $F_E = T_E/R_E = 1313/3 = 438$ LB



$F_{Ex} = F_E \cos 30° = 379$ LB → ; $F_{Ey} = F_E \sin 30° = 219$ LB ↑









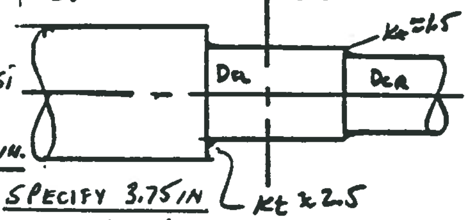
AT C: $T = 2625$ LB·IN ; USE $N = 4$ SHOCK

$M = \sqrt{4980^2 + 9708^2} = 10911$ LB·IN

CHOOSE 1137 CD – $S_y = 82000$ PSI

$S_u = 98$ KSI ; $S_n = 37$ KSI ; $S_n' = 0.8(0.81)(37) = 24.0$ KSI

$$D_{CL} = \left[\frac{32(4)}{\pi}\sqrt{\left(\frac{2.5(10911)}{24000}\right)^2 + \frac{3}{4}\left(\frac{2625}{82000}\right)^2}\right]^{1/3} = 3.59 \text{ IN.}$$





SPECIFY 3.75 IN

$C_s = 0.76$ OK

WITH $K_t = 1.5$, $D_{CR} = 3.03$ IN. SPECIFY 3.20 IN

$C_s = 0.77$ OK

FIGURE P12-6

27 TORQUE ON SHEAVE A AND GEAR E: $T = 63000(20)/310 = 4065$ LB·IN

FORCES ON SHEAVE A: $F_1 - F_2 = T_A/R_A = 4065/11 = 370$ LB

$F_A = F_1 + F_2 = 1.5(F_1 - F_2) = 554$ LB.

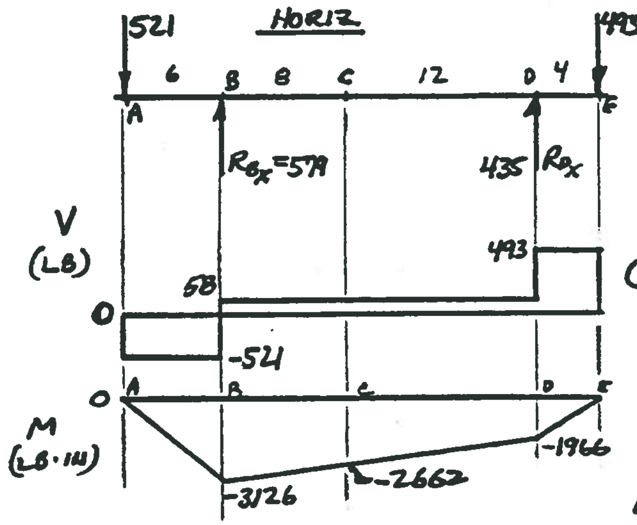

$F_{Ax} = F_A \cos 20° = 521$ LB ; $F_{Ay} = F_A \sin 20° - 120$ LB $= 189$ LB ↑ $-120$ LB ↓ =

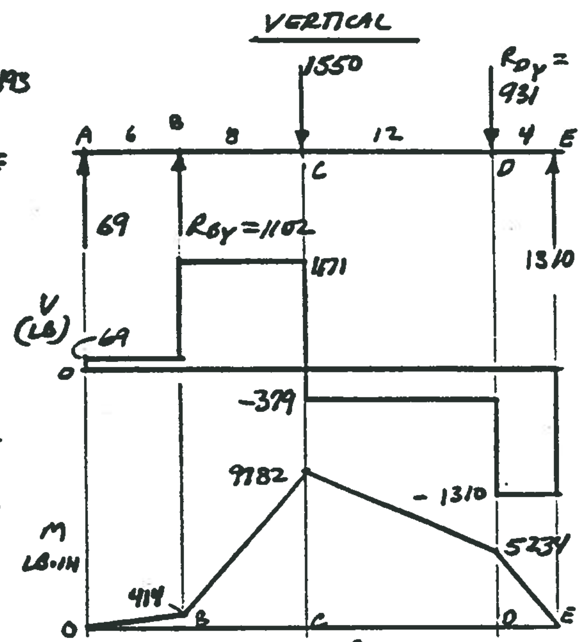
$F_{Ay} = 69$ LB ↑

FORCES ON GEAR E: $W_{t_E} = T/R_E = 4065/3 = 1355$ LB ↑

$W_{r_E} = W_{t_E} \tan 20° = 493$ LB → $= F_{Ex}$

$F_{Ey} = F_{tE} - 45 = 1310$ LB ↑




AT C: $T = 4065$ LB·IN

$M = [2662^2 + 9782^2]^{1/2} = 10138$ LB·IN

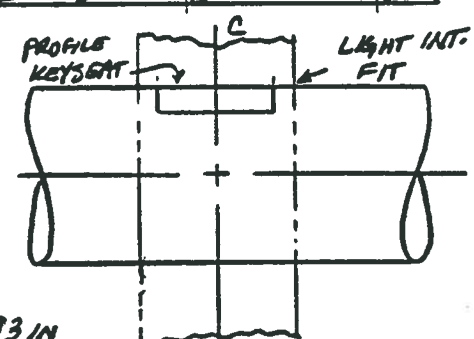
USE $K_t = 2.0$ ; $N = 3$

SAE 1050 CD: $S_y = 84$ ksi

$S_u = 100$ ksi; $S_n = 38$ ksi

$S_n' = 0.8(0.81)(38) = 24.6$ ksi

$C_s \quad C_R$




$$D_C = \left[\frac{32(3)}{\pi}\sqrt{\left(\frac{2.0(10138)}{24600}\right)^2 + \frac{3}{4}\left(\frac{4065}{84000}\right)^2}\right]^{1/3} = 2.93 \text{ IN}$$

SPECIFY $D_C = 3.00$ IN $C_s = 0.78$ OK

28 TORQUE ON GEAR A: $T_A = 63000(30)/480 = 3938$ LB·IN

TORQUE ON GEAR C: $T_C = 63000(50)/480 = 6563$ LB·IN

TORQUE ON SHEAVES D AND E: $T_D = T_E = 63000(10)/480 = 1313$ LB·IN

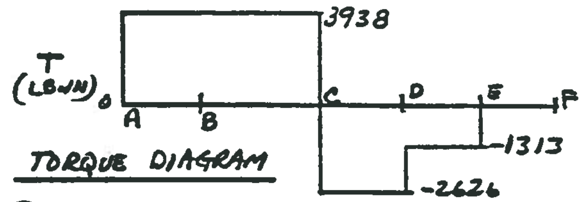
TORQUE IN SHAFT: $T_{AC} = 3938$ LB·IN ; $T_{C\text{-}D} = 2626$ LB·IN; $T_{D\text{-}E} = 1313$ LB·IN

$T_{E\text{-}F} = 0$

(CONTINUED ON NEXT PAGE)

28 (FIGURE P12-7)

(CONTINUED)




TORQUE DIAGRAM

FORCES ON GEAR A:

$W_{t_A} = T_A/R_A = 3938/2.5 = 1575\ LB \uparrow = F_{AY}$

$W_{rA} = W_{tA} \tan 20° = 1575 \tan 20° = 573\ LB \rightarrow = F_{AX}$

FORCES ON GEAR C: $W_{tC} = T_C/R_C = 6563/5 = 1313\ LB \rightarrow = F_{CX}$

$W_{rC} = W_{tC} \tan 20° = 1313 \tan 20° = 478\ LB \uparrow = F_{CY}$

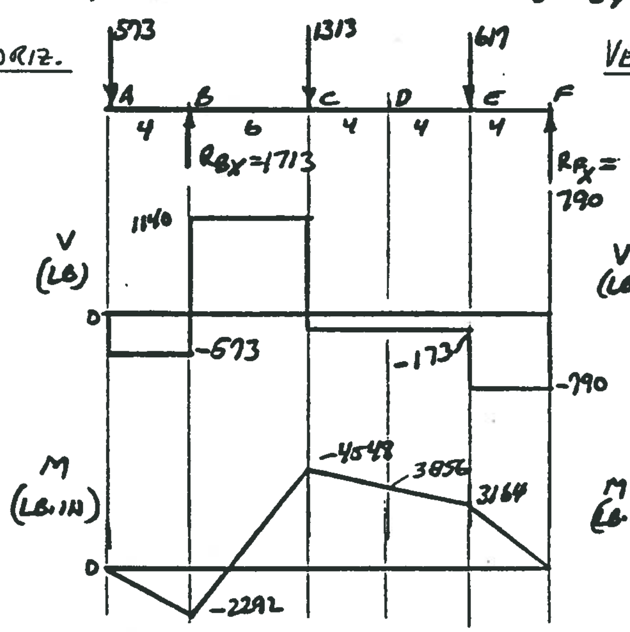
FORCES ON SHEAVE D: $F_1 - F_2 = T_D/R_D = 1313/3 = 438\ LB$

$F_D = F_{DY} = F_1 + F_2 = 1.5(F_1 - F_2) = 1.5(438) = 657\ LB \uparrow \ ; F_{DX} = 0$

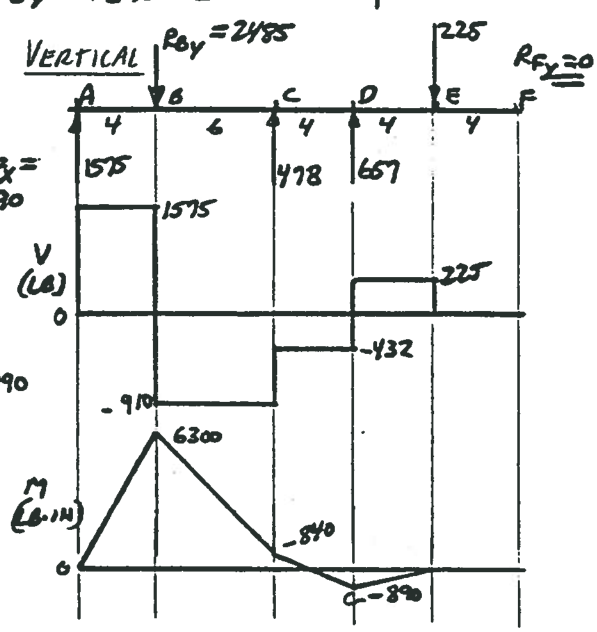
FORCES ON SHEAVE E: $F_E = F_D = 657\ LB$ 20°

$F_{EX} = F_E \cos 20° = 617\ LB \longrightarrow \ ; \ F_{EY} = F_E \sin 20° = 225\ LB \downarrow$

HORIZ.




VERTICAL




AT B: $T = 3938\ LB \cdot IN$ ; $N = 3$

$M_B = \sqrt{(2292)^2 + (6300)^2} = 6704\ LB \cdot IN$

$$D_{BR} = \left[\frac{32(3)}{\pi}\sqrt{\left(\frac{2.5(6704)}{35\,800}\right)^2 + \frac{3}{4}\left(\frac{3938}{133\,000}\right)^2}\right]^{1/3} = 2.43\ IN$$

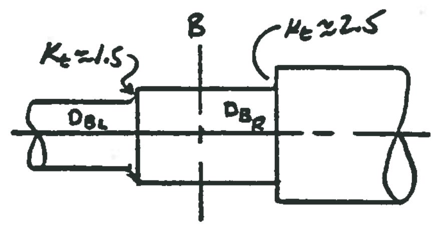
SPECIFY D = 2.50 IN

$C_S = 0.78$ OK

$D_{BL} = 2.05\ IN$ WITH $K_t = 1.5$

SPECIFY D = 2.20 IN

$C_S = 0.80$ OK




SAE 3140 OQT 1000

$S_y = 133\ ksi$ ; $S_u = 152\ ksi$

$S_n = 52\ ksi$ ; $S_n' = C_s C_R S_n$

$S_n' = (0.85)(0.81)(52) = 35.8\ ksi$

$\boxed{29}$ FIGURE P12-9

TORQUE ON SHEAVE B: $T_B = 63000(2.5)/220 = 716$ LB·IN

" ON GEARS C AND F: $T_C = T_F = 63000(5)/220 = 1432$ LB·IN

" ON GEAR D: $T_D = 63000(12.5)/220 = 3580$ LB·IN

TORQUE IN SHAFT: $T_{A-B} = 0$; $T_{BC} = 716$ LB·IN; $T_{CD} = 2148$ LB·IN

$T_{D-F} = 1432$ LB·IN

FORCES ON SHEAVE B: $F_1 - F_2 = T_B/R_B = 716/3 = 239$ LB

$F_B = F_1 + F_2 = 1.5(F_1 - F_2) = 1.5(239) = 358$ LB  30°

$F_{Bx} = F_B \sin 30° = 179$ LB ←; $F_{By} = F_B \cos 30° = 310$ LB ↓

FORCES ON GEAR C: $W_{tc} = T_C/R_C = 1432/3 = 477$ LB ← $= F_{cx}$

$W_{rc} = W_{tc} \tan 20° = (477)\tan 20° = 174$ LB ↓ $= F_{cy}$

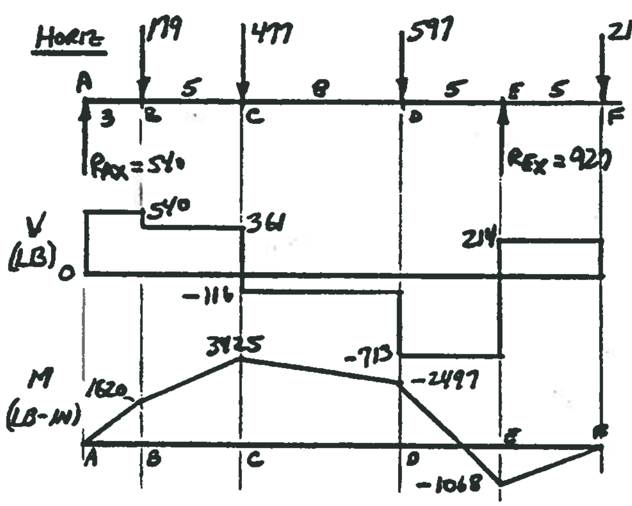
FORCES ON GEAR D: $W_{tD} = T_D/R_D = 3580/6 = 597$ LB ← $= F_{Dx}$

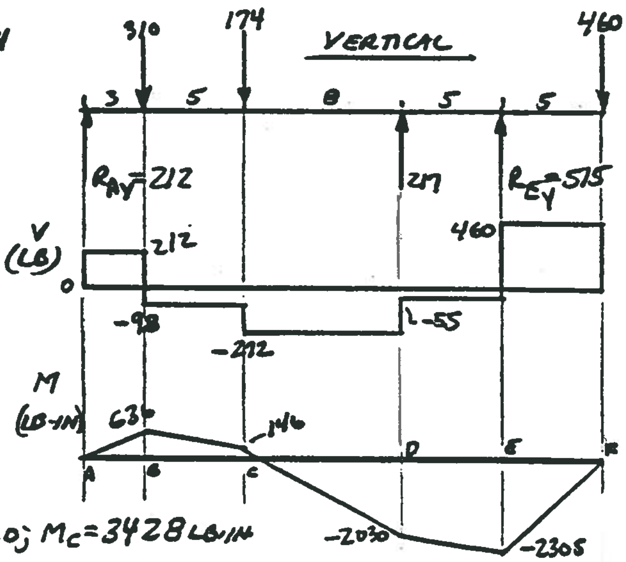
$W_{rD} = W_{tD} \tan 20° = (597)\tan 20° = 217$ LB ↑ $= F_{Dy}$

FORCES ON GEAR F: $W_{tF} = W_{tc} = 477$ LB  45° ‖ $W_{rF} = W_{rc} = 174$ LB  45°

$F_{Fx} = \underleftarrow{477 \sin 45°} - \underrightarrow{174 \cos 45°} = 214$ LB ←

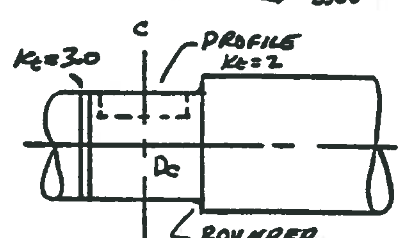
$F_{Fy} = 477 \cos 45° + 174 \sin 45° = 460$ LB ↓




AT C: $T = 2148$ LB·IN TO RIGHT, $K_t = 2.0$; $M_C = 3428$ LB·IN

$T = 716$ LB·IN TO LEFT, $K_t = 3.0$ " "




$$D_{C_R} = \left[\frac{32(3)}{\pi}\sqrt{\left(\frac{2.0(3428)}{15100}\right)^2 + \frac{3}{4}\left(\frac{2148}{51000}\right)^2}\right]^{1/3} = 2.40 \text{ IN}$$

$$D_{C_L} = \left[\frac{32(3)}{\pi}\sqrt{\left(\frac{3.0(3428)}{15100}\right)^2 + \frac{3}{4}\left(\frac{716}{51000}\right)^2}\right]^{1/3} = 2.75 \text{ IN}$$

CRITICAL

SAE 1020 CD: $S_y = 51$ KSI; $S_u = 61$ KSI

$S_n = 22$ KSI

$S_n' = (.85)(.81)(22) = 15.1$ KSI

$C_s$  $C_R$

INCREASE BY 6%: $D_c = 1.06(2.75)$

$D_c = 2.91$ IN

SPECIFY $D_c = 3.00$ IN $C_s = 0.78$ OK

30 FIGURE P12-17

TORQUE AT FAN: $T_A = 63000(12)/475 = 1592$ LB·IN

" ON PULLEY D: $T_D = 63000(3.5)/475 = 464$ LB

" ON SHEAVE C: $T_C = 63000(15.5)/475 = 2056$ LB·IN

TORQUE IN SHAFT: $T_{AC} = 1592$ LB·IN; $T_{CD} = 464$ LB·IN; $T_{DE} = 0$

FORCES ON A: $F_{Ax} = 0$; $F_{Ay} = 34$ LB ↓ THE FAN WOULD ALSO PRODUCE AN AXIAL THRUST FORCE BUT ITS EFFECT ON SHAFT DIA. IS SMALL.

FORCES ON C: (V-BELT)

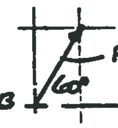
$F_1 - F_2 = T_C/R_C = 2056/5.0 = 411$ LB

$F_C = F_{Cx} = F_1 + F_2 = 1.5(F_1 - F_2) = 1.5(411) = 617$ LB ←; $F_{Cy} = 0$

FORCES ON D: (FLAT BELT)

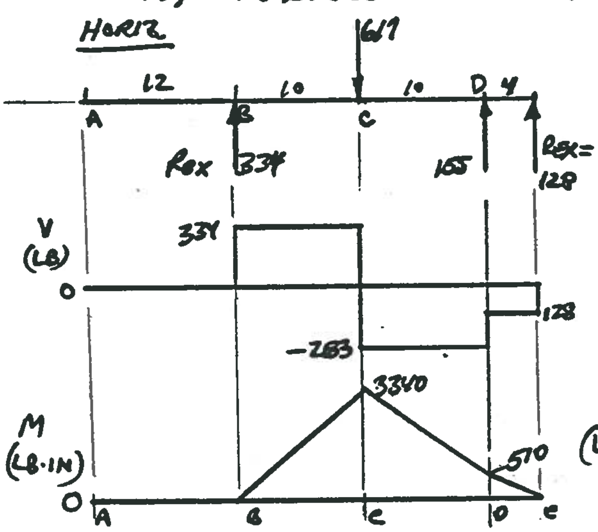
$F_1 - F_2 = T_D/R_D = 464/3 = 155$ LB

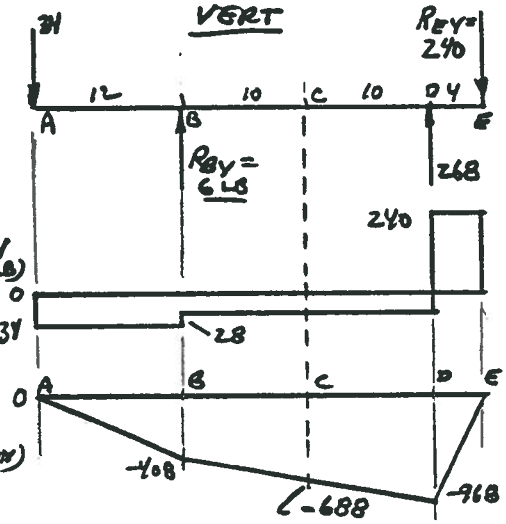
$F_D = F_1 + F_2 = 2.0(F_1 - F_2) = 2.0(155) = 310$ LB




$F_{Dx} = F_D \cos 60° = 155$ LB →

$F_{Dy} = F_D \sin 60° = 268$ LB ↑




AT C: $T_{C\,LEFT} = 1592$ LB·IN ; $T_{C\,RIGHT} = 464$ LB·IN

$M_C = \sqrt{3340^2 + 688^2} = 3410$ LB·IN

AT C & TO LEFT: $K_t = 2.0$ KEYSEAT

$$D = \left[\frac{32(3)}{\pi}\sqrt{\left(\frac{2.0(3410)}{.00}\right)^2 + \frac{3}{4}\left(\frac{1592}{90000}\right)^2}\right]^{1/3} = 2.00 \text{ IN}$$

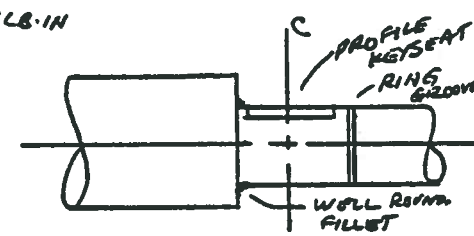
AT RIGHT OF C: $K_t = 3.0$ – RING GROOVE

$$D = \left[\frac{32(3)}{\pi}\sqrt{\left(\frac{3.0(3410)}{00}\right)^2 + \frac{3}{4}\left(\frac{464}{90000}\right)^2}\right]^{1/3} = \underline{2.29}$$

CRITICAL

INCREASE BY 6%: $D = 1.06(2.29) = \underline{2.42 \text{ IN.}}$ MIN

SPECIFY $\underline{D = 2.50 \text{ IN}}$ $\underline{C_S = 0.79}$ $\underline{OK}$




SAE 1144 CD: $S_y = 90$ KSI

$S_u = 100$ KSI; $S_n = 38$ KSI

$S_n' = (0.85)(0.81)(38) = 26.2$ KSI

$C_S$ $C_R$

FIGURE P12-31

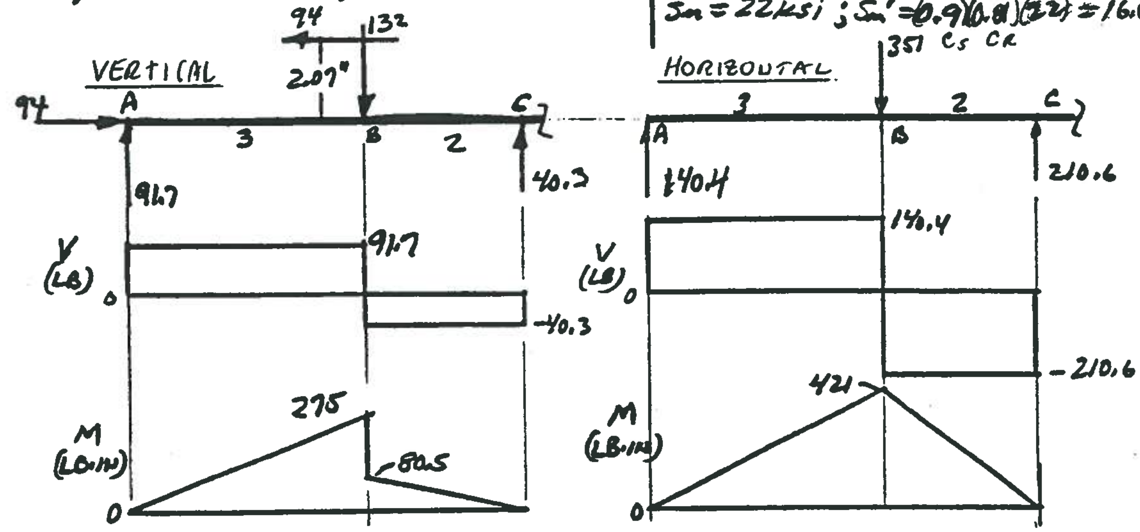
**31 AND 32**

TORQUE ON GEAR B AND IN SHAFT FROM B TO COUPLING:

$T = 63000(7.5)/650 = 727$ LB·IN

USE SAE 1020 CD: $S_y = 51$ ksi; $S_u = 61$ ksi

$S_n = 22$ ksi; $S_n' = (0.9)(0.81)(22) = 16.0$ ksi




AT B: $M = \sqrt{275^2 + 421^2} = 503$ LB·IN; USE $N = 3$; $K_t = 2.0$ (KEYSEAT)

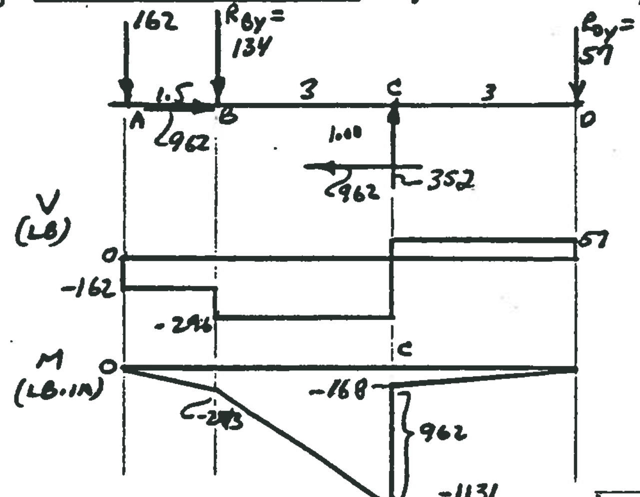
$$D_B = \left[\frac{32(3)}{\pi}\sqrt{\left(\frac{2.0(503)}{16000}\right)^2 + \frac{3}{4}\left(\frac{727}{51000}\right)^2}\right]^{1/3} = 1.25\ \text{IN} \quad C_s > 0.9 \quad \text{OK}$$

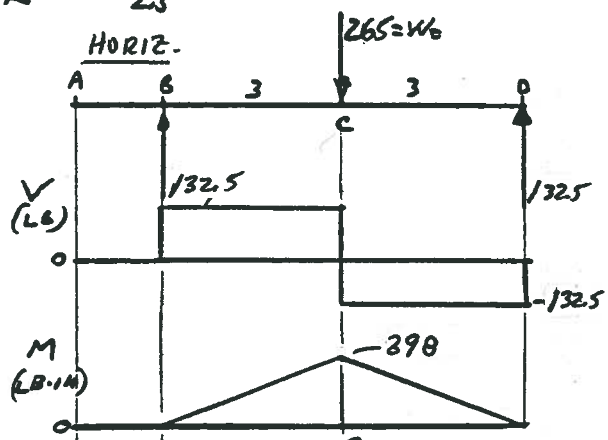
SPECIFY

**33 AND 34**

TORQUE $= 63000(7.5)/1750 = 270$ LB·IN

FORCES ON SHEAVE: $F_{Ay} = 1.5(F_1 - F_2) = \frac{1.5T}{R} = \frac{1.5(270)}{2.5} = 162$ LB




$M_c = \sqrt{1131^2 + 398^2} = 1199$ LB·IN

ASSUME TORQUE IS NEGLIGIBLE; USE $N = 4$

$$\sigma = \frac{K_t M}{S} = \frac{1.5(1199)}{\pi(1.614)^3/32} = 4357\ \text{psi} = \frac{S_n'}{N}$$

$S_n' = C_s C_R S_n = (0.83)(0.81) S_n = 0.67 S_n$

REQD $S_n' = N\sigma = 4(4357) = 17428$ psi $= 0.67 S_n$

REQD $S_n = S_n'/0.67 = 26000$ PSI

FROM FIG. 5-8; $S_u \approx 68$ KSI

SPECIFY SAE 1040 CD; $S_u = 80$ ksi; $S_y = 70$ ksi

$S_n = 30$ ksi; $S_n' = 0.67(30) = 20.1$ ksi

CHECK WITH EQ. 12-24 WITH M AND T.

$$D_{c\,min} = \frac{32(4)}{\pi}\sqrt{\left(\frac{(1.5)(1199)}{20100}\right)^2 + \frac{3}{4}\left(\frac{270}{70000}\right)^2} = 1.54\ \text{IN.} \quad \text{OK}$$

THIS IS LESS THAN ACTUAL $D_c = 1.614$ IN

FIGURE P12-35

35 TORQUES ON GEARS AND IN SHAFTS:

GEAR P, SHAFT 1: $T_1 = 63000(5)/1800 = 175$ LB·IN

GEARS B & C, SHAFT 2: $T_2 = 63000(5)/900 = 350$ LB·IN

GEAR Q, SHAFT 3: $T_3 = 63000(5)/300 = 1050$ LB·IN

SHAFT 2: (SEE CH. 10) $\phi_n = 14\tfrac{1}{2}°$; $\psi = 45°$

GEAR B

$W_{tB} = T_2/R_B = 350/1.5 = 233$ LB → (EQ 10-2)

$W_{xB} = W_{tB} \tan\psi = 233 \tan 45° = 233$ LB (EQ 10-8)

$\phi_t = \tan^{-1}\left(\dfrac{\tan\phi_n}{\cos\psi}\right) = 20.1°$ OR $\tan\phi_t = \dfrac{\tan\phi_n}{\cos\psi} = 0.366$ (EQ 10-1)

$W_{rB} = W_t \tan\phi_t = 233(0.366) = 85$ LB (EQ 10-7)

SIMILARLY:

GEAR C

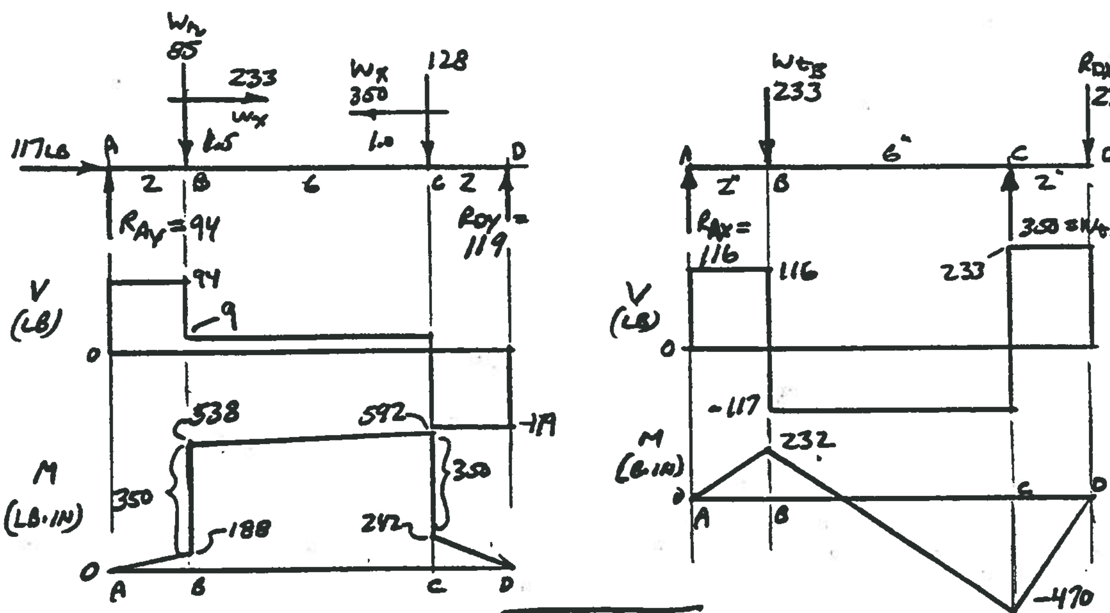
$W_{tC} = T_2/R_C = 350/1 = 350$ LB ←

$W_{xC} = W_{tC} \tan\psi = 350 \tan 45° = 350$ LB

$W_{rB} = W_t \tan\phi_t = 350(.366) = 128$ LB

FOR SAE 4140 OQT 1200; $S_y = 114$ KSI; $S_u = 130$ KSI; $S_n = 46$ KSI (FIG. 5-8)

$S_n' = C_s C_R S_n = (0.9)(0.81)(46\text{ KSI}) = 33.5$ KSI




AT C: $T = 350$ LB·IN ; $M_C = \sqrt{592^2 + 470^2} = 756$ LB·IN : USE $K_t = 2.0$ (KEY)

$$D_C = \left[\frac{32(3)}{\pi}\sqrt{\left(\frac{2.0(756)}{33500}\right)^2 + \frac{3}{4}\left(\frac{350}{114000}\right)^2}\,\right]^{1/3} = 1.11 \text{ IN}$$

SHAFTS 1 AND 3 CAN BE DESIGNED SIMILAR TO PROBLEM 33.

SPECIFY $D_C = 1.25$ IN. ; $C_S = 0.85$ OK

**36** DATA FROM FIGURE 10-8 TO 10-12.

PINION: $T = 263$ LB·IN; $M_E = \sqrt{220^2 + 95.8^2} = 240$ LB·IN; $N=3$, $K_t = 2.0$ (KEY)

SAE 1040 OQT 1200; $S_y = 63$ ksi; $S_u = 93$ ksi; $S_n = 36$ ksi; $S_n' = \underset{C_s}{(.86)}\underset{C_R}{(0.81)}(36) =$

$S_n' = 25.1$ ksi

$$D_E = \left[\frac{32(3)}{\pi}\sqrt{\left(\frac{2.0(240)}{25100}\right)^2 + \frac{3}{4}\left(\frac{263}{63000}\right)^2}\right]^{1/3} = 0.84 \text{ IN}$$

GEAR: $T = 788$ LB·IN; $M_F = 390$ LB·IN; $N = 3$; $K_t = 2.0$ (KEY)

$$D_F = \left[\frac{32(3)}{\pi}\sqrt{\left(\frac{2.0(390)}{25100}\right)^2 + \frac{3}{4}\left(\frac{788}{63000}\right)^2}\right]^{1/3} = 1.00 \text{ IN.} \quad C_s = 0.88 \text{ OK}$$

**37.** $T_{GEAR} = 63000(4.0)/600 = 420$ LB·IN; $T_{SP} = 420/2 = 210$ LB·IN

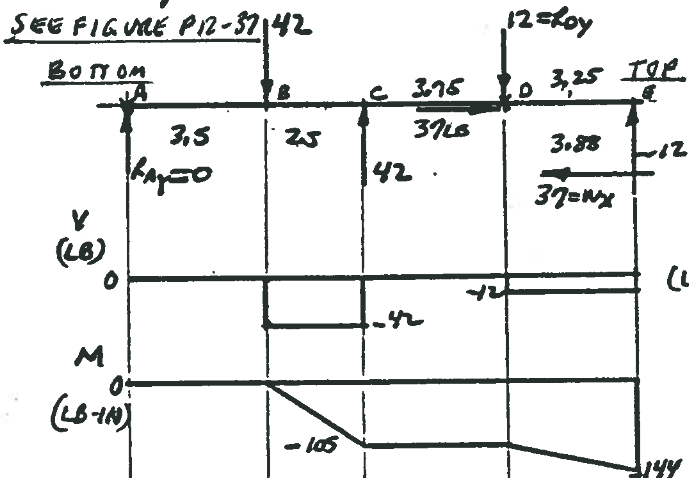
$F_{C1} = F_{C2} = T_{SP}/5 = 210/5 = 42$ LB $\qquad \Gamma = \tan^{-1}(45/15) = 71.6°$

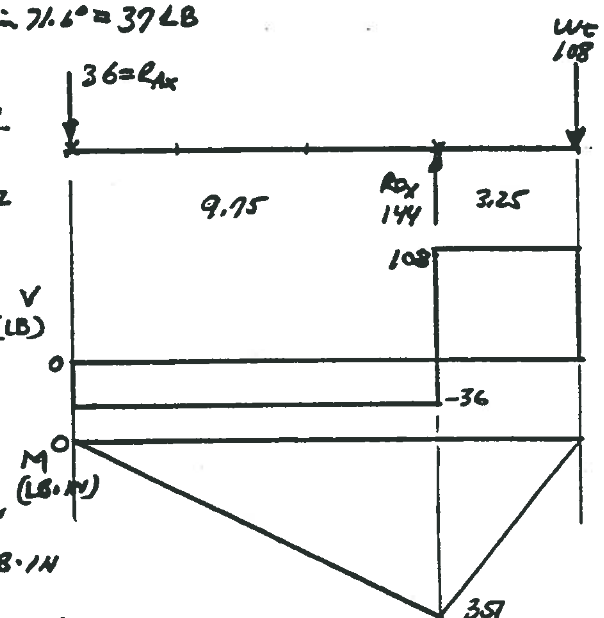
$W_{tG} = T_G/R_m = 420/3.88 = 108$ LB $\qquad \gamma = \tan^{-1}(15/45) = 18.4°$

$W_{RG} = W_t \tan\phi \cos\Gamma = (108)\tan 20° \cos 71.6° = 12$ LB

$W_{XG} = W_t \tan\phi \sin\Gamma = (108)\tan 20° \sin 71.6° = 37$ LB

SEE FIGURE P12-37




BEARING D: $M = \sqrt{105^2 + 357^2} = 366$ LB·IN

USE $N = 3$, $K_t = 2.5$ (FILLET)

$$D_D = \left[\frac{32(3)}{\pi}\sqrt{\left(\frac{2.5(366)}{44600}\right)^2 + \frac{3}{4}\left(\frac{420}{152000}\right)^2}\right]^{1/3} = 0.86 \text{ IN}$$

SAE 4140 OQT 1000

$S_y = 152$ ksi; $S_u = 168$ ksi

$S_n = 58$ ksi; $S_n' = \underset{C_s}{(.95)}\underset{C_R}{(0.81)}(58)$

$S_n' = 44.6$ ksi

SPECIFY $D = 1.00$ IN $\quad C_s = 0.88 \quad$ OK

38

$P = 15.0\ hp.$: $n_1 = 1725\ RPM$; $n_2 = 575\ RPM$; $n_3 = 287.5\ RPM$

TORQUE ON SHAFT 1 = $T_1 = T_A = 63000(15)/1725 = 548\ LB \cdot IN.$

TORQUE ON SHAFT 2 = $T_2 = T_B = T_C = 63000(15)/575 = 1643\ LB \cdot IN$

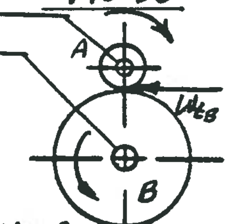
TORQUE ON SHAFT 3 = $T_3 = T_D = 63000(15)/287.5 = 3287\ LB \cdot IN.$

$W_{tB} = T_A/r_A = \dfrac{548\ LB \cdot IN}{0.90\ IN.} = 609\ LB \leftarrow$

$W_{rB} = W_{tB} \tan\phi = 609\ LB \tan 20° = 222\ LB \downarrow$

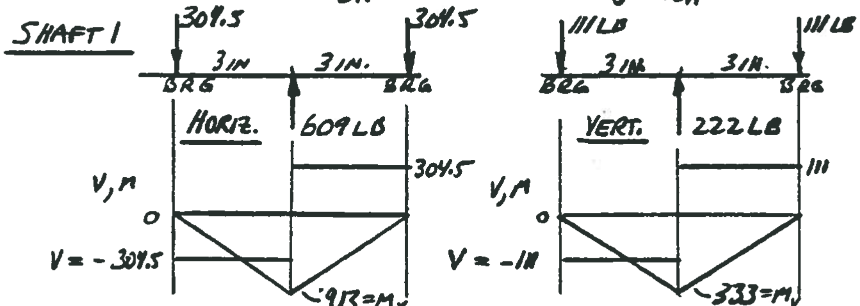
REACTIONS: $W_{tA} = 609\ LB \rightarrow$ ; $W_{rA} = 222\ LB \uparrow$

FIGURE P12-38



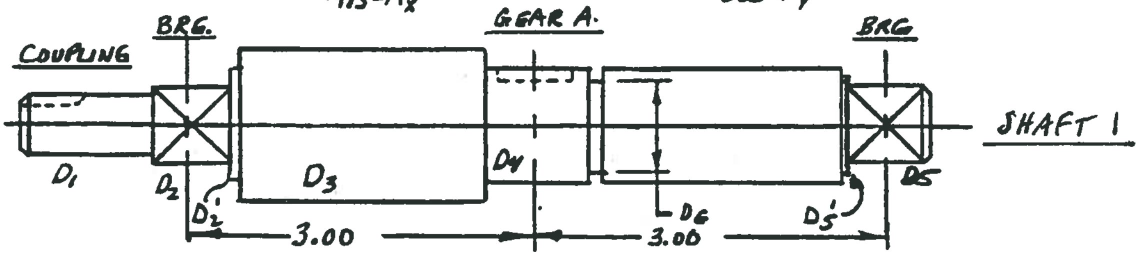

A DRIVES B

SHAFT 1




AT MIDDLE OF SHAFT:
$M = \sqrt{913^2 + 333^2} = 972\ LB \cdot IN$

$T = 548\ LB \cdot IN$ FROM COUPLING TO GEAR A.




EQUATION (12-24) USED TO COMPUTE ALL DIAMETERS.

DIAMETER $D_1$: $T = 548\ LB \cdot IN$; $M = 0$: DESIGN FOR 0.999 RELIABILITY – $C_R = 0.75$

USE SAE 1040 CD STEEL: $S_y = 71{,}000\ PSI$; $S_u = 80{,}000\ PSI$; 12% ELONGATION

$S_n = 30{,}000\ PSI$; $S_n' = S_n C_S C_R = (30\,000)(0.9)(0.75) = 20{,}250\ PSI$; LET $N = 3$.

THEN $D_{1min} = 0.589\ IN.$

DIAMETER $D_2$: SAME CONDITIONS AS $D_1$; $D_{2min} = 0.589\ IN.$

DIAMETER $D_3$: DEPENDS ON $D_4$.

DIAMETER $D_4$: $M = 972\ LB \cdot IN$; $T = 548\ LB \cdot IN$ AT SHOULDER AND KEYSEAT.
($K_t \approx 2.5$) ($K_t \approx 2.0$)

AT SHOULDER: $D_{4min} = 1.54\ IN.$

AT RING GROOVE: $T = 0$, $K_t \approx 3.0$: $D_{6min} = 1.64\ IN.$

INCREASE BY ≈ 6% FOR $D_4$: $D_4 \cong 1.06(1.64) = \underline{1.74\ IN}$ GOVERNING VALUE

DIAMETER $D_5$: $M = 0$; $T = 0$: VERY SMALL DIA. REQD TO RESIST SHEAR.

BEARING SEATS $D_2$, $D_5$: ASSUME BEARINGS WITH BORE = 0.7874 IN. (20 mm)
CAN BE FOUND TO CARRY RADIAL LOADS. CHAPTER 14, TABLE 14-3
BEARING NO. 6204

SPECIFICATIONS:

$D_1 = 0.750\ IN.$ $D_6$ SPECIFIED BY RETAINING RING MFGR.

$D_2 = D_5 = 0.7874\ IN.$

$\left.\begin{array}{l} D_3 = 2.00\ IN. \\ D_4 = 1.80\ IN. \end{array}\right\}$ RELIEF PROVIDED ON LEFT SIDE OF $D_3$ AND RIGHT END OF $D_4$ TO ENSURE THAT OUTER RACE OF BEARING DOES NOT CONTACT ROTATING SHAFT. $D_2' = D_4' = 0.969\ IN$ (SHAFT SHOULDER)

$C_S$ CHECKED FOR ALL DIAMETERS – OK

(CONTINUED)

38 (CONTINUED) SHAFT 2  $T_2 = 1643$ LB·IN

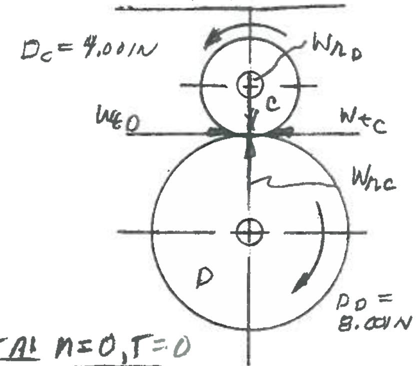
$W_{tC} = \frac{T_2}{R_C} = \frac{1643 \text{ LB·IN}}{2.00 \text{ IN}} = 822$ LB ←

$W_{tD} = 822$ LB →

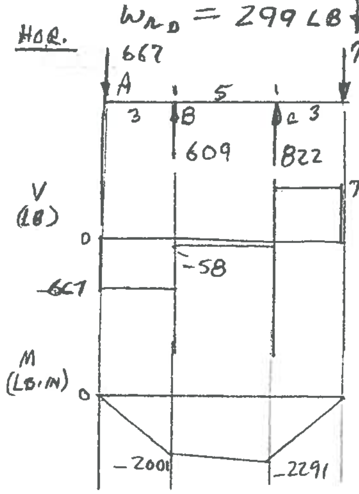
$W_{rC} = W_{tC} \tan\phi = 822 \tan 20° = 299$ LB ↑

$W_{rD} = 299$ LB ↓

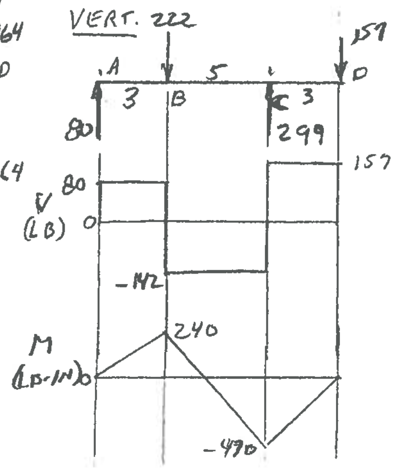
C DRIVES D




HOR.




VERT. 222




AT A: $M=0, T=0$

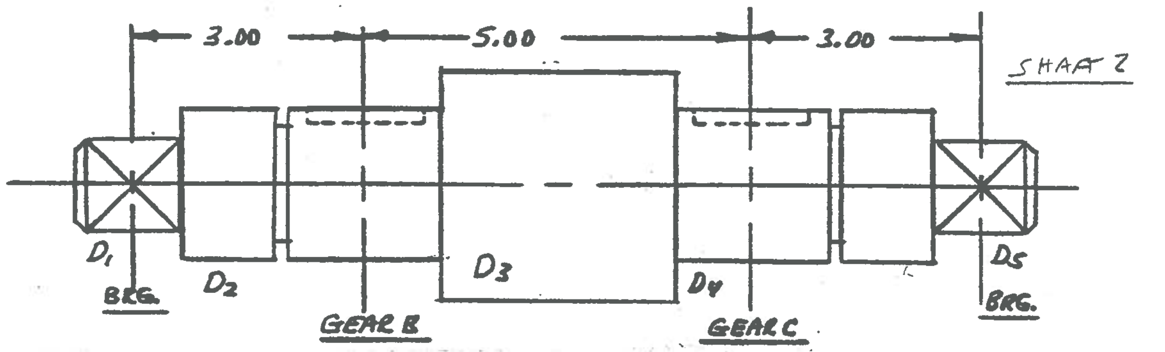
$V_A = \sqrt{667^2 + 80^2} = 671$ LB

B: $M_B = \sqrt{2001^2 + 240^2} = 2015$ LB·IN

C: $M_C = \sqrt{2291^2 + 470^2} = 2339$ LB·IN

D: $M=0, T=0,$

$V_D = \sqrt{764^2 + 157^2} = 780$ LB




SAE 1040 CD STEEL: $S_y = 71000$ PSI; $S_u = 80000$ PSI; 12% EL.

$S_n = 30000$ PSI; $S_n' = 20250$ PSI [SAME AS SHAFT 1]

$D_2$: $M_B = 2015$ LB·IN; $T = 1643$ LB·IN AT KEYSEAT + SHOULDER ($K_t = 2.5$) ($K_t = 2.0$)

$D_{2MIN} = 1.968$ IN. AT SHOULDER

AT RING GROOVE: $D_{2MIN} = (1.06)(2.089) = 2.215$ IN, GOVERNS

$D_4$: $M_C = 2339$ LB·IN; $T = 1643$ LB·IN AT KEYSEAT AND SHOULDER ($K_t = 2.5$)

$D_{4MIN} = 2.068$ IN

AT RING GROOVE: $T = 0$, $K_t = 3.0$; $M = 2339$ LB·IN

$D_{4MIN} = (1.06)(2.1959) = 2.328$ IN. GOVERNS

$D_5$: RIGHT BRG.: $M=0$; $T=0$, $V_D = 780$ LB.; $K_t = 2.5$

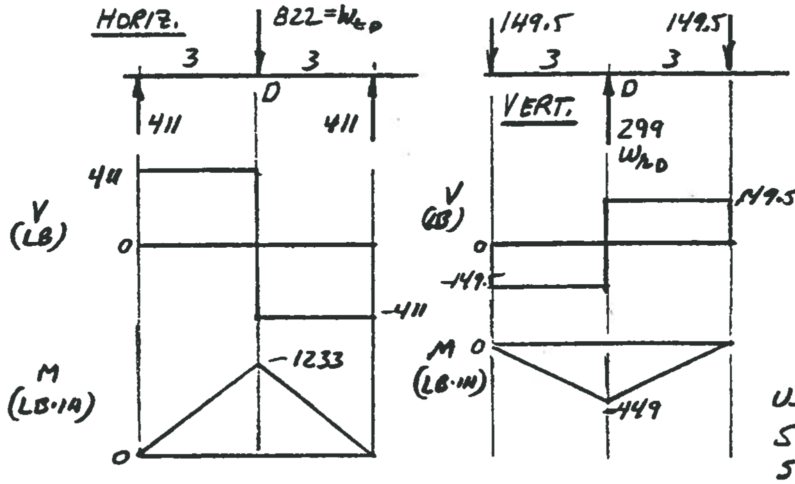
$D_{5MIN} = \sqrt{2.94 K_t V N / S_n'} = \sqrt{2.94(2.5)(780)(3)/20250} = 0.922$ IN.

$D_1$: $V_A = 671$ LB → $D_{1MIN} = 0.855$ IN.

LET: $D_2 = 2.250$ IN; $D_4 = 2.400$ IN, $D_3 = 2.600$ IN

$D_1$ AND $D_5$ DEPEND ON BEARING SELECTION.

38 (CONTINUED) SHAFT 3



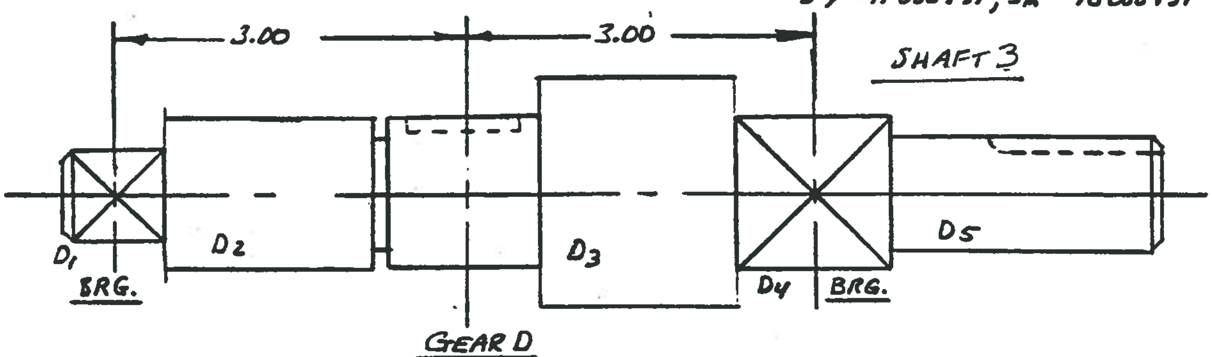

$M_D = \sqrt{1233^2 + 449^2}$

$M_D = 1312$ LB·IN.

$T = 3287$ LB·IN FROM KEYSEAT TO RIGHT END OF SHAFT.

USE SAME MATERIAL AS FOR SHAFTS 1 AND 2. $C_s \doteq 0.80$

$S_y = 71000$ PSI; $S_n' = 18000$ PSI




DIAMETER $D_2$: $M = 1312$ LB·IN; $T = 3287$ LB·IN, $K_t = 2.5$ AT SHOULDER.

$D_2 = 1.79$ IN. AT SHOULDER.

AT GROOVE: $D_G = 1.88$ IN FOR $K_t = 3.0$; $T = 0$.

INCREASE BY 6%: $D_2 \approx 1.06(1.88) = $ 2.00 IN. GOVERNS

DIAMETER $D_{4_{MIN}}$; $D_{5_{MIN}}$: $M = 0$; $T = 3287$ LB·IN.; $D_{4_{MIN}} = 1.07$ IN.

SPECIFY:

$D_1 = 1.3780$ IN (35 mm), BRG 6207

$D_2 = 2.000$ IN

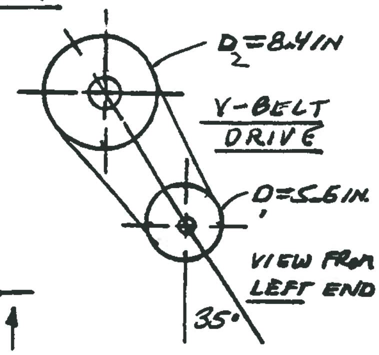
$D_3 = 2.250$ IN.

$D_4 = 1.3780$ IN (35 mm) BRG. 6207

$D_5 = 1.25$ IN

[CHECKED $C_s$ OK]

**39** $P = 12.0\ hp$, $n_{MOTOR} = 1150\ RPM$, FIGURE P12-39




**MOTOR SHAFT:**

NET DRIVING FORCE $= F_N = T/D/2$

$T = 63000(P)/n = 63000(12)/1150 = 657\ LB{\cdot}IN.$

$F_N = 657\ LB{\cdot}IN/(5.6\ IN/2) = 235\ LB.$

BENDING FORCE $= F_B = 1.5\ F_N = 352\ LB$ ↖

$F_{BX} = F_B \cdot \sin 35° = (352\ LB)(\sin 35°) = 202\ LB$ ←

$F_{BY} = F_B \cos 35° = (352\ LB)(\cos 35°) = 288\ LB$ ↑

FORCES ACT ON MOTOR SHAFT. DIRECTIONS AS VIEWED FROM BEHIND THE LEFT END OF THE MOTOR.

**REDUCER INPUT SHAFT:** FORCES SHOWN AS VIEWED FROM RIGHT END FOR CONSISTENCY WITH VIEWS OF GEAR SYSTEM IN FIGURE P12-38.

SHAFT SPEED: $n_1 = n_{MOTOR}\left(\frac{D_1}{D_2}\right) = 1150\ RPM\left(\frac{5.6}{8.4}\right) = 767\ RPM$

$T_1 = 63000(12)/767 = 986\ LB{\cdot}IN$

FORCES AT SHEAVE: $F_N = T_1/(D_2/2) = (986\ LB{\cdot}IN)/(8.4\ IN/2) = 235\ LB.$

$F_B = 1.5 F_N = 352\ LB$ ↙; $F_{BX} = 202\ LB$ ←; $F_{BY} = 288\ LB$ ↓

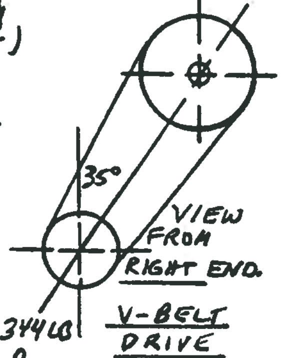
(SEE MOTOR SHAFT ANALYSIS)

FROM PROBLEM 38:

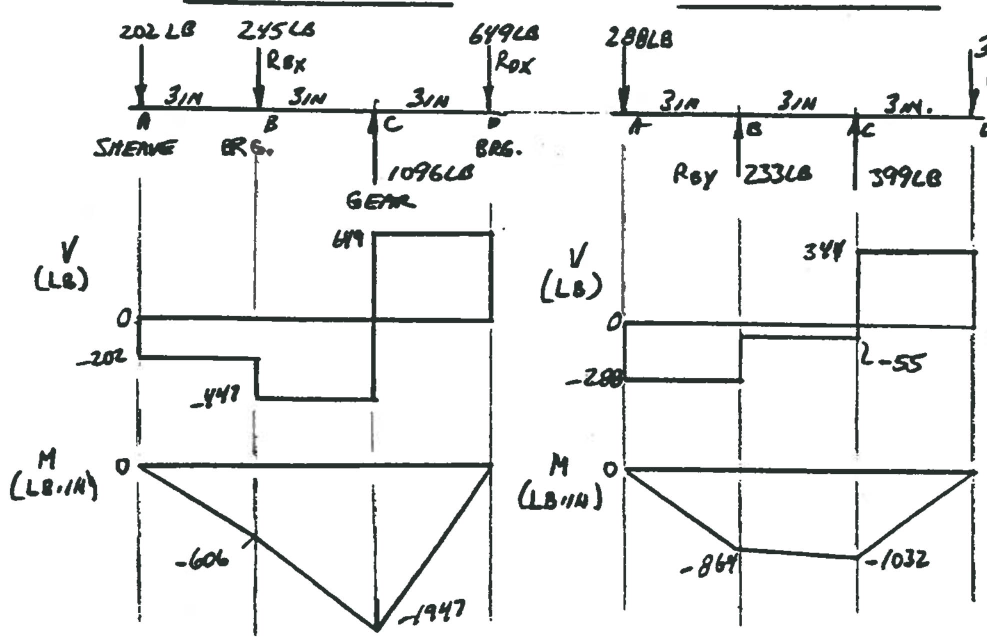
GEAR A: $W_{tA} = T_1/r_A = 986\ LB{\cdot}IN/0.9\ IN. = 1096\ LB$ →

$W_{rA} = W_{tA} \tan\phi = (1096\ LB)\tan 20° = 399\ LB$ ↑

POSITION OF SHEAVE ON SHAFT 1 ASSUMED




HORIZONTAL PLANE | VERTICAL PLANE




RESULTANT MOMENTS:

$M_B = \sqrt{606^2 + 864^2}$

$M_B = 1055\ LB{\cdot}IN.$

$M_C = \sqrt{1947^2 + 1032^2}$

$M_C = 2204\ LB{\cdot}IN.$

BEARING FORCES:

$R_B = \sqrt{245^2 + 233^2}$

$R_B = 338\ LB.$

$R_D = \sqrt{649^2 + 344^2}$

$R_D = 735\ LB.$

39 CONTINUED

DESIGN OF SHAFT 1 WOULD BE COMPLETED IN A MANNER SIMILAR TO THAT SHOWN IN PROBLEM 38.

SHAFT 2: SEE PROBLEM 38 FOR ANALYSIS AND DESIGN PROC.

FORCES: $W_{t_B} = W_{t_A} = 1096\ LB \leftarrow$ ; $W_{r_B} = W_{r_A} = 399 \downarrow$ GEAR B

SHAFT 2 SPEED $= n_2 = n_1 \dfrac{N_A}{N_B} = 767 \times \dfrac{18}{54} = 256\ RPM$

$T_2 = 63000(12)/256 = 2958\ LB{\cdot}IN$

$W_{t_C} = T_2/r_C = 2958\ LB{\cdot}IN / 2.00\ IN = 1479\ LB \rightarrow$ GEAR C

$W_{r_C} = W_{t_C} \tan 20° = 538\ LB \downarrow$

SHAFT 3: SEE PROBLEM 38 FOR ANALYSIS AND DESIGN PROCEDURE.

FORCES: $W_{t_D} = W_{t_C} = 1479\ LB \leftarrow$ ; $W_{r_D} = W_{r_C} = 538\ LB \uparrow$ GEAR D

CHAIN SPROCKET AT END OF SHAFT 3:

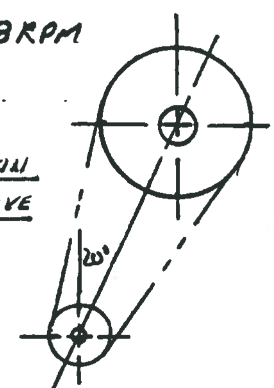
SPEED: $n_3 = n_2 \cdot N_C/N_D = 256\ RPM \cdot 24/48 = 128\ RPM$

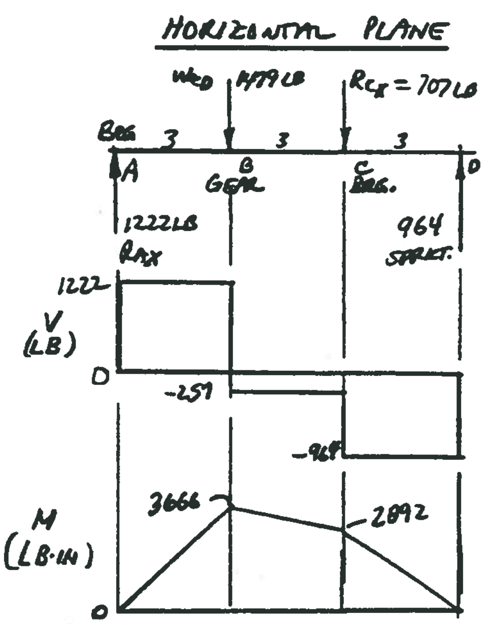
$T_3 = 63000(12)/128 = 5917\ LB{\cdot}IN$

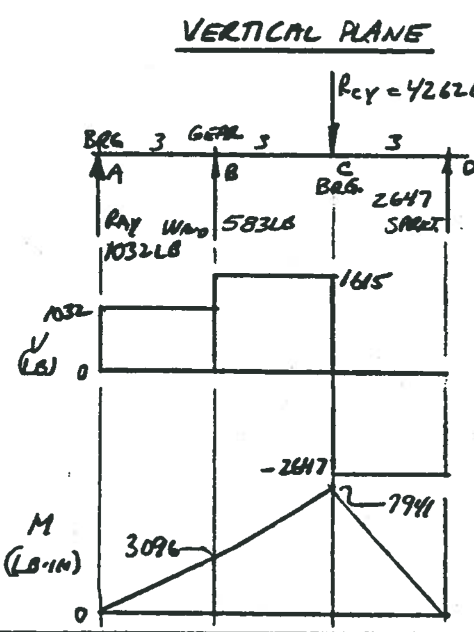
$F_N = F_B = \dfrac{T_3}{R} = \dfrac{5917\ LB{\cdot}IN}{(4.2\ IN/2)} = 2817\ LB \uparrow$

$F_{Bx} = F_B \sin 20° = (2817\ LB)\sin 20° = 964\ LB \rightarrow$

$F_{By} = F_B \cos 20° = (2817\ LB)\cos 20° = 2647\ LB \uparrow$




RESULTANTS:

$M_B = \sqrt{3666^2 + 3096^2}$

$M_B = 4798\ LB{\cdot}IN.$

$M_C = \sqrt{2892^2 + 7941^2}$

$M_C = 8451\ LB{\cdot}IN.$

BEARING FORCES:

$R_A = \sqrt{1222^2 + 1032^2}$

$R_A = 1600\ LB$

$R_C = \sqrt{707^2 + 4262^2}$

$R_C = 4320\ LB$

CONVEYOR SHAFT FORCES: $F_{Bx} = 964\ LB \leftarrow$ ; $F_{By} = 2647\ LB \downarrow$

SPEED: $n_C = n_3 \cdot 4.2/10.6 = 128 \cdot 4.2/10.6 = 50.6\ RPM$ ; $T_C = 63000(12)/50.6 = 14{,}932\ LB{\cdot}IN$

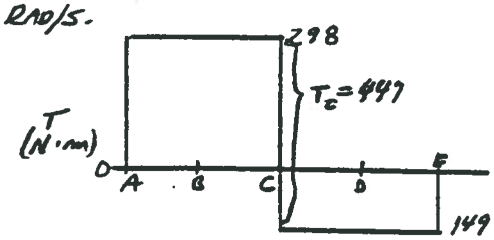
**40** FIGURE P12-40: SHAFT 2: $n = 480$ RPM : POWER IN AT C = 22.5 kW.
POWER OUT AT A = 15 kW : POWER OUT AT E = 7.5 kW.
COAL CRUSHER – USE N = 4 BECAUSE OF IMPACT AND SHOCK.

$$n = 480\,\frac{\text{REV}}{\text{MIN.}} \times \frac{2\pi\,\text{RAD}}{\text{REV.}} \times \frac{1\,\text{MIN.}}{60\,\text{SEC.}} = 50.27\ \text{RAD/s.}$$

$$T_{AC} = \frac{P}{n} = \frac{15\times10^3\ \text{N·m/s}}{50.27\ \text{RAD/s}} = 298\ \text{N·m}$$

$$T_{CE} = \frac{P}{n} = \frac{7.5\times10^3\ \text{N·m/s}}{50.27\ \text{RAD/s}} = 149\ \text{N·m}$$

$$\text{TORQUE ON GEAR C} = \frac{22.5\times10^3\ \text{N·m/s}}{50.27\ \text{RAD/s}} = 447\ \text{N·m}$$




TORQUE IN SHAFT

FORCES:

GEAR A: $W_{tA} = \frac{T_A}{r_A} = \frac{298\ \text{N·m}}{50\ \text{mm}} \times \frac{10^3\ \text{mm}}{\text{m}} = 5960\ \text{N} \leftarrow$

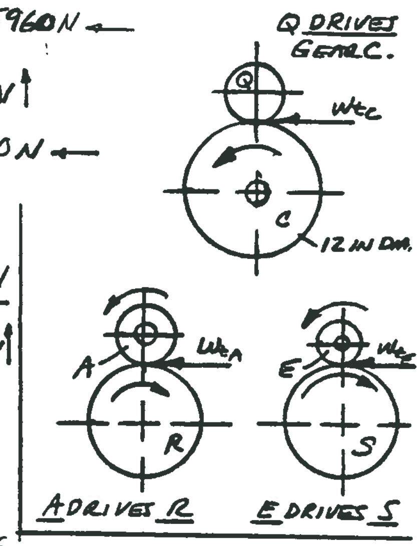
$W_{rA} = W_{tA}\tan 20° = (5960\ \text{N})\tan 20° = 2169\ \text{N} \uparrow$

GEAR C: $W_{tC} = \frac{T_C}{r_C} = \frac{447\ \text{N·m}}{150\ \text{mm}} \times 10^3 = 2980\ \text{N} \leftarrow$

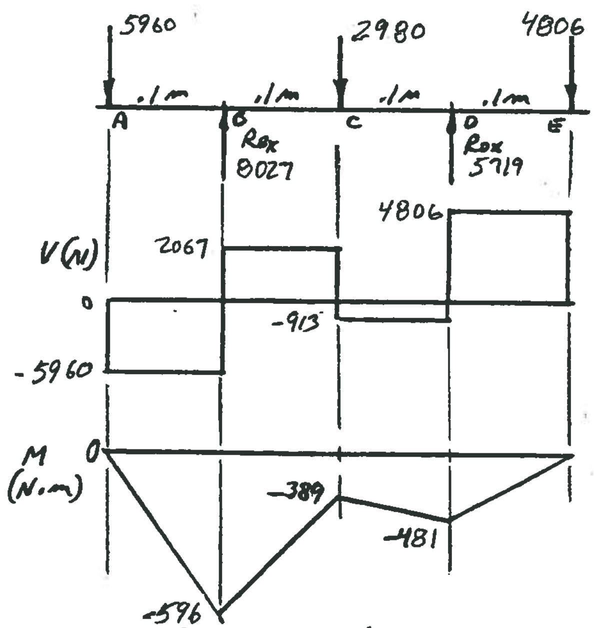
$W_{rC} = W_{tC}\tan 20° = (2980\ \text{N})\tan 20° = 1065\ \text{N} \downarrow$

GEAR E: $W_{tE} = \frac{T_E}{r_E} = \frac{149\ \text{N·m}}{31\ \text{mm}} \times 10^3 = 4806\ \text{N} \leftarrow$

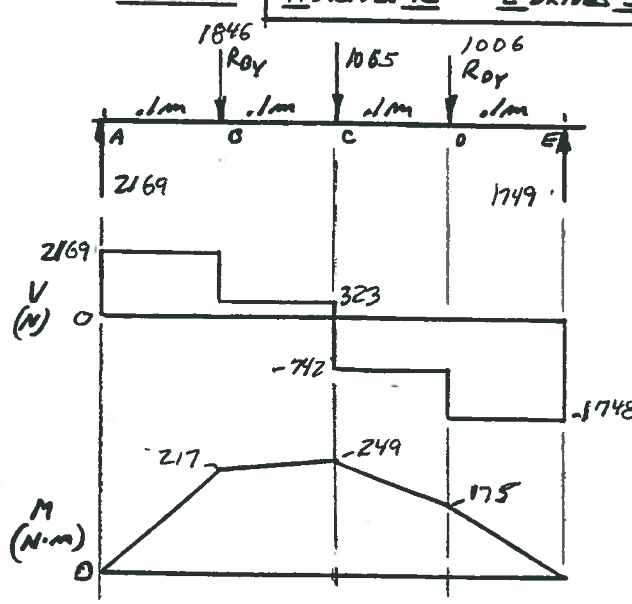
$W_{rE} = W_{tE}\tan 20° = (4806\ \text{N})\tan 20° = 1749\ \text{N} \uparrow$




HORIZONTAL PLANE




VERTICAL

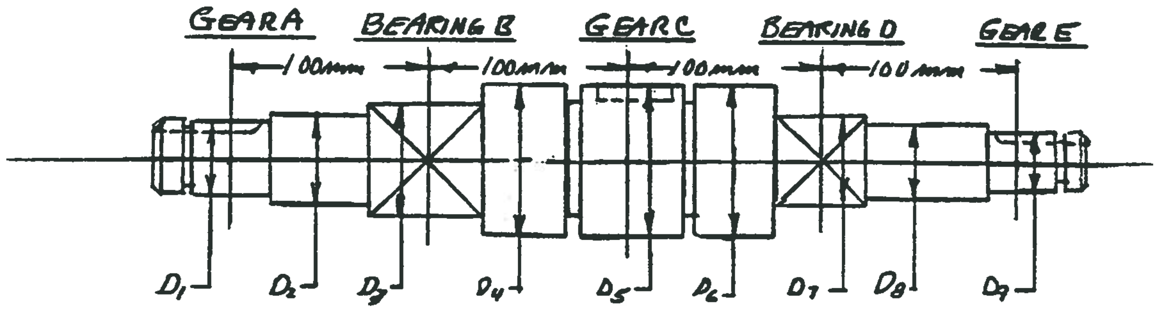



$M_B = [596^2 + 217^2]^{1/2} = \underline{634\ \text{N·m}}$ ; $M_C = [389^2 + 249^2] = \underline{462\ \text{N·m}}$ ; $M_D = [481^2 + 175^2]^{1/2} = \underline{512\ \text{N·m}}$

40 CONTINUED

BEARING FORCES: $R_B = \sqrt{8027^2 + 1846^2} = 8237\ N$; $R_D = \sqrt{5719^2 + 1006^2} = 5807\ N$

PROPOSED SHAFT DESIGN:




ASSUME ALL FILLETS ARE SMALL RADIUS WITH $K_t \approx 2.5$, EXCEPT $r_2, r_6$
USE $K_t = 3.0$ AT RING GROOVES WITH SHAFT DIA. $\approx 1.06 \times$ GROOVE DIA.
USE $K_t = 1.5$ AT $r_2$ AND $r_6$ (WELL ROUNDED)

MATERIAL SELECTION: SAE 4140 OQT 1000, $S_u = 1160\ MPa$, $S_y = 1050\ MPa$
17% ELONGATION — GOOD STRENGTH AND DUCTILITY.
FROM FIG. 5-9: $S_n = 400\ MPa$.
SELECT $C_s = 0.80$ (FOR $D \approx 65$ mm OR LESS); $C_R = 0.81$ (0.99 RELIABILITY)
$S_n' = C_s C_R S_n = (0.80)(0.81)(400\ MPa) = 259\ MPa = 259\ N/mm^2$

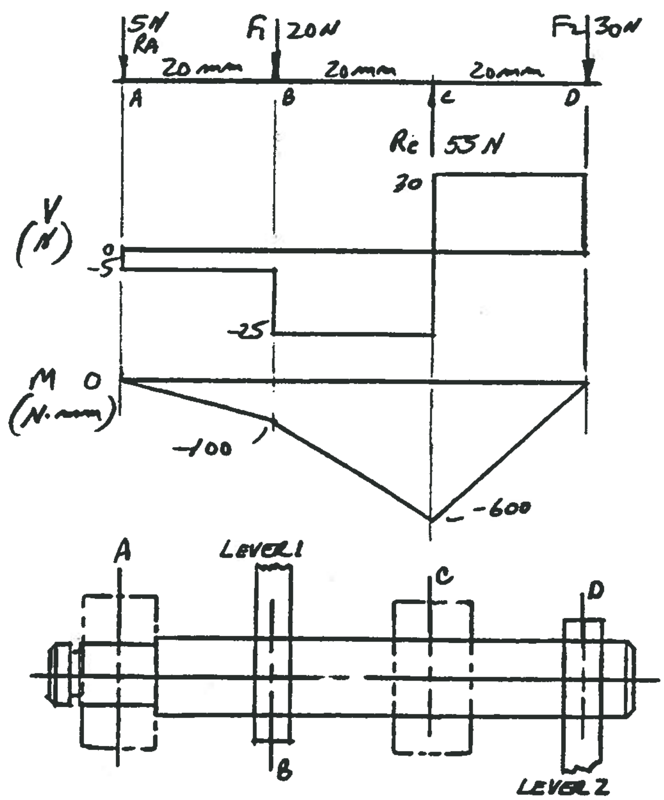
SOLUTION FOR DIAMETERS USING EQ. 9-22 – SUMMARY: (N = 4)

| LOCATION | $K_t$ | M (N·mm) | T (N·mm) | $D_{MIN}$ | SPECIFIED D |
|---|---|---|---|---|---|
| $D_1$ | 2.5 | 0 | 298,000 | 21.56 | 50.0 mm |
| $D_2$ | 1.5 | 634,000 | 298,000 | 53.13 | 60.0 mm |
| $D_3$ | 2.5 | 634,000 | 298,000 | 62.96 | 65.0 mm |
| $D_4$ | 3.0 | 525,000* | 298,000 | 62.82 × | 1.06 = 66.6 |
| $D_5$ | 2.5 | 462,000 | 298,000 | 56.67 | 80.0 mm |
| $D_6$ | 3.0 | 462,000 | 149,000 | 60.19 | × 1.06 = 63.8 |
| $D_7$ | 2.5 | 512,000 | 149,000 | 58.62 | 60.0 mm |
| $D_8$ | 1.5 | 512,000 | 149,000 | 49.45 | 55.0 mm |
| $D_9$ | 2.5 | 0 | 149,000 | 17.11 | 45.0 mm |

NOTES: $D_3$ AND $D_7$ ARE STANDARD BEARING BORES FROM TABLE 14-3.
* MOMENT AT $D_4$ ESTIMATED BETWEEN POINTS B AND C.
80.0 mm USED FOR $D_4$, $D_5$, AND $D_6$ TO PROVIDE SHOULDER FOR BEARINGS AT B AND D.
$D_2$ AND $D_8$ PROVIDE EASE OF INSTALLATION FOR BEARINGS.
$D_1$ AND $D_9$ MADE LARGER THAN REQUIRED FOR COMPATIBILITY WITH ADJACENT DIAMETERS AND TO WITHSTAND MOMENTS AT SHOULDERS.
FINAL STRESSES MUST BE CHECKED AFTER SPECIFYING FILLET RADII, GROOVE GEOMETRY, AND BEARING SELECTION.

41 FIGURE P12-41 DESIGN SHAFT AND LEVERS. $F_1 = 20N$

$F_2 = F_1(60mm)/40mm = 30N$ : TORQUE $= F_1 \cdot 60 = 1200 N\cdot mm$ FROM B TO D.




USE SAE 1137 CD STEEL

$S_u = 676 MPa$; $S_y = 565 MPa$

$S_n = 260 MPa$ (FIG. 5-8)

LET $C_s = 0.9$, $C_R = 0.75$

$S_n' = (0.9)(0.75)(260) = 175 MPa$

WIPER MECHANISM HAS AN OSCILLATING MOTION. BOTH BENDING AND TORSION WILL BE VARYING.

SHAFT RESTRAINED IN BEARING AT A BUT CAN FLOAT IN BEARING C.

ASSUME $K_t = 2.5$ AT FILLET TO RIGHT OF A. ASSUME $K_t = 1.0$ AT C.

ASSUME LEVERS ARE INSTALLED WITH A LIGHT PRESS FIT AND TACK WELDED IN POSITION. USE $K_t = 3.0$ FOR WELD AREA.

DESIGN EQUATION 12-24 MUST BE MODIFIED FOR VARYING TORQUE. ADD $K_t$ FOR TORSION. SUBSTITUTE $S_n'$ FOR $S_y$. THEN:

$$D = \left[\frac{32N}{\pi}\sqrt{\left(\frac{K_{tB}\,M}{S_n'}\right)^2 + \frac{3}{4}\left(\frac{K_{tT}\,T}{S_n'}\right)^2}\right]^{1/3}$$

AT C:

$$D = \left[\frac{32(3)}{\pi}\sqrt{\left(\frac{1.0(600)}{175}\right)^2 + \frac{3}{4}\left(\frac{1.0(1200)}{175}\right)^2}\right]^{1/3} = 5.94 mm$$

AT B:

$$D = \left[\frac{32(3)}{\pi}\sqrt{\left(\frac{3.0(100)}{175}\right)^2 + \frac{3}{4}\left(\frac{3.0(1200)}{175}\right)^2}\right]^{1/3} = 8.18 mm$$

USE D = 10.0 mm AT B, C, D.
USE D = 8.0 mm AT A.

LEVER DESIGN: USE FLAT STOCK, 4.0 mm THICK = b

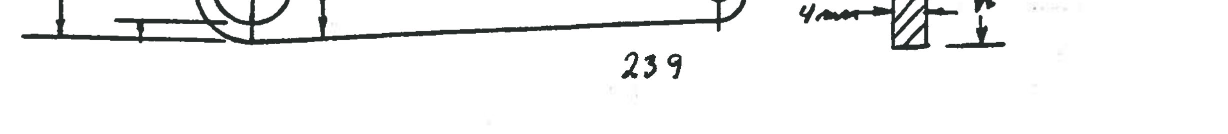
SAME MATERIAL AS SHAFT: $S_n' = 175 MPa$; $\sigma_d = \frac{S_n'}{N} = \frac{175}{3} = 58.3 MPa$

$M_{max} = (20N)(60mm) = 1200 N\cdot mm$ : $\sigma = M/S$

REQ'D $S = \frac{M}{\sigma_d} = \frac{1200 N\cdot mm}{58.3 N/mm^2} = 20.57 mm^3 = bh^2/6$

REQ'D $h = \sqrt{6S/b} = \sqrt{6(20.57 mm^3)/4.0mm} = 5.55 mm$ AT SHAFT.

BECAUSE SHAFT IS 10.0 mm DIA., LEVER IS ADEQUATE.



# CHAPTER 13
# TOLERANCES AND FITS

**1.** LOOSE – <u>RC8</u> : HOLE $^{+5}_{-0}$ ; SHAFT $^{-7}_{-10.5}$ ; CLEARANCE $^{+7}_{+15.5}$

LIMITS : HOLE $\frac{3.5050}{3.5000}$ SHAFT $\frac{3.4930}{3.4895}$ CL 0.0070 TO 0.0155

**2.** PRECISION – <u>RC2</u> : HOLE $^{+0.4}_{-0}$ ; SHAFT $^{-0.25}_{-0.55}$ ; CL $^{+0.25}_{+0.95}$

LIMITS : HOLE $\frac{0.50040}{0.50000}$ SHAFT $\frac{0.49975}{0.49945}$ CL 0.00025 TO 0.00095

**3.** LOOSE – <u>RC8</u> : HOLE $^{+2.8}_{-0}$ ; SHAFT $^{-3.5}_{-5.1}$ ; CL $^{+3.5}_{+7.9}$

ADJUST TOLERANCES FOR <u>BASIC SHAFT</u> SYSTEM <u>– ADD 3.5.</u>

HOLE $^{+6.3}_{+3.5}$ ; SHAFT $^{0}_{-1.6}$ ; CL $^{+3.5}_{+7.9}$

LIMITS : HOLE $\frac{0.6313}{0.6285}$ SHAFT $\frac{0.6250}{0.6234}$ CL 0.0035 TO 0.0079

**4.** CLOSE FIT – RELIABLE MOTION <u>– RC 5</u>

HOLE $^{+1.2}_{-0}$ ; SHAFT $^{-1.6}_{-2.4}$ ; CL $^{+1.6}_{+3.6}$

LIMITS : HOLE $\frac{0.8012}{0.8000}$ SHAFT $\frac{0.7984}{0.7976}$ CL 0.0016 TO 0.0036

**5.** LOOSE – <u>RC8</u> : HOLE $^{+4.0}_{-0}$ ; PIN $^{-5.0}_{-7.5}$ ; CL $^{+5.0}_{+11.5}$

LIMITS : HOLE $\frac{1.2540}{1.2500}$ PIN $\frac{1.2450}{1.2425}$ CL 0.0050 TO 0.0115

**6.** LOOSE <u>– RC8</u> : HOLE $^{+5.0}_{-0}$ ; PIN $^{-7.0}_{-10.5}$ ; CL $^{+7.0}_{+15.5}$

ADJUST TOLERANCES FOR <u>BASIC SHAFT</u> SYSTEM – ADD 7.0

HOLE $^{+12.0}_{+7.0}$ ; PIN $^{0}_{-3.5}$ ; CL $^{+7.0}_{+15.5}$

LIMITS : HOLE $\frac{4.0120}{4.0070}$ PIN $\frac{4.0000}{3.9965}$ CL 0.0070 TO 0.0155

**7.** PRECISION WITH WIDE TEMPERATURE VARIATIONS WOULD TYPICALLY CALL FOR RC3 OR RC4. RC2 PROBABLY TOO TIGHT FOR TEMP. CHANGE; RC5 PROBABLY TOO LOOSE FOR REQ'D PRECISION. RC3 OR RC4 NOT AVAILABLE IN TABLE 13-6.

ILLUSTRATE LIMITS WITH RC5:

HOLE: $^{+1.2}_{-0}$ ; PIN $^{-1.6}_{-2.4}$ ; CL $^{+1.6}_{+3.6}$

LIMITS: HOLE $\frac{0.7512}{0.7500}$ PIN $\frac{0.7484}{0.7476}$ CL 0.0016 TO 0.0036

**8.** LOOSE – RC8: HOLE $^{+4.0}_{-0}$ ; SHAFT $^{-5.0}_{-7.5}$ ; CL $^{+5.0}_{+11.5}$

ADJUST TOLERANCES FOR BASIC SHAFT SYSTEM – ADD 5.0.

HOLE $^{+9.0}_{+5.0}$ ; SHAFT $^{0}_{-2.5}$ ; CL $^{+5.0}_{+11.5}$

LIMITS: HOLE $\frac{1.5090}{1.5050}$ SHAFT $\frac{1.5000}{1.4975}$ CL 0.0050 TO 0.0115

**10.** $a = 0$; $b = 3.25/2 = 1.625$ IN; $c = 4.000/2 = 2.000$ IN : BOTH STEEL $E = 30 \times 10^6$ psi

USE FN5 – HEAVY FORCE FIT

HOLE: $^{+2.2}_{-0}$ SHAFT: $^{+8.4}_{+7.0}$ INTERFERENCE: $^{4.8}_{8.4}$

LIMITS: HOLE $\frac{3.2522}{3.2500}$ SHAFT $\frac{3.2584}{3.2570}$ $\delta_{MAX} = 0.0084$ IN.

(EQ. 13-2) $$p = \frac{E\delta}{2b}\left[\frac{(c^2-b^2)(b^2-a^2)}{2b^2(c^2-a^2)}\right] = \frac{(30\times10^6)(0.0084)}{2(1.625)}\left[\frac{(2.00^2-1.625^2)(1.625^2-0)}{2(1.625)^2(2.00^2-0)}\right]$$

$p = 13175$ psi

(EQ. 13-4) $$\sigma_o = p\left(\frac{c^2+b^2}{c^2-b^2}\right) = 13175\left[\frac{2.00^2+1.625^2}{2.0^2-1.625^2}\right] = 64363 \text{ psi}$$ NOT ACCEPTABLE FOR 1020 HR; $S_u = 55$ KSI

**11.** $a = 3.50/2 = 1.750$ IN ; $b = 4.0/2 = 2.000$ IN ; $C = 4.50/2 = 2.250$ IN

FN3: HOLE $^{+1.4}_{-0}$ SHAFT $^{+4.9}_{+4.0}$ INT. $^{2.6}_{4.9}$

LIMITS: STEEL SLEEVE ID = $\frac{4.0014}{4.0000}$ : BRONZE BUSHING OD = $\frac{4.0049}{4.0040}$ : $\delta_{MAX} = 0.0049$ IN

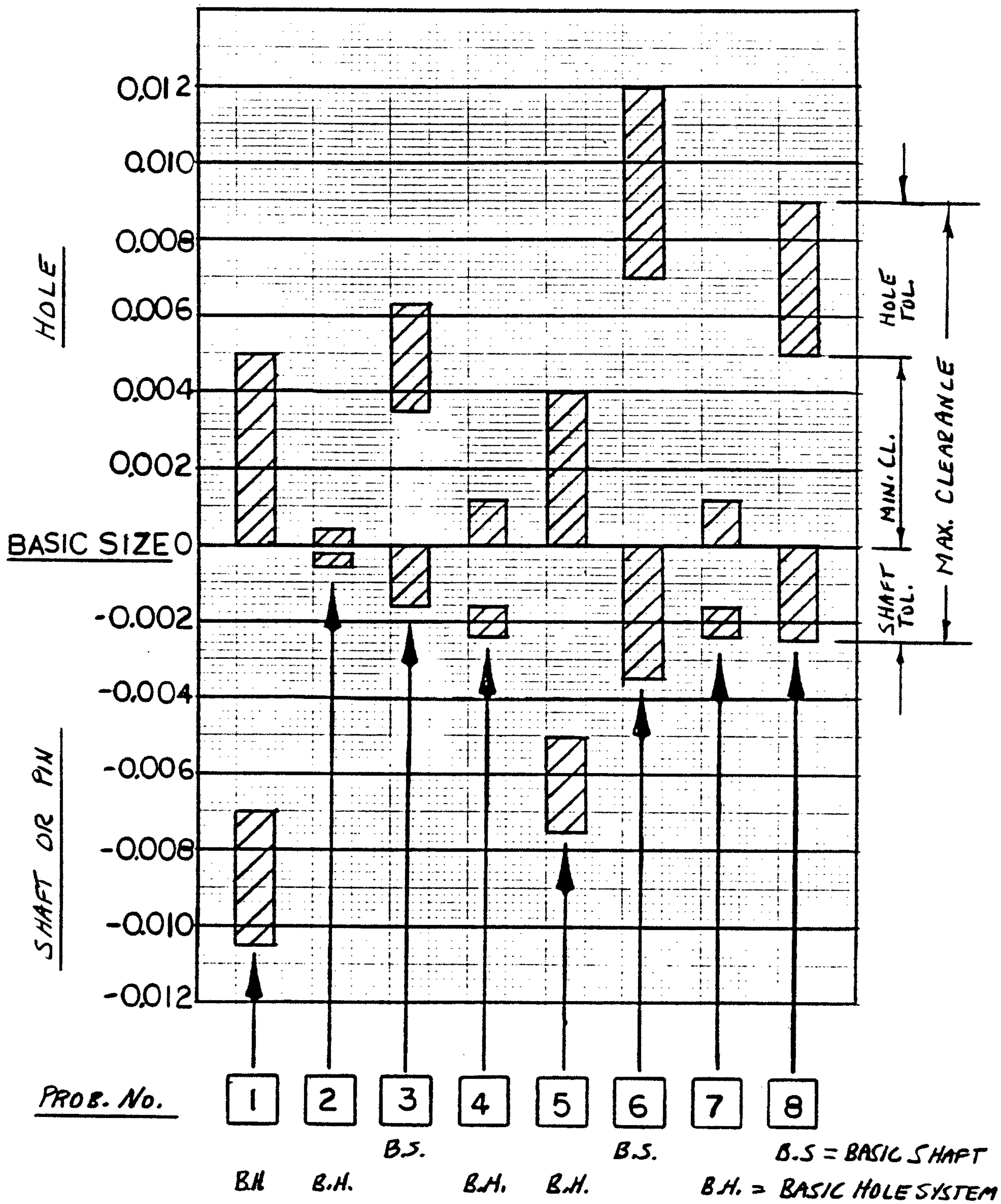
$p = 1575$ psi – (EQ. 13-3) USING $E_o = 30 \times 10^6$ psi $E_i = 15 \times 10^6$ psi; $V_o = 0.27$ $V_i = 0.27$

$\sigma_o = 13438$ psi – (EQ 13-4) INNER SURFACE OF STEEL SLEEVE

$\sigma_i = -11869$ psi – (EQ. 13-5) OUTER SURFACE OF BRONZE BUSHING

NOTE: APPENDIX A-12, BEARING BRONZE HAS A YIELD STRENGTH OF 18 000 psi. $N = \frac{S_y}{\sigma_i} = \frac{18000}{11869} = 1.52$ LOW

9 TOLERANCE DIAGRAMS FOR PROBLEMS 1-8.



| STRESSES FOR FORCE FITS<br>Refer to Figure 13-6 for geometry | Problem identification: Problem 12 | | | |
|---|---|---|---|---|
| ***Input Data:*** | Numerical values in *italics* must be inserted for each problem | | | |
| Inside radius of inner member = | $a$ = | *0.0000* | in | |
| Outside radius of inner memeber = | $b$ = | *1.5000* | in | |
| Outside radius of outer memeber = | $c$ = | *2.5000* | in | |
| Total interference = | $\delta$ = | *0.0072* | in | |
| Modulus of outer member = | $E_o$ = | *1.00E+07* | psi | ALUMINUM |
| Modulus of inner member = | $E_i$ = | *3.00E+07* | psi | STEEL |
| Poisson's ratio for outer member = | $\nu_o$ = | *0.33* | | |
| Poisson's ratio for inner member = | $\nu_i$ = | *0.27* | | |
| ***Computed results:*** | | | | |
| ***Pressure at Mating Surface:*** | $p$ = | 8894 | psi | Using Equation (13-3) |
| ***Tensile Stress in the Outer Member:*** | $\sigma_o$ = | 18901 | psi | Using Equation (13-4) |
| ***Compressive Stress in the Inner Member:*** | $\sigma_i$ = | -8894 | psi | Using Equation (13-5) |
| ***Increase in Diameter of Outer Member:*** | $\delta_o$ = | 0.00655 | in | Using Equation (13-6) |
| ***Decrease in Diameter of Inner Member:*** | $\delta_i$ = | 0.00065 | in | Using Equation (13-7) |

EVALUATE FN5 FIT FOR MAXIMUM INTERFERENCE.

HOLE TOLERANCE: $^{+0.0018}_{-0}$  SHAFT: $^{+0.0072}_{+0.0060}$  INTERFERENCE: $^{0.0072}_{0.0072}$

MAX. INTERFERENCE = 0.0072 − 0 = 0.0072 SMALLEST HOLE
LARGEST SHAFT

STRESS IN ALUMINUM = 18901 psi TENSION

ASSUMING NO ADDITIONAL LOADS AND STATIC CONDITION:

LET N=2. REQ'D $S_y$ = 2(18901 psi) = 37802 psi

SPECIFY ANY ALUMINUM ALLOY WITH $S_y$ > 37800 psi

EXAMPLES: 2014-T4, $S_y$ = 42 ksi OR 6061-T6, $S_y$ = 40 ksi

ANY STEEL FOR ROD WOULD BE SATISFACTORY, PROVIDED

$S_y$ > 2(8894 psi) = 17788

ANY CARBON OR ALLOY STEEL WITH $S_y$ > 18000 psi

FROM APP. 3. EXAMPLES AISI 1020 HR, $S_y$ = 30 ksi

AISI 1040 HR, $S_y$ = 42 ksi

13 STEEL SLEEVE ON ALUMINUM TUBE

$a = [2.00 - 2(0.065)]/2 = 0.935\text{ IN}$; $b = 2.00/2 = 1.000\text{ IN}$; $c = 3.00/2 = 1.500\text{ IN}$

EQ. 13-5 $\sigma_i = -p\left(\frac{b^2+a^2}{b^2-a^2}\right) = -p\left(\frac{1.00+.8742}{1.00-.8742}\right) = -p(14.898)$

FOR $\sigma_i = -8500\text{ psi}$; $p = \frac{-8500\text{ psi}}{14.898} = 570\text{ psi}$ MAX ALLOWABLE

FROM EQ 13-3 SOLVE FOR $\delta$.

$$\delta_{MAX} = p\left\{2b\left[\frac{1}{E_o}\left(\frac{c^2+b^2}{c^2-b^2}+\nu_o\right)+\frac{1}{E_i}\left(\frac{b^2+a^2}{b^2-a^2}-\nu_i\right)\right]\right\}$$

$E_o = 30\times10^6\text{ psi}$; $E_i = 10\times10^6\text{ psi}$; $\nu_o = 0.27$; $\nu_i = 0.33$

$\delta_{MAX} = 0.00177\text{ IN.}$

14 NOMINAL DIA. = 3.250 IN. ; ASSUME MAX INTERFERENCE = 0.0084 IN

FOR FINAL CLEARANCE = 0.002, CHANGE IN DIA. = 0.0084 + 0.002 = 0.0104

$\Delta t = \frac{\delta}{\alpha L} = \frac{0.0104\text{ in}}{(6.5\times10^{-6})\,°F^{-1}(3.250\text{ in})} = 492°F$ ; $t_f = 75° + 492 = 567°F$

15 BRONZE - SHRINK FROM +75 TO -20° ; $\Delta t = 95°F$

$\delta = \alpha L(\Delta t) = (10.0\times10^{-6})(4.00)(-95) = -0.0038\text{ IN.}$

MAX INTERFERENCE = 0.0049 IN.

-0.0038

0.0011

+0.0040 DESIRED CLEARANCE

0.0051 IN REQ'D EXPANSION OF STEEL.

$\Delta t = \frac{\delta}{\alpha L} = \frac{0.0051}{(6.1\times10^{-6})(4.000)} = 209°F$

75°F AMBIENT

284°F REQ'D STEEL TEMP.

16 EQ. 13-7

$$\delta_i = \frac{-2bp}{E_i}\left[\frac{b^2+a^2}{b^2-a^2}-\nu_i\right] = \frac{-2(2.00)(1575)}{(17\times10^6)}\left[\frac{2.00^2+1.75^2}{2.00^2-1.75^2}-0.27\right]$$

$\delta_i = -0.00269\text{ IN}$

FINAL ID = 3.500 - 0.00269 = 3.4973 IN.

# CHAPTER 14
# ROLLING CONTACT BEARINGS

$\boxed{1.}$ EQ 14-2: $L_d = \left(\frac{C}{P_d}\right)^k (10^6) = \left(\frac{2350}{1675}\right)^{3.0} (10^6) = 2.76 \times 10^6$ REV.

$\boxed{2.}$ $L_d =$ (20 000 HR)(880 RPM)(60 MIN/HR) $= 1.06 \times 10^9$ REV.

EQ. 14–3: $C = P_d \left(L_d/10^6\right)^{1/k} = 1250 \left(1.06 \times 10^9/10^6\right)^{1/3} = 12\,745$ LB

$\boxed{3.}$ EQ. 14-2 (a) $L_d = \left(\frac{C}{P_d}\right)^k (10^6) = \left(\frac{3150}{2200}\right)^3 (10^6) = 2.94 \times 10^6$ REV.

(b) $L_d = \left(\frac{3150}{4500}\right)^3 (10^6) = 0.343 \times 10^6 = 3.43 \times 10^5$ REV.

$\boxed{4.}$ USE $L_d =$ (5000 HR)(1150 RPM)(60 MIN/HR) $= 1.04 \times 10^9$ REV

$C = P_d \left(L_d/10^6\right)^{1/k} = 1450 \left(1.04 \times 10^9/10^6\right)^{1/3} = 14\,667$ LB

$\boxed{5.}$ FROM FIGURE 12-12:

REACTION AT B: $= R_B = \sqrt{R_{BX}^2 + R_{BY}^2} = \sqrt{458^2 + 4620^2} = 4643$ LB

REACTION AT D: $= R_D = \sqrt{R_{DX}^2 + R_{DY}^2} = \sqrt{1223^2 + 1680^2} = 2078$ LB

FROM EX. PROB. 12-1: DIA. AT B (MIN.) $= D_3 = 3.55$ IN

DIA. AT D (MIN.) $= D_6 = 1.09$ IN

INDUSTRIAL BLOWER; USE $L_d =$ (10 000 HR)(600 RPM)(60) $= 3.60 \times 10^8$ REV.

REQ'D. $\underline{\underline{C}}$ VALUE AT B: $C = R_B (3.6 \times 10^8/10^6)^{1/3} = 4643\,(7.114) = 33\,029$ LB.

AT D: $C = R_D (3.6 \times 10^8/10^6)^{1/3} = 2078\,(7.114) = 14\,782$ LB.

FROM TABLE 14-3: BRG. 6319 HAS $C = 34\,397$ LB; BORE $= 3.7402$ IN.

BRG. 6311 HAS $C = 16\,076$ LB; BORE $= 2.1654$ IN.

$\boxed{6.}$ DATA OF EX. PROB. 12-2 : FROM FIG. 12-16

$R_B = \sqrt{589^2 + 164^2} = 611$ LB ; $R_D = \sqrt{393^2 + 188^2} = 436$ LB

$D_{B_{MIN}} = 2.02$ IN ; $D_{D_{MIN}} = 1.98$ IN. FROM TABLE 12-2.

TABLE 14-4 : AGRICULTURAL EQ. - LET $L_d = 5000$ HR

$L_d = (5000)(1700\ \text{RPM})(60) = 5.1 \times 10^8$ REV

REQ'D $\underline{C}$ VALUE AT B: $C = 611\left(5.1\times10^8/10^6\right)^{1/3} = 4882$ LB

AT D: $C = 436\left(5.1\times10^8/10^6\right)^{1/3} = 3483$ LB

AT B: FROM TABLE 14-3 : BEARING 6011 HAS $C = 6317$ LB AND A BORE OF 2.1654 IN. C IS HIGHER THAN REQ'D BUT SHAFT DIA. MUST BE > 2.02 IN.
SPECIFY BRG. 6011 FOR BOTH B AND D.

$\boxed{7.}$ DATA OF EX. PROB. 12-3 AND FIGS. 12-17 AND 12-18

$R_A = \sqrt{507^2 + 41^2} = 509$ LB ; $R_C = \sqrt{1697^2 + 393^2} = 1742$ LB RADIAL

$R_A$ PURELY RADIAL ; BRG. C CARRIES 265 LB THRUST LOAD

$D_{A_{MIN}} = 0.59$ IN ; $D_{C_{MIN}} = 2.26$ IN

USE $L_d = (20\,000\ \text{HR})(101\ \text{RPM})(60) = 1.2 \times 10^8$ REV.

REQ'D. $\underline{C}$ VALUE AT A: $C_A = 509\left(1.2\times10^8/10^6\right)^{1/3} = 2519$ LB

BRG. 6302 HAS $C = 2563$ LB AND A BORE OF 0.5906 IN.
SHOULD BE COMPATIBLE WITH DIA. $D_2$ (FIG 12-16) TO PROVIDE A SHOULDER FOR THE BEARING. A LIGHTER BEARING WITH A LARGER BORE MAY BE PREFERRED.

BEARING C : COMBINED RADIAL & THRUST LOAD. (EQ. 14-5)

ASSUME $Y = 1.5$ : $P = (1.0)(0.56)(1742) + (1.5)(265) = 1373$ LB.

$C_c = 1373\left(1.2\times10^8/10^6\right)^{1/3} = 6772$ LB.

BRG. 6212 HAS $C = 10\,679$ LB, BORE $= 2.3622$ IN, $C_0 = 7307$ LB

CHECK : $T/C_0 = 265/7307 = 0.0363 \rightarrow e \cong 0.24$

$T/R = 265/1742 = 0.152 < e$ - USE EQ. 14-5 ; $P = 1.0 R_c = 1742$ LB

$C_c = 1742\left(1.2\times10^8/10^6\right)^{1/3} = 8592$ LB $\rightarrow$ BRG. 6212 OK.

ROLLING CONTACT BEARING DESIGN CALCULATIONS - CHAPTER 14 | Summary of Design Calculations

USING DATA FROM TABLE 14-3 | INNER RACE ROTATES IN ALL CASES | See manual solutions for details of calculations

NS = Not specified

| PROB NO. & BRG | RADIAL LOAD, R (LB) | THRUST LOAD, T (LB) | SPEED (RPM) | LIFE (HR) | LIFE (REV.) | EQUIV LOAD, P (LB) | DYNAMIC LOAD, C (LB) | BRG. NO. | RATED LOAD, C (LB) | BEARING BORE (mm) | BEARING BORE (in) | MIN. BORE (in) | X | Y | $C_o$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 5-BRG. B | 4643 | 0 | 600 | 10000 | 3.60E+08 | 4643 | 33029 | 6319 | 34397 | 95 | 3.7402 | 3.55 | 1 | 0 | 48561 |
| 5-BRG. C | 2078 | 0 | 600 | 10000 | 3.60E+08 | 2078 | 14782 | 6311 | 16076 | 55 | 2.1654 | 1.09 | 1 | 0 | 48561 |
| 6-BRG. B | 611 | 0 | 1700 | 5000 | 5.1E+08 | 611 | 4882 | 6011 | 6317 | 55 | 2.1654 | 2.00 | 1 | 0 | 4766 |
| 6-BRG. D | 436 | 0 | 1700 | 5000 | 5.1E+08 | 436 | 3483 | 6011 | 6317 | 55 | 2.1654 | 1.98 | 1 | 0 | 4766 |
| 7-BRG. A | 509 | 0 | 101 | 20000 | 1.21E+08 | 509 | 2519 | 6302 | 2563 | 15 | 0.5906 | 0.59 | 1 | 0 | 1214 |
| 7-BRG. C | 1742 | 265 | 101 | 20000 | 1.21E+08 | 1742 | 8621 | 6212 | 10679 | 60 | 2.3622 | 2.26 | 1 | 0 | 7307 |
| 9 | 455 | 0 | 1150 | 20000 | 1.38E+09 | 455 | 5066 | 6306 | 6317 | 30 | 1.1811 | NS | 1 | 0 | 10 |
| 10 | 857 | 0 | 450 | 30000 | 8.1E+08 | 857 | 7989 | 6308 | 9218 | 40 | 1.5748 | NS | 1 | 0 | 10 |
| 11 | 1265 | 645 | 210 | 5000 | 6.30E+07 | 1579 | 6284 | 6307 | 7464 | 35 | 1.3780 | NS | 0.56 | 1.35 | 4272 |
| 12 | 235 | 88 | 1750 | 20000 | 2.1E+09 | 301 | 3860 | 6305 | 5058 | 25 | 0.9843 | NS | 0.56 | 1.93 | 2608 |
| 13 | 2875 | 1350 | 600 | 15000 | 5.4E+08 | 3919 | 31909 | 6318 | 32149 | 90 | 3.5433 | NS | 0.56 | 1.71 | 24281 |
| 14(M-->lb) | 854 | 0 | 3450 | 15000 | 3.11E+09 | 854 | 12459 | 6310 | 13894 | 50 | 1.9685 | NS | 1 | 0 | 10 |
| 14(kN) | 3.8 | 0 | 3450 | 15000 | 3.11E+09 | 3.80 | 55.44 | 6310 | 61.8 | 50 | 1.9685 | NS | 1 | 0 | 10 |
| 15(kN) | 5.6 | 2.8 | 450 | 2000 | 5.40E+07 | 6.78 | 25.61 | 6306 | 28.1 | 30 | 1.1811 | NS | 0.56 | 1.3 | 16 |
| 16(kN) | 10.5 | 0 | 1150 | 20000 | 1.38E+09 | 10.50 | 116.90 | 6316 | 124.0 | 80 | 3.1496 | NS | 1 | 0 | 10 |
| 16(M-->lb) | 2361 | 0 | 1150 | 20000 | 1.38E+09 | 2361 | 26286 | 6316 | 27878 | 80 | 3.1496 | NS | 1 | 0 | 10 |
| 17(kN) | 1.2 | 0.85 | 860 | 20000 | 1.03E+09 | 2.04 | 20.62 | 6305 | 22.5 | 25 | 0.9843 | NS | 0.56 | 1.61 | 11.6 |
| 24-1 | 1750 | 350 | 101 | | 0 | 1750 | 0 | 6211 | 9802 | 55 | 2.1654 | NS | 1 | 0 | 6520 |
| 24-2 | 600 | 250 | 101 | | 0 | 809 | 0 | 6211 | 9802 | 55 | 2.1654 | NS | 0.56 | 1.89 | 6520 |
| 24-3 | 280 | 110 | 101 | | 0 | 403 | 0 | 6211 | 9802 | 55 | 2.1654 | NS | 0.56 | 2.24 | 6520 |

See manual solution for combined loading and overall life for Problem 24.

**19.** VARYING LOADS: (BRG. 6324) $n = 600\,rpm$; $C = 46783$

| | | |
|---|---|---|
| 1 | 4500 LB | 25 MIN |
| 2 | 2500 LB | 15 MIN |
| | | 40 MIN. |

$$F_m = \left(\frac{25(4500)^3 + 15(2500)^3}{40}\right)^{1/3} = 3975\,LB$$

$$L = \left(C/F_m\right)^3 = \left(46783/3975\right)^3 = 1630 \times 10^6\,rev \times \frac{MIN}{600\,rev} \times \frac{h}{60\,MIN} =$$

$L = 45285\,h$

**20.** BEARING 6314, $n = 600\,rpm$; $C = 23381\,LB$.

| | | |
|---|---|---|
| 1. | 2500 LB | 25 MIN |
| 2. | 1500 LB | 15 MIN |
| | | 40 MIN |

$$F_m = \left(\frac{25(2500)^3 + 15(1500)^3}{40}\right)^{1/3} = 2226\,LB$$

$$L = \left(C/F_m\right)^3 = \left(23381/2226\right)^3 = \frac{1159 \times 10^6\,rev}{(600\,rev/min)(60\,min/h)} = 32,189\,h$$

**21.** BEARING 6209, $n = 1700\,rpm$, $C = 7464\,LB$

| | | |
|---|---|---|
| 1. | 600 LB | 480 MIN |
| 2. | 200 LB | 115 MIN |
| 3. | 100 LB | 45 MIN |
| | | 640 MIN |

$$F_m = \left(\frac{480(600)^3 + 115(200)^3 + 45(100)^3}{640}\right)^{1/3} = 547\,LB$$

$$L = \left(C/F_m\right)^3 = \left(7464\,LB/547\,LB\right)^3 = \frac{2580 \times 10^6\,rev}{(1700\,rev/min)(60\,min/h)} = 24,909\,h$$

**22.** BEARING 6209, $n = 1700\,rpm$, $C = 7464\,LB$

| | | |
|---|---|---|
| 1. | 450 LB | 480 MIN |
| 2. | 180 LB | 115 MIN |
| 3. | 50 LB | 45 MIN |
| | | 640 MIN |

$$F_m = \left(\frac{480(450)^3 + 115(180)^3 + 45(50)^3}{640}\right)^{1/3} = 411\,LB.$$

$$L = \left(\frac{C}{F_m}\right)^3 = \left(\frac{7464}{411}\right)^3 = \frac{5989 \times 10^6\,rev}{(1700\,rev/min)(60\,min/h)} = 58,720\,h$$

**23.** BEARING 6205, $n = 101$ rpm, $C = 3147$ LB

| | | |
|---|---|---|
| 1. | 500 LB | 6.75 h |
| 2. | 800 LB | 0.40 h |
| 3. | 100 LB | 0.85 h |
| | | 8.00 h |

$$F_m = \left(\frac{6.75(500)^3 + 0.40(800)^3 + 0.85(100)^3}{8.0}\right)^{1/3} = 508\text{ LB}$$

$$L = \left(\frac{C}{F_m}\right)^3 = \left(\frac{3147}{508}\right)^3 = \frac{237.7 \times 10^6 \text{ rev}}{(101 \text{ rev/min})(60 \text{ min/h})} = \underline{39231 \text{ h}}$$

**24.** BEARING 6211, $n = 101$ rpm, $C = 9802$ LB, $C_0 = 6520$ LB

COMBINED RADIAL AND THRUST LOADS:

COMPUTE EQUIVALENT LOAD P AS IN SECTION 14-10.

| | | R | T | T/R | $T/C_0$ | e | Y | P |
|---|---|---|---|---|---|---|---|---|
| 1. | 6.75 h | 1750 LB | 350 LB | 0.20 | 0.053 | 0.26 | — | 1750 LB = R |
| 2. | 0.40 h | 600 LB | 250 LB | 0.417 | 0.0383 | 0.234 | 1.89 | 809 LB |
| 3. | 0.85 h | 280 LB | 110 LB | 0.393 | 0.017 | 0.20 | 2.24 | 403 LB |
| | 8.00 h | | | | | | | |

$[P = 0.56\,R + Y\,T]$

USE EQUIVALENT LOADS TO COMPUTE $F_m$.

$$F_m = \left(\frac{6.75(1750)^3 + 0.40(809)^3 + 0.85(403)^3}{8.00}\right)^{1/3} = 1658\text{ LB}$$

$$L = \left(\frac{C}{F_m}\right)^3 = \left(\frac{9802}{1658}\right)^3 = \frac{206.7 \times 10^6 \text{ rev}}{(101 \text{ rev/min})(60 \text{ min/h})} = \underline{34,115 \text{ h}}$$

$\boxed{25.}$ $P_d = 1450$ LB. $n = 1150$ RPM. $L_d = 15000$ h. $R = 0.95 \Rightarrow C_R = 0.62$

ACTUAL $L_d = (15000\ h)(1150\ RPM)(60\ MIN/h) = 1.04 \times 10^9$ REV

ADJUSTED $L_{da} = L_d / C_R = 1.04 \times 10^9 / 0.62 = 1.68 \times 10^9$ REV

EQ 14-3: $C = P_d \left(\frac{L_{da}}{10^6}\right)^{1/k} = 1450 \left(\frac{1.68 \times 10^9}{10^6}\right)^{1/3} = \underline{17\,229\ LB.}$

$\boxed{26.}$ $P_d = 509$ LB. $n = 101$ RPM. $L_d = 20\,000$ h. $R = 0.99 \Rightarrow C_R = 0.21$

ACTUAL $L_d = (20000)(101)(60) = 1.21 \times 10^8$ REV.

ADJUSTED $L_{da} = L_d / C_R = 1.21 \times 10^8 / 0.21 = 5.77 \times 10^8$ REV

EQ. 14-3: $C = 509 \left(\frac{5.77 \times 10^8}{10^6}\right)^{1/3} = \underline{4238\ LB}$

$\boxed{27.}$ $P_d = 436$ LB. $n = 1700$ RPM. $L_d = 5000$ h. $R = 0.97 \Rightarrow 0.44$

ACTUAL $L_d = (5000)(1700)(60) = 5.10 \times 10^8$ REV.

ADJUSTED $L_{da} = L_d / C_R = 5.10 \times 10^8 / 0.44 = 1.16 \times 10^9$ REV

EQ. 14-3: $C = 436 \left(\frac{1.16 \times 10^9}{10^6}\right)^{1/3} = \underline{4580\ LB}$

$\boxed{28.}$ $P_d = 1250$ LB. $n = 880$ RPM. $L_d = 20000$ h. $R = 0.95 \Rightarrow C_R = 0.62$

ACTUAL $L_d = (20\,000\ h)(880\ RPM)(60\ MIN/h) = 1.06 \times 10^9$ REV

ADJUSTED $L_d = L_d / C_R = 1.06 \times 10^9 / 0.62 = 1.70 \times 10^9$ REV

EQ 14-3: $C = 1250 \left(\frac{1.70 \times 10^9}{10^6}\right)^{1/3} = \underline{14\,928\ LB}$

# CHAPTER 16
# PLAIN SURFACE BEARINGS

All of the problems in this chapter are design problems with no unique solutions. A sample of each type of design problem is shown here.

**1.** $F = 225$ LB; $D = 3.00$ IN; $n = 1750$ RPM

BOUNDARY LUBRICATED BEARINGS

LET $L = 1.5D = 1.5(3.00) = 4.50$ IN.

$$p = \frac{F}{LD} = \frac{225\text{ LB}}{(4.50)(3.00)\text{ IN}^2} = 16.67\text{ psi}$$

$$V = \pi D n/12 = \pi(3.00)(1750)/12 = 1374\text{ FT/min}$$

$$pV = (16.67)(1374) = 22\,900\text{ psi-fpm}$$

REQ'D $pV$-RATING $= 2(pV) = 2(22\,900) = 45\,800$ psi-fpm

POROUS BRONZE BEARING MATERIAL/OIL IMPREGNATED OR BU OR DU DRY LUBRICATED BEARING

**4.** $F = 75$ LB; $D = 0.50$ in; $n = 600$ rpm

LET $L = 1.5D = 1.5(0.50) = 0.75$ in

$p = F/LD = 75/(.75)(.50) = 200.0$ psi

$V = \pi(.50)(600)/12 = 78.5$ ft/min

$\Big\}\ pV = 15\,708$ psi-fpm

REQ'D $pV = 2(15\,708) = 31\,416$ psi-fpm POROUS BRONZE OR BU BEARING

**7.** $F = 800$ LB; $D = 3.00$ in; $n = 350$ rpm

LET $L = 1.50D = 1.50(3.00) = 4.50$ in

$p = F/LD = 800/(4.50)(3.00) = 59.3$ psi

$V = \pi(3.00)(350)/12 = 275$ fpm

$\Big\}\ pV = 16\,290$ psi-fpm

REQ'D $pV = 2(16\,290) = 32\,580$ psi-fpm POROUS BRONZE OR BU BEARING

**8.** $F = 60$ LB; $D = 0.75$ IN; $n = 750$ RPM: TRY $L = 1.25D = 1.25(0.75) = 0.938$ IN

LET $L = 1.00$ IN.; $L/D = 1.00/0.75 = 1.33$ OK

$p = F/LD = 60/(1.00)(0.75) = 80$ psi

$V = \pi D n/12 = \pi(0.75)(750)/12 = 147.3$ ft/min

$\Big\}\ pV = 11,784$ psi-fpm

REQD $pV$ RATING $= 2(11\,784) = 23\,568$ psi-fpm — USE BABBITT-HIGH TIN

**9.** $F = 1250\ LB$; $D_{MIN} = 2.60\ IN.$; $n = 1750\ RPM$; ELECTRIC MOTOR

LET $\underline{D = 2.75\ IN}$, $R = D/2 = 1.375\ IN.$

FOR $p = 300\ PSI$; $L = \dfrac{F}{pD} = \dfrac{1250\ LB}{(300\ LB/IN^2)(2.75\ IN)} = 1.515\ IN$

$L/D = 1.515/2.75 = 0.55$ : LET $\underline{L = 0.5D = 1.375\ IN}$; $\underline{L/D = 0.50}$

$p = \dfrac{F}{LD} = \dfrac{1250\ LB}{(1.375)(2.75)\ IN^2} = \underline{331\ psi}$ OK

$\underline{C_d = 0.0036\ IN}$ ; $C_r = 0.0018\ IN$ ; $R/C_r = 1.375/0.0018 = 754$
(FIG 16-3)

$\underline{\text{SURFACE FINISH : } 16\text{-}32\ \mu IN\ \text{AVG}}$

$h_o \doteq 0.00025(2.75\ IN) = 0.00069\ IN$ : USE $\underline{h_o = 0.0007\ IN.}$

$h_o/C_r = 0.0007/0.0018 = 0.389 \longrightarrow S = 0.29$
(FIG. 16-7)

$n_s = n/60 = 1750/60 = 29.17\ rev/s$

$\text{REQ'D}\ \mu = \dfrac{Sp}{n_s (R/C_r)^2} = \dfrac{(0.29)(331)}{(29.17)(754)^2} = 5.79 \times 10^{-6}\ \text{REYNS}$

$\underline{\text{SAE 50}}$ OIL HAS $\mu = 6.5 \times 10^{-6}$ REYNS @ 160°F

S IS PROPORTIONAL TO $\mu$ : $S = 0.29\,(6.5/5.79) = 0.326$

FROM FIG 16-8: $f(R/C_r) = 8.0$

$f = \dfrac{f(R/C_r)}{(R/C_r)} = \dfrac{8.0}{754} = 0.0106$

$T_f = fFR = (0.0106)(1250)(1.375) = 18.2\ lb\cdot in$

$P_f = \dfrac{T_f n}{63000} = \dfrac{(18.2\ lb\cdot in)(1750\ RPM)}{63000} = 0.506\ hp$

13.

$F = 500\ LB$ ; $D_{MIN} = 1.15\ IN$ ; $n = 2500\ RPM$ ; PRECISION SPINDLE

LET $\underline{D = 1.200\ IN}$ ; $R = D/2 = 0.600\ IN.$

FOR $p = 200$ psi ; $L = \dfrac{F}{pD} = \dfrac{500\ LB}{(200\ LB/IN^2)(1.200\ IN)} = 2.08$

$L/D = 2.08/1.200 = 1.73$ ; LET $\underline{L/D = 1.50}$ ; $\underline{L = 1.50\,D} = (1.5)(1.2) = \underline{1.800\ IN}$

$p = \dfrac{F}{LD} = \dfrac{500}{(1.800)(1.200)} = \underline{232\ psi}$ OK

$\underline{C_d = 0.0014\ IN}$ ; $C_r = 0.0007\ IN$ ; $R/C_r = 0.600/0.0007 = 857$

$\underline{\text{SURFACE FINISH: 8–16 } \mu\text{IN AVG.}}$

$h_o \approx 0.00025(1.20) = 0.00030\ IN$ ; $h_o/C_r = 0.0003/0.0007 = 0.429$

$S = 0.11$ ; $n_s = 2500/60 = 41.67\ rev/s$

REQ'D $\mu = \dfrac{S\,p}{n_s\,(R/C_r)^2} = \dfrac{(0.11)(232)}{(41.67)(857)^2} = 0.832 \times 10^{-6}$ REYNS

$\underline{\text{SAE 5W}}$ HAS $\mu = 0.91 \times 10^{-6}$ @ 160°F

$S$ IS PROPORTIONAL TO $\mu$ : $S = 0.11(.91/.832) = 0.120$

$f(R/C_r) = 2.80$ FROM FIG. 16-8 :

$f = \dfrac{f(R/C_r)}{(R/C_r)} = \dfrac{2.80}{857} = 0.0033$

$T_f = f\,F\,R = (0.0033)(500\ lb)(0.60\ in) = 0.98\ LB{\cdot}IN$

$P_f = T_f\,n/63000 = (0.98\ lb{\cdot}in)(2500\ rpm)/63000 = 0.039\ hp$

$\boxed{16.}$

$F = 18.7\ kN$; $D_{MIN} = 100mm$; $n = 500\ RPM$; CONVEYOR

$\underline{D = 100\ mm;\ R = 50\ mm}$

FOR $p = 2.0\ MPa = 2.0\ N/mm^2$; $L = \frac{F}{pD} = \frac{18.7\times10^3 N}{(2.0\ N/mm^2)100mm} = 93.5mm$

LET $\underline{L/D = 1.0}$; $\underline{L = D = 100\ mm}$

$\underline{p} = \frac{F}{LD} = \frac{18.7\times10^3 N}{(100)(100)\ mm^2} = 1.87\ N/mm^2 = \underline{1.87\ MPa}$ OK

$\underline{C_d = 0.15 mm}$ ; $C_r = 0.075\ mm$ ; $R/C_r = 50/0.075 = 667$
(LARGE CLEARANCE DESIRED)

$\underline{\text{SURFACE FINISH}}$: NOTE: $1.0\ \mu in = 1.0\times10^{-6}\ in \times \frac{.0254\ m}{in} = .0254 \frac{\mu m}{in}$

THEN $8\ \mu in = 0.20\ \mu m$; $16\ \mu in = 0.40\ \mu m$; $32\ \mu in = 0.80\ \mu m$
$63\ \mu in \approx 1.60\ \mu m$

SPECIFY $\underline{\text{SURFACE FINISH} = 0.8 \text{ TO } 1.6\ \mu m \text{ AVG.}}$

$h_o \doteq 0.00025(100) = 0.025\ mm$; $h_o/C_r = 0.025/0.075 = 0.333$

$S = 0.096$ ; $n_s = n/60 = 500/60 = 8.33\ REV/S$

REQ'D $\mu = \frac{S p}{n_s (R/C_r)^2} = \frac{(0.096)(1.87\times10^6 Pa)}{(8.33 REV/S)(667)^2} = 0.0485\ Pa\cdot S$

AT $70°C$, $\underline{\text{SAE 50}}$ HAS $\mu = 0.046\ Pa\cdot S$; $h_o$ SLIGHTLY $< 0.025mm$

$S = (0.096)\frac{0.046}{0.0485} = 0.091 \longrightarrow f(R/C_r) = 2.6$

$f = \frac{f(R/C_r)}{R/C_r} = \frac{2.6}{667} = 0.0039$

$T_f = fFR = (0.0039)(18.7\times10^3 N)(50\times10^{-3} m) = 3.65\ N\cdot m$

$P_f = Tn = 3.65\ N\cdot m \times 8.33 \frac{REV}{S} \times \frac{2\pi\ RAD}{REV} = 191 \frac{N\cdot m}{S} = 191$ WATTS

$\boxed{17.}$

$F = 2.25\,kN$; $D_{min} = 25\,mm$; $n = 2200\,RPM$; MACHINE TOOL

LET $\underline{D = 25\,mm}$; $R = D/2 = 12.5\,mm$ | $C_d = 0.036\,mm$; $C_r = 0.018\,mm$; $R/C_r = \dfrac{12.5}{0.018} = 694$

FOR $p = 2.0\,MPa$; $L = \dfrac{F}{pD} = \dfrac{2.25 \times 10^3\,N}{(2.0\,N/mm^2)(25\,mm)} = 45\,mm$

LET $\underline{L = 2D = 50\,mm}$; $\underline{L/D = 2.0}$

$$p = \frac{F}{LD} = \frac{2.25 \times 10^3\,N}{(50)(25)\,mm^2} = 1.80\,N/mm^2 = 1.80\,MPa \quad \underline{ok}$$

<u>SURFACE FINISH: 16-32 $\mu$in AVG. (0.4 TO 0.8 $\mu$m AVG.)</u>

$h_o \cong 0.00025(25) = 0.00625\,mm \cong 0.006\,mm$

$h_o/C_r = 0.006/0.018 = 0.333 \longrightarrow S = 0.057$

$n_s = n/60 = 2200/60 = 36.7\,REV/S$

$$\text{REQ'D. } \mu = \frac{S\,p}{n_s (R/C_r)^2} = \frac{(0.057)(1.80 \times 10^6\,Pa)}{(36.7\,REV/s)(694)^2} = 0.0058\,Pa \cdot s$$

<u>SAE 5W</u> HAS $\mu = 0.0066\,Pa \cdot s$ @ $70\,°C$

$S = 0.057\,(.0066/0.0058) = 0.065 \longrightarrow f(R/C_r) = 1.6$

$$f = \frac{f(R/C_r)}{R/C_r} = \frac{1.6}{694} = 0.0023$$

$T_f = f\,F\,R = (0.0023)(2.25 \times 10^3\,N)(12.5 \times 10^{-3}\,m) = 0.065\,N \cdot m$

$$P_f = T_f\,n = (0.065\,N \cdot m)\left(36.7\,\frac{REV}{s}\right)\left(\frac{2\pi\,RAD}{REV}\right) = 15.0\,\frac{N \cdot m}{s}$$

$P_f = 15.0$ WATTS

19. HYDROSTATIC LUBRICATION

$F = 1250\ LB;\ p_s = 300\ psi$ : LET $p_r = 250\ psi$

LET $R_r/R = 0.50$ ; $a_f = 0.55$ ; $q_f = 1.40$

$$A_p = \frac{F}{a_f p_r} = \frac{1250\ LB}{(0.55)(250\ LB/IN^2)} = 9.09\ IN^2 = \pi D^2/4$$

$$D = \sqrt{4A_p/\pi} = \sqrt{4(9.09)/\pi} = 3.40\ IN. \quad OK$$

$R = D/2 = 1.70\ IN$ ; $R_r = 0.5R = 0.5(1.7) = 0.85\ IN.$

LET $h = 0.005\ IN$ ; SAE 30 OIL @ 120°F — $\mu = 8.3\times10^{-6}\ LB\cdot S/IN^2$

$$Q = q_f \frac{F\,h^3}{A_p\,\mu} = \frac{(1.40)(1250\ LB)(.005)^3 IN^3}{(9.09\ IN^2)(8.3\times10^{-6}\ LB\cdot S/IN^2)} = 2.90\ IN^3/S$$

$$P = p_r Q = (250\ LB/IN^2)(2.90\ IN^3/S) = 725\ LB\cdot IN/S \times \frac{1\ hp}{6600\ LB\cdot IN/S} = 0.11\ hp$$

21.

$F = 3500\ LB;\ p_s = 500\ psi$ : LET $p_r = 350\ psi$

LET $R_r/R = 0.60$; $a_f = 0.62$ ; $q_f = 1.60$

$$A_p = \frac{F}{a_f p_r} = \frac{3500\ LB}{(.62)(350\ LB/IN^2)} = 16.13\ IN^2$$

$$D = \sqrt{4A_p/\pi} = \sqrt{4(16.13)/\pi} = 4.53\ IN \text{ — USE } D = 4.50\ IN.$$

$$A_p = \pi D^2/4 = \pi(4.50)^2/4 = 15.90\ IN^2$$

$$p_r = \frac{F}{a_f A_p} = \frac{3500\ LB}{(0.62)(15.90\ IN^2)} = 355\ psi \quad OK$$

$R = D/2 = 2.25\ IN.$; $R_r = 0.60R = 1.35\ IN.$

LET $h = 0.008\ IN.$ : SAE 40 OIL @ 140°F; $\mu = 7.0\times10^{-6}\ LB\cdot S/IN^2$

$$Q = \frac{q_f F h^3}{A_p \mu} = \frac{(1.60)(3500\ LB)(0.008)^3 IN^3}{(15.90\ IN^2)(7.0\times10^{-6}\ LB\cdot S/IN^2)} = 25.8\ IN^3/S$$

$$P = p_r Q = (355\ LB/IN^2)(25.8\ IN^3/S) = \frac{9145\ LB\cdot IN/S}{(6600\ LB\cdot IN/S)/hp} = 1.39\ hp$$

**25.** HYDROSTATIC LUBRICATION – METRIC DATA

$F = 22.5\,kN$; $p_s = 2.0\,MPa$ : LET $p_r = 1.60\,MPa = 1.6\times10^6\,N/m^2 = 1.60\,N/mm^2$

$R_r/R = 0.60$; $a_f = 0.62$; $q_f = 1.60$

$$A_p = \frac{F}{a_f p_r} = \frac{22.5\times10^3\,N}{(.62)(1.60\,N/mm^2)} = 2.27\times10^4\,mm^2$$

$D = \sqrt{4A_p/\pi} = \sqrt{4(2.27\times10^4)/\pi} = 170\,mm$ OK

$R = D/2 = 85\,mm$; $R_r = 0.6R = 51.0\,mm$

LET $h = 0.15\,mm$ : at 50°C, SAE 30 OIL HAS $\mu = 0.054\,Pa\cdot s$

$$Q = \frac{q_f F h^3}{A_p \mu} = \frac{(1.60)(22500\,N)(.15)^3\,mm^3}{(2.27\times10^4\,mm^2)(0.054\,N\cdot s/m^2)}\times\frac{1\,m}{10^3\,mm} = 9.91\times10^{-5}\,m^3/s$$

$$P = p_r Q = \left(1.60\times10^6\,\frac{N}{m^2}\right)\left(9.91\times10^{-5}\,\frac{m^3}{s}\right) = 159\,\frac{N\cdot m}{s} = 159\text{ WATTS}$$

**26.** $F = 1.20\,kN$; $p_s = 750\,kPa = 0.75\,MPa$

LET $p_r = 600\,kPa = 0.60\,MPa = 0.60\times10^6\,N/m^2 = 0.60\,N/mm^2$

$R_r/R = 0.60$; $a_f = 0.62$; $q_f = 1.60$

$$A_p = \frac{F}{a_f p_r} = \frac{1.20\times10^3\,N}{(.62)(.60\,N/mm^2)} = 3226\,mm^2$$

$D = \sqrt{4A_p/\pi} = \sqrt{4(3226)/\pi} = 64\,mm$ OK

$R = D/2 = 32\,mm$; $R_r = 0.6R = .6(32) = 19.2\,mm$

LET $h = 0.10\,mm$ : at 60°C, SAE 10W OIL HAS $\mu = 0.014\,Pa\cdot s$

$$Q = \frac{q_f F h^3}{A_p \mu} = \frac{(1.60)(1200\,N)(0.10)^3\,mm^3}{(3226\,mm^2)(0.014\,N\cdot s/m^2)}\cdot\frac{1\,m}{10^3\,mm} = 4.25\times10^{-5}\,m^3/s$$

$$P = p_r Q = 0.60\times10^6\,\frac{N}{m^2}\cdot 4.25\times10^{-5}\,\frac{m^3}{s} = 25.5\,\frac{N\cdot m}{s} = 25.5\text{ WATTS}$$

# CHAPTER 17
# LINEAR MOTION ELEMENTS

**5.** REQ'D. $A_t = \frac{F}{\sigma_a} = \frac{30\,000\ \text{LB}}{10\,000\ \text{LB/IN}^2} = 3.0\ \text{IN}^2$ USE 2½-3 ACME THREAD.
$A_t = 3.802\ \text{IN}^2$

**6.** REQ'D $A_s = \frac{F}{\tau_a} = \frac{30\,000\ \text{LB}}{6000\ \text{LB/IN}^2} = 5.0\ \text{IN}^2$

FOR AN ACME 2½-3 : $A_s = 4.075\ \text{IN}^2/\text{IN. OF LENGTH}$
REQ'D. $L = 5.0/4.075 = 1.23\ \text{IN.}$

**7.** $\lambda = \tan^{-1}\left(\frac{L}{\pi D_p}\right) = \tan^{-1}\left(\frac{0.333}{\pi(2.2939)}\right) = \tan^{-1}(0.0463) = 2.65°$ ← tan λ

$\cos\phi = \cos 14.5° = 0.968$
(EQ. 18-10)

$$T_u = \frac{FD_p}{2}\left[\frac{\cos\phi\tan\lambda + f}{\cos\phi - f\tan\lambda}\right] = \frac{30000(2.2939)}{2}\left[\frac{(.968)(.0463)+.15}{.968-.15(.0463)}\right]$$

$T_u = 6974\ \text{lb·in}$

**8.** $$T_d = \frac{FD_p}{2}\left[\frac{f - \cos\phi\tan\lambda}{\cos\phi + f\tan\lambda}\right] = \frac{30000(2.2939)}{2}\left[\frac{.15-(.968)(.0463)}{.968+.15(.0463)}\right]$$

$T_d = 3712\ \text{lb·in}$

**9.** SQUARE THREAD: 3/4-6; $F = 4000\ \text{LB}$, $L = p = 1/n = 1/6 = 0.1667\ \text{IN.}$

$$T_u = \frac{FD_p}{2}\left[\frac{L + \pi f D_p}{\pi D_p - fL}\right] = \frac{4000(.6424)}{2}\left[\frac{.1667+\pi(.15)(.6424)}{\pi(.6424) - .15(.1667)}\right]$$

$T_u = 303\ \text{lb·in}$

**10.** $$T_d = \frac{FD_p}{2}\left[\frac{\pi f D_p - L}{\pi D_p + fL}\right] = \frac{4000(.6424)}{2}\left[\frac{\pi(.15)(.6424) - .1667}{\pi(.6424) + .15(.1667)}\right] = 86\ \text{LB·IN}$$

**11.** $\lambda = \tan^{-1}(L/\pi D_p) = \tan^{-1}(.1667/\pi(.6424)) = 4.72°$
$\tan\lambda = 0.083 < f$ → SELF LOCKING

12. $e = \frac{FL}{2\pi T_u} = \frac{(4000)(0.1667)}{2\pi(303)} = \underline{0.35}$ OR $\underline{35\%}$

13. $n = \frac{0.50\text{ IN}}{\text{S}} \times \frac{1\text{ REV.}}{0.1667\text{ IN}} \times \frac{60\text{ S}}{\text{MIN}} = \underline{180\text{ RPM}}$

$P = Tn/63000 = (303)(180)/63000 = \underline{0.866\text{ hp}}$

14. $\text{TRAVEL} = \frac{24\text{ in}}{\text{CYCLE}} \times \frac{10\text{ CYCLES}}{\text{HR}} \times \frac{24\text{ HR}}{\text{DAY}} \times \frac{365\text{ DAYS}}{\text{YR}} \times 10\text{ YRS} = 2.10 \times 10^7\text{ INCHES}$

AT 600 LB; $\underline{3/4-2\text{ SCREW REQ'D.}}$; $L = 0.50\text{ IN}$

15. $T = \frac{FL}{2\pi e} = \frac{(600)(.50)}{2\pi(.9)} = \underline{53.1\text{ lb-in}}$

16. $n = \frac{1\text{ REV}}{0.50\text{ IN}} \times \frac{10.0\text{ IN}}{\text{MIN.}} = \underline{20.0\text{ RPM}}$

$P = \frac{Tn}{63000} = \frac{53.1(20.0)}{63000} = \underline{0.017\text{ hp.}}$

17. FOR THE 3/4-2 SCREW AT 600 LB, TRAVEL LIFE $= 1.04 \times 10^8$ IN.

$1.04 \times 10^8\text{ IN.} \times \frac{\text{CYCLE}}{24\text{ IN}} \times \frac{\text{HR}}{20\text{ CYCLES}} \times \frac{\text{DAY}}{24\text{ HR}} \times \frac{\text{YR}}{365\text{ DAYS}} = \underline{24.7\text{ YRS}}$

METRIC TRAPEZOIDAL POWER SCREWS – TABLE 17-1 M

PROBLEMS 18 TO 23 USE SAME DATA

18. SPECIFY A SIZE: LOAD = 125 kN; $\sigma_a = 75\text{ MPa} = \frac{F}{A_T}$

REQ'D. $A_T = \frac{F}{\sigma_a} = \frac{125\,000\text{ N}}{75\text{ N/mm}^2} = 1667\text{ mm}^2$ – USE M55 X 9 SCREW

$A_T = 1791\text{ mm}^2$

NOMINAL $D_o = 55\text{ mm}$; LEAD = PITCH = 9.0 mm FOR SINGLE THREAD

$D_p = 50.5\text{ mm}$

19. FIND TORQUE TO RAISE 125 kN LOAD FOR $f = 0.15$.

$T_U = \frac{FD_p}{2}\left[\frac{L + \pi f D_p}{\pi D_p - fL}\right] = \frac{125\,000\text{ N}(50.5\text{ mm})}{2}\left[\frac{9.0 + \pi(0.15)(50.5)}{\pi(50.5) - (0.15)(9.0)}\right]$ EQ 17-2

$\underline{T_U = 658\,559\text{ N}\cdot\text{mm}} = \underline{658.6\text{ N}\cdot\text{m}}$

20. FIND TORQUE TO LOWER LOAD. $F = 125\text{ kN}$, $f = 0.15$

$T_d = \frac{FD_p}{2}\left[\frac{\pi f D_p - L}{\pi D_p + fL}\right] = \frac{125\,000\text{ N}(50.5\text{ mm})}{2}\left[\frac{\pi(0.15)(50.5) - 9}{\pi(50.5) + 0.15(9)}\right]$ EQ 17-4

$T_d = 291\,905\text{ N}\cdot\text{mm} = 291.9\text{ N}\cdot\text{m}$

**21.** FIND POWER TO RAISE 125 kN – 4250 mm IN 7.5 S. <u>M55 X 9 SCREW</u>

$P = T_U n$. $T_U = 658.6$ N·m FROM PROB. 19.

$$n = \frac{v}{L} = \frac{d/t}{L} = \frac{4250\ mm}{7.5\ s} \times \frac{1\ REV}{9\ mm} \times \frac{2\pi\ RAD}{REV} = 395.6\ RAD/s$$

$P = T_U n = 658.6\ N\cdot m \times 395.6\ RAD/s = 260\ 547\ N\cdot m/s = 260\ 547\ W$ (VERY HIGH)

$P = 260.5$ kW

**22.** FIND LEAD ANGLE $\lambda$: $\lambda = Tan^{-1}\left(\frac{L}{\pi D_p}\right) = Tan^{-1}\left(\frac{9}{\pi(50.5)}\right) = 3.25°$ EQ. 17-3

FOR $\lambda < 5°$ – SELF LOCKING.

**23.** FIND EFFICIENCY: $\underline{e} = \frac{FL}{2\pi T_U} = \frac{125000\ N(9\ mm)}{2\pi(658\ 559\ N\cdot mm)} = 0.272$; <u>27.2%</u>

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**24.** SPECIFY A POWER SCREW SIZE: $F = 8500\ N$; $\sigma_a = 110\ MPa$

PROBLEM 24-29 USE SAME DATA.

REQD $A_T = \frac{F}{\sigma_a} = \frac{8500\ N}{110\ N/mm^2} = 77.3\ mm^2$ — M14 X 3 SCREW

$A_T = 103.9\ mm^2$

$D_p = 12.5\ mm$, $L = p = 3.0\ mm$

**25.** FIND TORQUE TO RAISE LOAD; $f = 0.15$

$$T_U = \frac{F D_p}{2}\left[\frac{L + \pi f D_p}{\pi D_p - fL}\right] = \frac{8500\ N(12.5\ mm)}{2}\left[\frac{3.0 + \pi(0.15)(12.5)}{\pi(12.5) - 0.15(3)}\right]$$

$\underline{T_U} = 12\ 167\ N\cdot mm = \underline{12.167\ N\cdot m}$

**26.** FIND TORQUE TO LOWER LOAD. $F = 8500\ N$; $f = 0.15$

$$T_d = \frac{F D_p}{2}\left[\frac{\pi f D_p - L}{\pi D_p + fL}\right] = \frac{8500\ N(12.5\ mm)}{2}\left[\frac{\pi(0.15)(12.5) - 3}{\pi(12.5) + 0.15(3)}\right]$$

$\underline{T_d} = 3956\ N\cdot mm = \underline{3.956\ N\cdot m}$

**27.** FIND POWER TO RAISE LOAD: 240 mm IN 3.5 S.

$P = T_U n$; $T_U = 12.167\ N\cdot m$ [PROB 25]

$$n = \frac{v}{L} = \frac{d/t}{L} = \frac{240\ mm}{3.5\ s} \times \frac{1\ REV}{3.0\ mm} \times \frac{2\pi\ RAD}{REV} = 143.62\ RAD/s$$

$\underline{P} = T \cdot n = 12.167\ N\cdot m\ (143.62\ RAD/s) = 1747\ N\cdot m/s = 1747\ W = \underline{1.747\ kW}$

**28.** LEAD ANGLE $\lambda = Tan^{-1}\left(\frac{L}{\pi D_p}\right) = Tan^{-1}\left(\frac{3.0}{\pi(12.5)}\right) = 4.37°$

**29.** EFFICIENCY = $\underline{e} = \frac{FL}{2\pi T_U} = \frac{8500\ N(3.0\ mm)}{2\pi(12167\ N\cdot mm)} = 0.334 = \underline{33.4\%}$

$\boxed{30.}$ BALL SCREW FROM PROBLEM 14, $3/4-2$, LENGTH = 28.0 IN
FIND ESTIMATE OF CRITICAL SPEED. EQ. 17-15

$$n_c\,(RPM) = \frac{4.76\times10^6\, d\, K_s}{(SF)\, L^2}$$

d = MINOR DIA. OF SCREW. THIS VALUE NOT AVAILABLE IN THIS BOOK, OR ON INTERNET SITE 10. AS AN ESTIMATE, USE MINOR DIA. FOR A 3/4 ACME SCREW – $d \cong 0.55$ IN – FROM MACHINERY'S HANDBOOK, 28TH ED., P. 1830.

L = 28.0 IN – BETWEEN SINGLE BEARINGS AT ENDS

$K_s = 1.00$

LET SF = 1.0 TO ESTIMATE CRITICAL SPEED.

$$n_c = \frac{4.76\times10^6(0.55\,\text{IN})(1.0)}{(1)\,(28.0\,\text{IN})^2} = 3339\ \text{RPM}$$

SAFE OPERATING SPEED – SF = 3.0

$$n_{OPER} = \frac{n_c}{SF} = \frac{3339\ \text{RPM}}{3} = 1113\ \text{RPM}$$

# CHAPTER 18
# SPRINGS

**1** $k = \Delta F/\Delta L = 12.0/(2.75-1.85) = \underline{13.3 \text{ LB/IN.}}$

**2** $k = \dfrac{F_o - 0}{L_f - L_o}$ ; $L_f - L_o = \dfrac{F_o}{k}$ ; $L_f = L_o + \dfrac{F_o}{k} = 1.25 + \dfrac{4.65 \text{ LB}}{18.8 \text{ LB/IN}} = \underline{1.497 \text{ IN.}}$

**3** $k = \dfrac{F_s - F_o}{L_o - L_s}$ : $F_s = k(L_o - L_s) + F_o = 76.7(0.830 - 0.626) + 32.2 = \underline{47.8 \text{ LB} = F_s}$

$L_f = L_o + F_o/k = 0.830 + 32.2/76.7 = \underline{1.25 \text{ IN} = L_f}$

**4** $k = \Delta F/\Delta L = 99.2 \text{ N}/(63.5 - 37.1)\text{ mm} = \underline{3.76 \text{ N/mm}}$

**5** $L_f = L_o + F_o/k = 39.47 \text{ mm} + 54.05 \text{ N}/1.47 \text{ N/mm} = \underline{76.24 \text{ mm}}$

**6** $F_s = k(L_o - L_s) + F_o = 8.95(29.4 - 21.4) + 134 \text{ N} = \underline{205.6 \text{ N} = F_s}$

$L_f = L_o + F_o/k = 29.4 + 134/8.95 = \underline{44.4 \text{ mm} = L_f}$

**7** $ID = OD - 2D_w = 1.100 - 2(0.085) = \underline{0.93 \text{ IN}} = ID$

$D_m = OD - D_w = 1.100 - 0.085 = \underline{1.015 \text{ IN} = D_m}$

$C = D_m/D_w = 1.015/0.085 = \underline{11.94 = C}$

$L_s = N D_w$ ; $N = L_s/D_w = 0.563/0.085 = \underline{6.6 \text{ COILS}}$

**8** $D_m = OD - D_w = 0.560 - 0.059 = 0.501 \text{ IN}$ : $C = D_m/D_w = 0.501/0.059 = \underline{8.49 = C}$

$L_f = p N_a + 2D_w$ : $N_a = N - 2 = 19 - 2 = 17$ ACTIVE COILS

$p = \dfrac{L_f - 2D_w}{N_a} = \dfrac{4.22 - 2(0.059)}{17} = \underline{0.241 \text{ IN} = p}$

$\lambda = \tan^{-1}\left(\dfrac{p}{\pi D_m}\right) = \tan^{-1}\left(\dfrac{0.241}{\pi(0.501)}\right) = \underline{8.7 \text{ DEGREES} = \lambda}$

$L_s = N D_w = 19(0.059) = \underline{1.12 \text{ IN.} = L_s}$

$\boxed{9}$ FROM EQ 19-6: $F_o = \frac{f_o G D_w}{8C^3 N_a} = \frac{(4.22-3.00)(11.85\times10^6)(0.059)}{8(8.49)^3(17)} = \underline{10.25\,LB = F_o}$

EQ. 7-4: $\tau_o = \frac{8KF_oC}{\pi D_w^2} = \frac{8(1.17)(10.25)(8.49)}{\pi(0.059)^2} = \underline{74\,500\,PSI = \tau_o}$

$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4(8.49)-1}{4(8.49)-4} + \frac{0.615}{8.49} = 1.17$

FROM FIGURE 19-9: MUSIC WIRE: $\tau_d = 132000\,PSI$ FOR AVG. SERVICE - $\underline{OK}$.

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$\boxed{10.}$ $L_f/D_m = 4.22/0.501 = 8.42$: FROM FIG 19-15, CURVE A, $(f_o/L_f)_{CR} = 0.18$

$(f_o)_{CR} = 0.18\,L_f = 0.18(4.22) = 0.76\,IN$

ACTUAL $f_o = 4.22 - 3.00 = 1.22\,IN > 0.76\,IN$ - $\underline{\text{BUCKLING SHOULD OCCUR}}$

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$\boxed{11.}$ (EQ. 19-3)

$(OD)_s = \sqrt{D_m^2 + \frac{p - D_w^2}{\pi^2}} + D_w = \sqrt{(.501)^2 + \frac{0.241-(0.059)^2}{\pi^2}} + .059 = 0.583\,IN$

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$\boxed{12.}$ $F_s = k(L_f - L_s) = 8.40(4.22 - 1.12) = 26.05\,LB$

FROM PROB. 9; $k = \frac{F_o}{L_f - L_o} = \frac{10.25}{4.22 - 3.00} = 8.40\,LB/IN$

$\tau_s = \tau_o \times F_s/F_o = 74\,500\,PSI \times 26.05/10.25 = \underline{189\,300\,PSI;}$ TOO HIGH

FROM FIG 19-9, LIGHT SERVICE, $\tau_d = 147\,000\,PSI$ = APPROX. $S_y$

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**Notes concerning Problems 13 - 35:** Most of these problems are design problems. No single unique solutions exist. Sample solutions are shown.

Problems 13 through 24 are compression springs. Problem 13 is done by both methods 1 and 2 as outlined in the text. Others are done by one or the other method. In some problems only summary results are shown.

Problems 25 through 31 are extension springs. Problem 25 is worked out in detail. Problems 26 through 30 were designed with the aid of a computer program using the same procedure. Only summary results are shown. Problem 31 is the stress analysis of the ends of the spring.

Problems 32 through 35 are torsion springs. In each case, the design of each end is required. It was assumed that ends would be straight with lengths $L_1$ and $L_2$ as shown. The effects of the ends on the spring rate were then included in the analysis.

**13** **METHOD 1** $F_o = 220$ LB; $F_i = 180$ LB; $\Delta F = 220 - 180 = 40$ LB; $\Delta L = 0.50$ IN.

STEP 1 — ASTM A229; SEVERE SERVICE; $G = 11.2 \times 10^6$ PSI

STEPS 2-5 — LET $L_i = 3.00$ IN; $D_m = 3.00$ IN. (DESIGN DECISIONS)

$$k = \Delta F/\Delta L = 40\text{ LB}/0.50\text{ IN} = 80\text{ LB/IN}$$
$$L_f = L_i + F_i/k = 3.00 + 180/80 = 5.25\text{ IN}$$
$$L_o = L_i - \Delta L = 3.00 - 0.50 = 2.50\text{ IN}$$
$$f_o = L_f - L_o = 5.25 - 2.50 = 2.75\text{ IN}$$

STEP 6 — ASSUME $\tau_d = 80\,000$ PSI

STEP 7 — (EQ. 18-7)

$$D_w = \left[\frac{(3.06)(F_o)(D_m)}{\tau_d}\right]^{1/3} = \left[\frac{3.06(220)(3.00)}{80\,000}\right]^{1/3} = 0.293\text{ IN.}$$

STEP 8 — SELECT $D_w = 0.3065$ IN U.S. GAGE 0; $\tau_d = 78$ KSI; $\tau_{MAX} = 104$ KSI

STEP 9 — $C = D_m/D_w = 3.00/0.3065 = 9.79$; $K = 1.15$ (FIG. 18-14)

STEP 10 —

$$\tau_o = \frac{8KF_oD_m}{\pi D_w^3} = \frac{8(1.15)(220)(3.00)}{\pi(0.3065)^3} = 67\,126\text{ PSI} \quad \text{OK}$$

$$\text{STEP 11} - N_a = \frac{GD_w}{8kC^3} = \frac{(11.2\times10^6)(0.3065)}{8(80)(9.79)^3} = 5.72\text{ COILS}$$

STEP 12 —
$$L_s = D_w(N_a + 2) = 0.3065(5.72 + 2) = 2.365\text{ IN}$$
$$F_s = k(L_f - L_s) = 80(5.25 - 2.365) = 231\text{ LB}$$
$$\tau_s = \tau_o(F_s/F_o) = 67\,126(231/220) = 70\,420\text{ PSI} \quad \text{OK}$$

STEP 13 —
$$D_o = D_m + D_w = 3.00 + 0.3065 = 3.3065\text{ IN.}$$
$$D_i = D_m - D_w = 3.00 - 0.3065 = 2.694\text{ IN.}$$

<u>SUMMARY</u>: $D_w = 0.3065$ IN; $L_f = 5.25$ IN; $L_o = 2.50$ IN; $D_m = 3.000$ IN.

**13** **METHOD 2** PROCESS STARTS SAME AS METHOD 1 WITHOUT $D_m$.

STEP 2 — (EQ. 18-10) $D_w = \sqrt{21.4\,F_o/\tau_d} = \sqrt{21.4(220)/80000} = 0.243$ IN

STEP 3 — TRY $D_w = 0.2625$ IN. U.S. GAGE 2, $\tau_d = 80\,000$ PSI

STEP 4 — (EQ. 18-11)

$$N_{a_{MAX}} = (L_o - 2D_w)/D_w = [2.50 - 2(0.2625)]/0.2625 = 7.5\text{ COILS}$$

USE $N_a = 6.0$ STEPS

$$\text{STEP 6} - (\text{EQ } 19\text{-}12)\ C = \left[\frac{GD_w}{8kN_a}\right]^{1/3} = \left[\frac{(11.2\times10^6)(0.2625)}{8(80)(6.0)}\right]^{1/3} = 9.15$$

$K = 1.16$ (FIG. 7-12)

(EQ 18-4)
$$\text{STEP 8}\quad \tau_o = \left(\frac{8}{\pi}\right)\frac{KCF_o}{D_w^2} = \frac{2.546(1.16)(9.15)(220)}{(0.2625)^2} = 86\,262\text{ PSI} \left\{\begin{matrix}\text{TOO}\\ \text{HIGH}\end{matrix}\right\}$$

REPEAT FROM STEP 3: TRY $D_w = 0.2830$ IN. U.S. GAGE 1, $\tau_d = 79{,}000$ PSI

$\tau_{MAX} = 104\,000$ PSI

(CONTINUED — NEXT PAGE)

**13.** (CONTINUED) STEP 4 – $Na_{MAX} = [2.50-2(0.2830)]/0.2830 = 6.83$

USE $Na = 6$ COILS

STEP 6 – $C = \left[\frac{(11.2\times10^6)(0.2830)}{8(80)(6.0)}\right]^{1/3} = 9.38$ ; $K = 1.155$ (STEP 5)

OPERATING STRESS – $\tau_0 = \frac{(2.546)(1.155)(9.38)(220)}{(0.2830)^2} = 75\,770$ PSI OK

– SOLID LENGTH & STRESS AT SOLID LENGTH

$L_s = D_w(N_a+2) = 0.2830(6.0+2) = 2.264$ IN

$F_s = k(L_f - L_s) = (80\text{ LB/IN})(5.25-2.264)\text{ IN} = 239$ LB

STRESS AT SOLID HEIGHT $\tau_s = \tau_0(F_s/F_0) = 75\,770(239/220) = 82\,272$ PSI OK

FINAL GEOMETRY – $D_m = CD_w = 9.38(0.2830) = 2.655$ IN.

$D_o = D_m + D_w = 2.655 + 0.2830 = 2.938$ IN

$D_i = D_m - D_w = 2.655 - 0.2830 = 2.372$ IN

$cc = (L_o - L_s)/N_a \doteq (2.50 - 2.264)/6.0 = 0.0393\text{ IN} > \frac{D_w}{10}$ OK

SUMMARY: $D_w = 0.2830$ IN, $L_f = 5.25$ IN, $L_o = 2.50$ IN., $D_m = 2.655$ IN.

**14.** METHOD 2 $L_o = 1.75$ IN ; $F_o = 22.0$ LB ; $L_i = 3.00$ IN ; $F_i = 5.0$ LB

ASTM A401 ; SEVERE SERVICE ; $\tau_d \approx 125$ KSI

$k = \frac{F_o - F_i}{L_i - L_o} = \frac{22.0-5.0}{3.00-1.75} = 13.6\text{ LB/IN} = k$ | $f_o = L_f - L_o = 3.368 - 1.75$ ; $f_o = 1.618$ IN

$L_f = L_i + F_i/k = 3.00 + 5.0/13.6 = 3.368$ IN

$D_w = \sqrt{21.4(F_o)/\tau_d} = \sqrt{21.4(22)/125\,000} = 0.061$ IN.

TRY 16 GA. ; $D_w = 0.0625$ IN $\rightarrow \tau_d = 130$ KSI ; $\tau_{MAX} = 188$ KSI

$Na_{MAX} = (L_o - 2D_w)/D_w = [1.75 - 2(0.0625)]/0.0625 = 26$ : TRY $Na = 20$

$C = \left[\frac{G D_w}{8k\,N_a}\right]^{1/3} = \left[\frac{(11.2\times10^6)(0.0625)}{8(13.6)(20)}\right]^{1/3} = 6.85 \rightarrow K = 1.225$

$\tau_o = \frac{2.546\,K\,C\,F_o}{D_w^2} = \frac{2.546(1.225)(6.85)(22.0)}{(0.0625)^2} = 120\,325$ PSI OK

$L_s = D_w(N_a+2) = 0.0625(22) = 1.375$ IN.

$F_s = k(L_f - L_s) = 13.6(3.368 - 1.375) = 27.1$ LB

$\tau_s = \tau_o(F_s/F_o) = 120\,325\text{ PSI}(27.1/22.0) = 148\,245$ PSI OK

$D_m = C D_w = 6.85(0.0625) = 0.428\text{ IN.} = D_m$

$D_o = D_m + D_w = 0.491$ ; $D_i = D_m - D_w = 0.366$ IN.

$cc = (L_o - L_s)/N_a = (1.75 - 1.375)/20 = 0.019\text{ IN} > D_w/10$ OK

BUCKLING: $L_f/D_m = 3.368/0.428 = 7.87$ : $(f_o/L_f)_{CR} = 0.21$ (FIG. 19-15)

$(f_o)_{CR} = 0.21 L_f = 0.21(3.368) = 0.711$ IN. : ACTUAL $f_o > (f_o)_{CR}$ – WILL BUCKLE

14. (CONTINUED) AFTER SEVERAL ITERATIONS THIS DESIGN WAS PRODUCED WHICH WILL NOT BUCKLE.

$D_w = 0.072$ IN ; $\tau_d = 129$ KSI ; $\tau_{MAX} = 185$ KSI

$Na_{MAX} = 22.3$ ; USED $Na = 12$ ; $C = 8.52$ ; $D_m = 0.613$ IN.

$\tau_o = 108200$ PSI ; $L_s = 1.008$ IN. ; $F_s = 321$ LB ; $\tau_s = 157.9$ KSI

CC = 0.062 IN (GOOD) : BUCKLING – $L_f/D_m = 5.49$ ; $(f_o/L_f)_{CR} = 0.53$

$(f_o)_{CR} = 0.53 L_f = 1.78$ IN ; ACTUAL $f_o = 1.62$ IN OK

15. METHOD 2 $L_o = 1.25$ IN ; $F_o = 14.0$ LB ; $L_i = 2.00$ IN. ; $F_i = 1.50$ LB
ASTM A313, TYPE 302, AVG. SERVICE ; $\tau_d \approx 100$ KSI

$$k = \frac{14-1.50}{2.00-1.25} = 16.67 \text{ LB/IN} : L_f = L_i + \frac{F_i}{k} = 2.00 + \frac{1.50}{16.67} = 2.09 \text{ IN.}$$

$$D_w = \sqrt{21.4(F_o)/\tau_d} = \sqrt{21.4(14.0)/100\,000} = 0.055 \text{ IN.}$$

TRY 16 GA. ; $D_w = 0.0625$ IN ; $\tau_d = 115$ KSI ; $\tau_M = 128$ KSI

$$Na_{MAX} = (L_o - 2D_w)/D_w = [1.25 - 2(.0625)]/.0625 = 18.0 \rightarrow \text{USE } Na = 16$$

$$C = \left[\frac{G D_w}{8(k) Na}\right]^{1/3} = \left[\frac{(10 \times 10^6)(.0625)}{8(16.67)(16)}\right]^{1/3} = 6.64 \text{ THEN } K = 1.23$$

$$\tau_o = \frac{2.546(1.23)(6.64)(14.0)}{(.0625)^2} = 74540 \text{ PSI OK}$$

$$L_s = D_w(Na+2) = 0.0625(18) = 1.125 \text{ IN.}$$

$$F_s = k(L_f - L_s) = 16.67(2.09 - 1.125) = 16.09 \text{ LB}$$

$$\tau_s = \tau_o (F_s/F_o) = 74540 (16.09/14.0) = 85650 \text{ PSI OK}$$

$$D_m = C D_w = 6.64(0.0625) = 0.415$$

$$L_f/D_m = 2.09/.415 = 5.04 \text{ NO BUCKLING}$$

$$CC = (L_o - L_s)/Na = (1.25 - 1.125)/16 = 0.008 \text{ IN} > D_w/10 \text{ OK}$$

$$D_o = D_m + D_w = 0.415 + .0625 = 0.478 \text{ IN}$$

$$D_i = D_m - D_w = 0.415 - 0.0625 = 0.353 \text{ IN.}$$

**16** METHOD 2: $L_o = 4.00\,IN$, $F_o = 250\,LB$, $L_i = 10.50\,IN$, $F_i = 60\,LB$

ASTM A231: $\tau_d = 90\,KSI$ FOR SEVERE SERVICE

$$k = \frac{F_o - F_i}{L_i - L_o} = \frac{250-60}{10.50-4.00} = 29.23\,LB/IN \;;\; L_f = 10.50 + 60\,LB/29.23\,LB/IN = 12.55\,IN$$

$$D_w = \sqrt{21.4\,F_o/\tau_d} = \sqrt{21.4(250)/90000} = 0.244\,IN.$$

TRIALS WITH GAGES 2 AND 1 FAILED - $\tau_0$ TOO HIGH

FOR $D_w = 0.3065\,IN$ (0 GAGE), $\tau_d = 90000\,PSI$, $\tau_{max} = 130\,KSI$

$$N_{a\,max} = (L_o - 2D_w)/D_w = [4.00 - 2(0.3065)]/0.3065 = 11.05;\ TRY\ N_a = 10$$

$$C = \left[\frac{G\,D_w}{8k\,N_a}\right]^{1/3} = \left[\frac{(11.2\times10^6)(0.3065)}{8(29.23)(10)}\right]^{1/3} = 11.37;\ THEN\ K = 1.125$$

$$\tau_o = \frac{(2.546)\,K\,C\,F_o}{D_w^2} = \frac{(2.546)(1.125)(11.37)(250)}{(0.3065)^2} = 86{,}680\,PSI\ \underline{OK}$$

$$L_s = D_w(N_a+2) = (0.3065)(12) = 3.678\,IN$$

$$F_s = k(L_f - L_s) = (29.23)[12.55 - 3.678] = 259\,LB$$

$$\tau_s = \tau_o\,F_s/F_o = (86\,680)\,259/250 = 89{,}900\,PSI < \tau_{max}\ \underline{OK}$$

$$D_m = C\,D_w = (11.37)(0.3065) = 3.485\,IN$$

$$L_f/D_m = 12.55/3.485 = 3.60\ \text{NO BUCKLING}$$

$$CC = (L_o - L_s)/N_a = (4.00 - 3.678)/10 = 0.032\,IN > D_w/10\ \underline{OK}$$

**17**

METHOD 2 $F_o = 14.0\,LB$; $L_o = 0.68\,IN$; $F_i = 0$; $L_i = L_f = 1.75\,IN$: $k = \frac{14}{1.75-.68} = 13.08\,LB/IN.$

MUSIC WIRE; AV. SERV.; $\tau_d = 120\,KSI$

$$D_w = \sqrt{21.4(14)/120\,000} = 0.050\,IN:\ USE\ D_w = 0.055\,IN,\ 24\,GAGE \begin{cases}\tau_d = 135\,KSI\\ \tau_m = 150\,KSI\end{cases}$$

$$N_{a\,max} = (L_o - 2D_w)/D_w = [0.68 - 2(.055)]/0.055 = 8.9\ COILS$$

USE $N_a = 8.0$

$$C = \left[\frac{(11.85\times10^6)(0.055)}{8(13.08)(8.0)}\right]^{1/3} = 9.20 \rightarrow K = 1.16$$

$$\tau_o = \frac{2.546(1.16)(9.20)(14)}{(.055)^2} = 125\,750\,PSI\ OK$$

$$L_s = D_w(N_a+2) = .055(10) = 0.550\,IN$$

$$F_s = k(L_f - L_s) = 13.08(1.75 - 0.55) = 15.70\,LB$$

$$\tau_s = \tau_o\,(F_s/F_o) = 125\,750\,(15.7/14.0) = 141\,000\,PSI\ \underline{OK}$$

$$D_m = C\,D_w = 9.20(0.055) = 0.506\,IN$$

$$L_f/D_m = 1.75/0.506 = 3.46\ \text{NO BUCKLING}$$

$$cc = (0.68 - 0.55)/8 = 0.016\,IN > D_w/10\ \underline{OK}$$

$$D_o = D_m + D_w = 0.561\,IN.$$

$$D_i = D_m - D_w = 0.451\,IN.$$

$$k = \frac{8.00}{1.75} = 4.57 \text{ LB/IN}$$

**18.** METHOD 2 $L_f = 2.75$; $f_o = 1.75 \text{ IN}$; $L_o = L_f - f_o = 1.00 \text{ IN}$; $F_o = 8.00 \text{ IN}$.

ASTM A313, TYPE 316, AVG. SERV.; $\tau_d \approx 90$ KSI

$D_w = \sqrt{21.4(8.0)/90\,000} = 0.044 \text{ IN} \rightarrow$ USE $D_w = 0.0475 \text{ IN}$; 18 GAGE

$\tau_d = 0.85(122 \text{ ksi}) = 104 \text{ ksi}$; $\tau_m = 0.85(135) = 115 \text{ ksi}$

$Na_{MAX} = (L_o - 2D_w)/D_w = [1.00 - 2(0.0475)]/.0475 = 19.05 \rightarrow$ USE $Na = 17$

$$C = \left[\frac{(10.0 \times 10^6)(0.0475)}{8(4.57)(17)}\right]^{1/3} = 9.14 \rightarrow K = 1.16$$

$$\tau_o = \frac{2.546(1.16)(9.14)(8.0)}{(0.0475)^2} = 95\,740 \text{ PSI} \quad \underline{\text{OK}}$$

$L_s = 0.0475(17+2) = 0.9025 \text{ IN}$ : $F_s = 4.57 \text{ LB/IN}(2.75 - .9025) = 8.44 \text{ LB}$

$\tau_s = \tau_o(F_s/F_o) = 95\,740(8.44/8.0) = 101\,050 \text{ PSI}$ OK

$D_m = C D_w = 9.14(0.0475) = 0.434 \text{ IN.}$; $L_f/D_m = 2.75/.434 = 6.33$ (HIGH)

$(f_o/L_f)_{CR} = 0.32$; $(f_o)_{CR} = 0.32(2.75) = 0.88 \text{ IN}$; $f_{o_{ACT}} = 1.75 > 0.88$ (WILL BUCKLE)

**19.** SAME AS 18 EXCEPT $D_{ROD} = 0.625 \text{ IN}$ — USE METHOD 1

LET $D_m = 0.75 \text{ IN}$; $\tau_d \approx 100$ KSI

$$D_w = \left[\frac{3.06(F_o)(D_m)}{\tau_d}\right]^{1/3} = \left[\frac{3.06(8.0)(0.75)}{100\,000}\right]^{1/3} = 0.057 \rightarrow \text{USE } D_w = 0.0625 \text{ IN.}$$

$\tau_d = 0.85(115 \text{ ksi}) = 97.8 \text{ KSI}$; $\tau_m = 0.85(128) = 109 \text{ KSI}$

$C = D_m/D_w = 0.75/0.0625 = 12.0 \rightarrow K = 1.12$

$$\tau_o = \frac{2.546(1.12)(12.0)(8.0)}{(0.0625)^2} = 70\,100 \text{ PSI} \quad \underline{\text{OK}}$$

$$Na = \frac{G D_w}{8 k C^3} = \frac{(10 \times 10^6)(0.0625)}{8(4.57)(12)^3} = 9.89 \text{ COILS}$$

$L_s = D_w(Na + 2) = 0.0625(11.89) = 0.743 \text{ IN}$

$F_s = k(L_f - L_s) = 4.57(2.75 - 0.743) = 9.17 \text{ LB}$

$\tau_s = \tau_o(F_s/F_o) = 70\,100(9.17/8.0) = 80\,400 \text{ PSI}$ $\underline{\text{OK}}$

$L_f/D_m = 2.75/0.75 = 3.67$ $\underline{\text{OK}}$ NO BUCKLING

$cc = (L_o - L_s)/Na = (1.00 - 0.743)/9.89 = 0.026 \text{ IN}$ $\underline{\text{GOOD}}$

$D_o = D_m + D_w = 0.75 + 0.0625 = 0.8125 \text{ IN.}$

$D_i = D_m - D_w = 0.75 - 0.0625 = 0.6875 \text{ IN} > D_{ROD}$ $\underline{\text{OK}}$

**20.** SAME AS 17 EXCEPT $D_{HOLE} = 0.75$ IN. METHOD 1

LET $D_m = 0.625$ IN; $\tau_d = 120$ KSI

$$D_w = \left[\frac{3.06(14.0)(0.625)}{120000}\right]^{1/3} = 0.061 \text{ IN.} \rightarrow \text{USE } D_w = 0.063 \text{ IN}; \quad \tau_d = 130 \text{ KSI}, \ \tau_M = 145 \text{ KSI}$$

$$C = D_m/D_w = 0.625/.063 = 9.92 \rightarrow K = 1.15$$

$$\tau_0 = \frac{2.546(1.15)(9.92)(14.0)}{(0.063)^2} = 102500 \text{ PSI} \quad \text{OK}$$

$$N_a = \frac{(11.85\times10^6)(0.063)}{8(13.08)(9.92)^3} = 7.31 \text{ COILS}: L_s = 0.063(7.31+2) = 0.586 \text{ IN}$$

$$F_s = k(L_f - L_s) = 13.08(1.75-0.586) = 15.22 \text{ LB}: \tau_s = \tau_0 \, F_s/F_0 = 111{,}500 \text{ PSI} \quad \text{OK}$$

$L_f/D_m = 1.75/.625 = 2.80$ OK NO BUCKLING

$$cc = (0.68 - 0.586)/7.31 = 0.013 \text{ IN} > D_w/10 \quad \text{OK}$$

$$D_0 = 0.625 + 0.063 = 0.688 \text{ IN}$$

CLEARANCE WITH HOLE $= 0.75 - 0.688 = 0.062$ IN OK $> D_w/10$

**21.** METHOD 2 $L_0 = 3.05$ IN; $F_0 = 45$ LB; $L_i = 3.50$ IN; $F_i = 22.0$ LB

$$k = \frac{45-22}{3.50-3.05} = 51.1 \text{ LB/IN.}; \quad L_f = L_i + F_i/k = 3.50 + 22/51.1 = 3.93 \text{ IN}$$

$$D_w = \sqrt{21.4(45)/90000} = 0.103 \text{ IN.} \rightarrow \text{USE } D_w = 0.1055 \text{ IN.}$$

ASTM A231: $\tau_d = 109$ ksi; $\tau_M = 157$ ksi; SEVERE SERVICE

$$N_{a_{MAX}} = (L_0 - 2D_w)/D_w = [3.05 - 2(0.1055)]/0.1055 = 26.9 \text{ COILS} \rightarrow \text{USE } N_a = 18$$

$$C = \left[\frac{G D_w}{8 k N_a}\right]^{1/3} = \left[\frac{(11.2\times10^6)(0.1055)}{8(51.1)(18)}\right]^{1/3} = 5.44 \rightarrow K = 1.285$$

$$\tau_0 = \frac{2.546(1.285)(5.44)(45)}{(0.1055)^2} = 71950 \text{ PSI} < \tau_d \quad \text{OK}$$

$$L_s = D_w(N_a+2) = 0.1055(20) = 2.11 \text{ IN}: F_s = 51.1 \text{ LB/IN}(3.93-2.11) = 93.0 \text{ LB}$$

$$\tau_s = \tau_0 (F_s/F_0) = 71950(93.0/45.0) = 148700 \text{ PSI} < \tau_{MAX} \quad \text{OK}$$

$$D_m = C D_w = 5.44(0.1055) = 0.574 \text{ IN}$$

$$L_f/D_m = 3.93/0.574 = 6.85 : (f_0/L_f)_{CR} = 0.27$$

$$(f_0)_{CR} = 0.27(3.93) = 1.061 \text{ IN.}$$

$$(f_0)_{ACT} = L_f - L_0 = 3.93 - 3.05 = 0.88 \text{ IN} < (f_0)_{CR} \quad \text{OK NO BUCKLING}$$

$$cc = (3.05 - 2.11)/18 = 0.052 \text{ IN} > \frac{D_w}{10} \quad \text{OK}$$

$$D_0 = D_m + D_w = 0.680 \text{ IN.}$$

$$D_i = D_m - D_w = 0.469 \text{ IN.}$$

**22** COMPRESSION SPRING: METHOD 1

ASTM A227 STEEL WIRE; $G = 11.5 \times 10^6$ PSI (TABLE 18-4)

STRESSES FROM FIG. 18-8.

$L_i = 2.50$ IN, $F_i = 20.0$ LB, $L_o = 2.10$ IN, $F_o = 35$ LB.

INSTALL SPRING AROUND 1.50 IN DIA. SHAFT.

$$\text{SPRING SCALE} = k = \frac{F_o - F_i}{L_i - L_o} = \frac{35-20}{2.50-2.10} = 37.5 \text{ LB/IN}$$

$$\text{FREE LENGTH} = L_f = L_i + F_i/k = 2.50 + 20.0/37.5 = 3.033 \text{ IN.}$$

TRY $D_m = 1.75$ IN.; $D_{w_{MAX}} = D_m - D_{SH.} = 1.75 - 1.50 = 0.25$ IN.

SEVERE SERVICE: $\tau_d \cong 85{,}000$ PSI

$$D_w \cong \left[\frac{(3.06)(F_o)(D_m)}{\tau_d}\right]^{1/3} = \left[\frac{(3.06)(35)(1.75)}{85000}\right]^{1/3} = 0.130 \text{ IN.}$$

TRY U.S. WIRE GA. 9: $D_w = 0.1483$ IN.

$\tau_d = 87\,000$ PSI (SEVERE SERV.); $\tau_{MAX} = 116\,000$ PSI (LT. SERV.)

$C = D_m/D_w = 1.75/0.1483 = 11.80$; $K = 1.125$ (FIG. 18-14)

$$\tau_o = \frac{8KF_oD_m}{\pi D_w^3} = \frac{8(1.125)(35)(1.75)}{\pi(0.1483)^3} = 53{,}800 \text{ PSI} \quad \underline{\text{OK}}$$

$$N_a = \frac{G D_w}{8kC^3} = \frac{(11.5 \times 10^6)(0.1483)}{8(37.5)(11.80)^3} = 3.45 \text{ COILS}$$

$L_s = D_w(N_a + 2) = (0.1483)(3.45+2) = 0.809$ IN.

$F_s = k(L_f - L_s) = (37.5)(3.033 - 0.809) = 83.4$ LB

$\tau_s = \tau_o\, F_s/F_o = (53\,800 \text{ PSI})(83.4/35) = 128{,}200$ PSI <u>TOO HIGH</u>

<u>TRY $D_w = 0.162$ IN - 8 GAUGE</u>: $\tau_d = 86{,}000$ PSI; $\tau_{MAX} = 114{,}000$ PSI

SUMMARY OF RESULTS:

$C = 10.80$; $K = 1.13$

$\tau_o = 41{,}455$ PSI <u>OK</u>

$N_a = 4.93$ COILS; $L_s = 1.122$ IN; $F_s = 71.7$ LB

$\tau_s = 84{,}900$ PSI <u>OK</u>

$ID = D_m - D_w = 1.75 - 0.162 = 1.588$ IN <u>OK</u>

23 COMPRESSION SPRING: METHOD 1 : METRIC

ASTM A227 STEEL WIRE: $G = 79.3\ GPa = 79.3 \times 10^3\ MPa$ (TABLE 18-4)

$L_i = 60\ mm$, $F_i = 90\ N$, $L_o = 50\ mm$, $F_o = 155\ N$

$$k = \frac{F_o - F_i}{L_i - L_o} = \frac{155-90}{60-50} = 6.5\ N/mm$$

$$L_f = L_i + F_i/k = 60\ mm + 90N/6.5N/mm = 73.85\ mm$$

SPRING INSTALLED ON 38 mm DIA. SHAFT: TRY $D_m = 45\ mm$

SEVERE SERVICE: $\tau_d \approx 550\ MPa$ (APP. A19-1)

$$\text{TRIAL } D_w = \left[\frac{(3.06)(F_o)(D_m)}{\tau_d}\right]^{1/3} = \left[\frac{(3.06)(155)(45)}{550}\right]^{1/3} = 3.39\ mm$$

TRY $D_w = 3.80\ mm$ (TABLE 18-2); $\tau_d = 600\ MPa$ $\tau_{MAX} = 800\ MPa$

$C = D_m/D_w = 45/3.80 = 11.84$; $K = 1.125$ (FIG. 18-14)

$$\tau_o = \frac{8KF_oD_m}{\pi D_w^3} = \frac{8(1.125)(155)(45)}{\pi(3.80)^3} = 364\ MPa \ \underline{OK}$$

$$N_a = \frac{G D_w}{8kC^3} = \frac{(79 \times 10^3)(3.80)}{8(6.5)(11.84)^3} = 3.49 \text{ COILS}$$

$$L_s = D_w(N_a+2) = (3.80)(3.49+2) = 20.86\ mm$$

$$F_s = k(L_f - L_s) = (6.5N/mm)(73.85 - 20.86)mm = 344\ N$$

$$\tau_s = \tau_o\ F_s/F_o = 364\ MPa\left(\frac{344\ N}{155\ N}\right) = 809\ MPa \text{ (JUST SLIGHTLY HIGH)}$$

$$ID = D_m - D_w = 45 - 3.80 = 41.2\ mm \ \underline{OK}$$

24 COMPRESSION SPRING: ANALYSIS : ASTM A229, $G = 11.2 \times 10^6\ PSI$

17 Ga.: $D_w = 0.054$ IN.; $\tau_d = 106$ KSI (SEVERE); $\tau_d = 128$ KSI (AVG.); $\tau_d = 141$ KSI (LT.)

$OD = 0.531$ IN: $D_m = OD - D_w = 0.531 - 0.054 = 0.477$ IN.

$C = D_m/D_w = 0.477/0.054 = 8.83$; $K = 1.17$; $N = 7.0$ COILS; $N_a = 7.0 - 2 = 5.0$

FOR $F_o = 10.0$ LB:

$$\tau_o = \frac{8KF_oC}{\pi D_w^2} = \frac{8(1.17)(10)(8.83)}{\pi(0.054)^3} = \underline{90250\ PSI} \text{ OK FOR } \underline{\text{SEVERE SERV.}}$$

$$f_o = \frac{8(F_o)(C^3)(N_a)}{G D_w} = \frac{(8)(10)(8.83)^3(5.0)}{(11.2 \times 10^6)(0.054)} = 0.456\ IN$$

$L_o = L_f - f_o = 1.25\ IN - 0.456 = 0.794\ IN$; $L_s = D_w(N) = 0.054(7) = 0.378\ IN.$

$$k = \frac{F_o}{L_f - L_o} = \frac{10.0\ LB}{(1.25 - 0.794)\ IN} = 21.93\ LB/IN.$$

$$F_s = k(L_f - L_s) = (21.93)(1.25 - 0.378) = 19.12\ LB.$$

$$\tau_s = \tau_o\ F_s/F_o = (90\,250\ PSI)\ 19.12\ LB/10.0\ LB = 172{,}600\ PSI \ \underline{\text{TOO HIGH}}$$

BUT $\tau_{MAX} \approx \tau_d$ FOR LIGHT SERVICE $= 141{,}000\ PSI$

25 EXTENSION SPRING $F_o = 7.75\ LB$; $L_o = 2.75\ IN$; $L_i = 2.25\ IN$, $F_i = 5.25\ LB$ INSTALLED

$D_o < .300\ IN$; MUSIC WIRE, SEVERE SERVICE.

LET $D_m = 0.250\ IN.$; $\tau_d \approx 120\,000\ PSI$

$$D_w = \left[\frac{8KF_oD_m}{\pi\,\tau_d}\right]^{1/3} = \left[\frac{8(1.20)(7.75)(0.250)}{\pi(120\,000)}\right]^{1/3} = 0.0367\ IN$$

USE 16 GA., $D_w = 0.037\ IN$; $\tau_d = 120\,000\ PSI$

$D_o = D_m + D_w = 0.287\ IN$ $\underline{OK}$; $D_I = D_m - D_w = 0.213\ IN.$

$C = D_m/D_w = 0.25/0.037 = 6.76 \rightarrow K = 1.225$

$$\tau_o = \frac{8KF_oD_m}{\pi D_w^3} = \frac{8KF_oC}{\pi D_w^2} = \frac{8(1.225)(7.75)(6.76)}{\pi(0.037)^2} = 119320\ PSI\ \underline{OK}$$

$$k = \frac{\Delta F}{\Delta L} = \frac{7.75-5.25}{2.75-2.25} = 5.0\ LB/IN.$$

$$N_a = \frac{GD_w}{8C^3k} = \frac{(11.85\times10^6)(0.037)}{8(6.76)^3(5.0)} = 35.5\ COILS$$

BODY LENGTH $\cong D_w(N_a+1) = 0.037(36.5) = 1.350\ IN. = B.L.$

ASSUME FULL LOOP AT EACH END

$L_f = B.L. + 2D_I = 1.350 + 2(0.213) = 1.777\ IN.$

DEFLECTION TO $L_o$: $f_o = L_o - L_f = 2.750 - 1.777 = 0.973\ IN$

INITIAL FORCE $= F_I = F_o - kf_o = 7.75 - 5.0(0.973) = 2.88\ LB$

INITIAL STRESS $= \tau_I = \tau_o \times F_I/F_o = 119320 \times 2.88/7.75 = 44340\ PSI$ $\underline{HIGH}$

FROM FIG 19-21 — STRESS SHOULD BE $\approx$ 14-21 KSI

USE SMALLER END LOOPS — SAY 0.06 EACH

$L_f = B.L. + 2(0.06) = 1.35 + 0.12 = 1.47\ IN$

$f_o = L_o - L_f = 2.750 - 1.47 = 1.28\ IN.$

$F_I = 7.75 - 5.0(1.28) = 1.35\ LB$

$\tau_I = \tau_o \times F_I/F_o = 119320 \times 1.35/7.75 = 20\,800\ PSI$ $\underline{OK}$

SUMMARY:

$D_w = 0.037\ IN$ — 16 GA. MUSIC WIRE

$D_m = 0.250\ IN$; $D_o = 0.287\ IN$; $D_i = 0.213\ IN.$

35.5 COILS

2 LOOPS @ 0.06 IN. EACH

$F_o = 7.75\ LB$ @ 2.75 IN — OPERATING

$F_i = 5.25\ LB$ @ 2.25 IN — INSTALLED

$F_I = 1.35\ LB$ @ 1.47 IN — INITIAL

**26** EXTENSION: MUSIC WIRE, AVG. SERVICE, $\tau_d \approx 130$ ksi
$F_o = 15.0$ LB; $F_I = 5.20$ LB; $L_o = 5.00$ in; $L_I = 3.75$ IN.
$k = 7.84$ LB/in; $D_{m_1} = 0.60$ IN; $D_{w_{TRIAL}} = 0.0596$ IN.; $D_w = 0.063$ (26 Ga.)
$\tau_d = 132{,}000$ PSI; $C = 9.52$; $K = 1.153$; $\tau_o = 105{,}600$ PSI OK, $N_a = 13.78$ COILS
BODY LENGTH = B.L. = 0.931 IN; FOR END LOOP SIZE = 0.537 in (ID); $F_I = -8.48$ LB
CHANGE E.L.S = 1.20 in → $F_I = 1.92$ LB; $\tau_I = 13500$ PSI (OK FIG. 18-21)
OD = 0.663 IN < 0.75 IN. OK

**27** EXTENSION: MUSIC WIRE, SEVERE SERVICE; $\tau_d \approx 100$ ksi; $F_o = 1.0$ LB; $L_o = 3.00$ in.,
$k = 6.80$ LB/in; $D_{m_1} = 0.65$ IN; $D_{w_{TRIAL}} = 0.058$ in → USED $D_w = 0.059$ IN (25 Ga.)
$\tau_d = 110$ ksi; OD = 0.709 IN; ID = 0.591; $C = 11.02$; $K = 1.131$; $\tau_o = 91.1$ ksi OK
$N_a = 9.61$ coils; B.L. = 0.626 IN.; E.L.S. = 0.591 in = ID; $F_I = 1.89$ LB; $\tau_I = 17.3$ ksi HIGH
CHANGE E.L.S. = 0.54 in; $F_I = 1.20$ LB; $\tau_I = 10.95$ ksi (OK FIG. 18-21)

**28** EXTENSION: MUSIC WIRE, SEVERE SERV.; $F_o = 10.0$ LB; $L_o = 6.10$ IN; $\tau_d = 100$ ksi
(SAME AS #24 FOR $D_m$, $D_w$, OD, ID, $C$, $K$, $\tau_o$): $N_a = 25.14$ COILS; B.L. = 1.572 in
E.L.S. = 0.591 IN. = ID; $F_I = 1.48$ LB; $\tau_I = 13500$ PSI (OK FIG. 18-21)

**29** EXTENSION: MUSIC WIRE; AVERAGE SERV.; $F_o = 10.0$ LB; $L_o = 9.40$ in, $k = 1.50$ LB/in.
$\tau_d \approx 130$ KSI; $D_m = 0.6875$ IN. (11/16); $D_{w_{TR}} = 0.0545$ IN; USED $D_w = 0.055$ IN,
$\tau_d = 135$ KSI; (24 Ga.): OD = 0.7425 IN; ID = 0.6325 IN; $C = 12.5$; $K = 1.114$
$\tau_o = 117.3$ KSI; $N_a = 27.81$ COILS; B.L. = 1.58 IN; E.L.S. = 0.6325 IN. = ID →
$F_I = -0.14$ LB: CHANGE E.L.S = 1.0 IN; $F_I = 0.96$ LB; $\tau_I = 113$ ksi (OK)

**30** EXTENSION: ASTM A313; TYPE 302: $F_o = 162$ LB; $L_o = 10.80$ in; $k = 38.0$ LB/in
$\tau_d \approx 80$ ksi (AVG. SERV.); $D_m = 1.50$ IN: $D_{w_{TR}} = 0.210$ in → $D_w = 0.2253$ IN (4 Ga.)
$\tau_d = 81$ ksi: OD = 1.7253 IN; ID = 1.2747 in; $C = 6.66$; $K = 1.225$
$\tau_o = 66.3$ ksi; $N_a = 25.11$ COILS; B.L. = 5.88 in; E.L.S. = 1.27 in = ID,
$F_I = 71.7$ LB; $\tau_I = 29300$ psi (TOO HIGH).
CHANGE E.L.S. = 0.90 in; $F_I = 43.6$ LB; $\tau_I = 17.8$ ksi (OK FIG. 18-21)

31 EXTENSION SPRING ENDS

19 Ga.: $D_w = 0.041$ IN : $D_m = 0.28$ IN; $R_1 = 0.25$ IN; $R_2 = 0.094$ IN

$\sigma_d = 146$ KSI; $\tau_d = 130$ KSI FOR AVERAGE SERVICE

BENDING (EQ.19-13) (FIG.18-22)

$$C_1 = 2R_1/D_w = 2(.25)/.041 = 12.20$$

$$K_1 = \frac{4C_1^2 - C_1 - 1}{4C_1(C_1 - 1)} = 1.065$$

$$\sigma_A = \frac{16(.28)(5.0)(1.065)}{\pi(0.041)^3} + \frac{4(5.0)}{\pi(.041)^2} = 114000 \text{ PSI OK}$$

TORSION (EQ.18-15)

$$C_2 = 2R_2/D_w = 2(.094)/0.041 = 4.59$$

$$K_2 = \frac{4C_2 - 1}{4C_2 - 4} = 1.209$$

$$\tau_B = \frac{8 D_m F_0 K_2}{\pi (D_w)^3} = \frac{8(.28)(5.0)(1.209)}{\pi(0.041)^3} = 62\,600 \text{ PSI OK}$$

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32 TORSION SPRING: ASTM A313, TYPE 302, AVG. SERVICE

ASSUME $D_m = 0.420$ IN.; $\sigma_d = 140\,000$ PSI; $K_b = 1.15$; $\theta_t = 180° = 0.50$ REV.

$$D_w = \left[\frac{32 M_0 K_b}{\pi \sigma_d}\right]^{1/3} = \left[\frac{32(1.0)(1.15)}{\pi(140\,000)}\right]^{1/3} = 0.0437 \text{ IN}$$

USE 18Ga. $D_w = .0475$ IN; $\sigma_d = 160\,000$ PSI

$$C = D_m/D_w = 0.420/0.0475 = 8.84 : K = \frac{4C^2 - C - 1}{4C(C-1)} = 1.092$$

$$\text{ACTUAL } \sigma = \frac{32 M_0 K_b}{\pi D_w^3} = \frac{32(1.0)(1.092)}{\pi(0.0475)^3} = 103\,800 \text{ PSI OK}$$

$k_\theta = 1.0$ LB-IN / 0.50 REV = 2.00 LB-IN/REV

$$N_a = \frac{E D_w^4}{10.2 D_m (k_\theta)} = \frac{(28\times10^6)(0.0475)^4}{10.2(0.420)(2.00)} = 16.64 \text{ COILS TOTAL}$$

ENDS: SPECIFY $L_1 = L_2 = 0.75$ IN

$$N_e = \frac{L_1 + L_2}{3\pi D_m} = \frac{0.75 + 0.75}{3\pi(0.420)} = 0.38 \text{ COIL (ENDS)}$$

$N_b = N_a - N_e = 16.64 - 0.38 = 16.26$ COILS IN BODY

$$\text{OPERATING } D_m = \frac{D_{mI} N_a}{N_a + \theta_t} = \frac{0.420(16.64)}{(16.64 + 0.50)} = 0.408 \text{ IN}$$

MINIMUM ID = 0.408 IN $- D_w = 0.408 - 0.0475 = 0.360$ IN

MAKE ROD 90% OF $ID_{min} \approx 0.9(0.360) = 0.324$ IN

USE $D_{Rod} = 0.300$ IN (STD. SIZE)

$L = D_w(N_a + 1 + \theta_t) = 0.0475(16.64 + 1 + 0.5) = 0.862$ IN IF COILS TOUCH.

$D_o = D_m + D_w = 0.420$ IN $+ 0.0475 = 0.4675$ IN. OK

33 TORSION SPRING: ASTM A313, TYPE 302; SEVERE SERVICE

ASSUME $D_m = 1.125$; $\sigma_d = 110\,ksi$; $K_b = 1.15$; $\theta_c = 270° = 0.75\,REV.$

$$D_w = \left[\frac{32 M_o K_b}{\pi \sigma_d}\right]^{1/3} = \left[\frac{32(12.0)(1.15)}{\pi(110\,000)}\right]^{1/3} = 0.1085\,IN \quad \begin{cases} USE\ 11\ GA.;\ D_w = 0.1205\,IN \\ \sigma_d = 118\,000\,psi \end{cases}$$

$$C = D_m/D_w = 1.125/0.1205 = 9.34;\ K_b = \frac{4C^2 - C - 1}{4C(C-1)} = 1.087$$

$$\sigma = \frac{32 M_o K_b}{\pi D_w^3} = \frac{32(12.0)(1.085)}{\pi(0.1205)^3} = 75\,920\,PSI \ \underline{OK}$$

$$k_\theta = M_o/\theta_c = 12.0\,LB\text{-}IN/0.75\,REV = 16.0\,LB\text{-}IN/REV$$

$$N_a = \frac{ED^4}{10.2 D_m (k_\theta)} = \frac{(28\times10^6)(0.1205)^4}{10.2(1.125)(16.0)} = 32.15\ \text{COILS TOTAL}$$

ENDS: LET $L_1 = L_2 = 1.50\,IN$; $N_e = \dfrac{L_1 + L_2}{3\pi(D_m)} = 0.28$ COIL (ENDS)

BODY $N_b = N_a - N_e = 32.15 - 0.28 = 31.87$ COILS

OPERATING $D_m = \dfrac{D_{mI} N_a}{N_a + \theta_c} = \dfrac{(1.125)(32.15)}{(32.15 + 0.75)} = 1.100\,IN$

$ID_{min} = 1.100 - 0.1205 = 0.9795\,IN$; $D_{ROD} \approx 0.9(.9795) = 0.882\,IN$

USE $D_{ROD} = 7/8\,IN = 0.875\,IN$; $L = D_w(N_a + 1 + \theta_c) = 0.1205(32.15 + 1.75)$

$L = 4.08\,IN$ : $OD = 1.125 + 0.1205 = 1.246\,IN$

34 TORSION SPRING: MUSIC WIRE; SEVERE SERVICE; $\theta_c = 1.0\,REV$

$D_m = 0.625\,IN.$; $\sigma_d \approx 140\,ksi$; $k_b \approx 1.15$

$$D_w = \left[\frac{32 M_o k_b}{\pi \sigma_d}\right]^{1/3} = \left[\frac{32(2.50)(1.15)}{\pi(140\,000)}\right]^{1/3} = 0.0594\,IN \quad \begin{cases} USE\ 26\ GA.;\ D_w = 0.063 \\ \sigma_d = 156\,000\,PSI \end{cases}$$

$$C = D_m/D_w = 0.625/0.063 = 9.92;\ K_b = \frac{4C^2 - C - 1}{4C(C-1)} = 1.081$$

$$\sigma = \frac{32(2.50)(1.081)}{\pi(.063)^3} = 110\,100\,PSI \ \ OK$$

$k_\theta = 2.50\,LB\text{-}IN/REV.$; $N_a = \dfrac{(29\times10^6)(0.063)^4}{10.2(0.625)(2.50)} = 28.66$ COILS

SPECIFY ENDS: $L_1 = L_2 = 1.50\,IN.$

$N_e = \dfrac{2(1.5)}{3\pi(0.625)} = 0.51$ COIL : $N_b = 28.66 - 0.51 = 28.15$ COILS

MINIMUM $D_m = \dfrac{D_{mI} N_a}{N_a + \theta_t} = \dfrac{0.625(28.66)}{28.66 + 1.0} = 0.604\,IN$

MIN I.D. $= 0.604 - 0.063 = 0.541\,IN$

$D_{ROD} \approx 0.9(0.541) = 0.487\,IN \rightarrow USE\ D_R = 7/16\,IN. = 0.4375\,IN.$

$OD = 0.625 + 0.063 = 0.688\,IN$

$L = 0.063(28.66 + 1 + 1.0)\ 1.93\,IN$

$\boxed{35}$

TORSION SPRING: $D_w = 0.038$; $D_o = 0.368$ IN; $N_b = 9.50$ COILS

$L_1 = 0.50$ IN; $L_2 = 1.125$ IN; A401 STEEL; $\theta = 180° = 0.50$ REV

FROM EQ. 7-21, SOLVE FOR M:

$$M_o = \frac{E D_w^4 \theta}{10.2 (D_m)(N_a)} = \frac{(29.5 \times 10^6)(0.038)^4(0.50)}{10.2(0.330)(10.02)} = \underline{0.91 \text{ LB-IN}}$$

WHERE: $D_m = D_o - D_w = 0.368 - 0.038 = 0.330$ IN

ENDS: $N_e = \dfrac{L_1 + L_2}{3\pi(D_m)} = \dfrac{0.50 + 1.125}{3\pi(0.330)} = 0.52$ COIL

$N_a = N_b + N_e = 9.50 + 0.52 = 10.02$ COILS

$$\sigma = \frac{32 M_o K_b}{\pi (D_w)^3} = \frac{32(0.91)(1.094)}{\pi(0.038)^3} = 184\,800 \text{ PSI}$$

WHERE $C = D_m/D_w = 0.330/0.038 = 8.68$

$$K_b = \frac{4C^2 - C - 1}{4C(C-1)} = 1.094$$

FOR SEVERE SERVICE $\sigma_d = 200\,000$ PSI; $\sigma = 184\,800$ PSI $\underline{\text{OK}}$

# CHAPTER 19
# FASTENERS

**4** LOAD PER BOLT = 6000 LB/4 = 1500 LB

SELECT GRADE 2 : $\sigma_a = .75\, S_{PROOF} = .75(55000) = 41250$ PSI [SAE GRADE 2] TABLE 19-2

$$A_t = \frac{1500}{41250} = 0.0364\ IN^2$$

USE 1/4-28 UNF OR 5/16-18 UNC

FOR 5/16-18 : $T = KDP = 0.15(.3125)(1500) = \underline{70.3\ LB\text{-}IN}$

**5** $F = A_t S_{PR.} = (0.0140\ IN^2)(85000\ LB/IN^2) = \underline{1190\ LB}$ TABLE 19-2

**6** M 4X.5 : $A_t = 9.79\ mm^2$; GRADE 8.6 (SECTION 19-2)

TENSILE STRENGTH ≈ 800 MPa

YIELD STRENGTH ≈ 0.6 (T.S.) = .6(800) = 480 MPa

PROOF STRENGTH ≈ 0.9 (Y.S.) = .9(480) = 432 MPa

$F = A_t \cdot S_{PR} = (9.79\ mm^2)(432\ N/mm^2) = 4229\ N = \underline{4.23\ kN}$ TABLE 19-5

**7** 7/8-14 UNF ; $D = 0.875\ IN \times 25.4\ mm/IN = 22.2\ mm$

$p = 1/n = 1/14 = 0.0714\ IN \times 25.4\ mm/IN = 1.81\ mm$

NEAREST METRIC THREAD = $\underline{M24 \times 2}$ TABLE 19-5

DIFFERENCE IN D = 24.0 − 22.2 = $\underline{1.8\ mm}$ = $\underline{.071\ IN}$

METRIC THREAD IS APPROX. $\underline{8\%\ \text{LARGER}}$ IN THIS SIZE

**8** PITCH = .630/20 = 0.0315 IN × 25.4 mm/IN = 0.800 mm

D = 0.196 IN × 25.4 mm/IN = 4.98 mm

<u>CLOSEST STANDARD THREAD IS M5X0.8</u>

AMERICAN STANDARD #10-32 IS SIMILAR BUT NOT AS CLOSE TO MEASURED DIMENSIONS.

**9** M 10×1.5 : $A_t = 58.0\ mm^2$

NYLON 6/6 : T.S. = 146 MPa : MAX. $\sigma_a = 0.75(TS) = 109.5$ MPa

$F = A_t \sigma_A = (58.0\ mm^2)(109.5\ N/mm^2) = 6351\ N = \underline{6.351\ kN}$

$\boxed{10}$

1/4-20 UNC ; $A_t = 0.0318\ in^2$ ; $\sigma_a = 0.5(TS)$

a) SAE GRADE 2: $\sigma_a = 0.5(74) = 37\ ksi$ : $F = A_t\sigma_A = (.0318)(37000)$

$\underline{F = 1177\ LB}$

b) SAE GRADE 5: $\sigma_a = .5(120) = 60\ ksi$; $\underline{F = 1908\ LB}$

c) SAE GRADE 8: $\sigma_a = .5(150) = 75\ ksi$ ; $\underline{F = 2385\ LB}$

d) ASTM A307 : $\sigma_a = .5(60) = 30\ ksi$; $\underline{F = 954\ LB}$

e) ASTM A574: $\sigma_a = .5(180) = 90\ ksi$; $\underline{F = 2862\ LB}$

f) METRIC GRADE 8.8; $\sigma_a = 0.5(830\ MPa) = 415\ MPa \left(\frac{1.0\ ksi}{6.895\ MPa}\right) = 60.2\ ksi$

$F = A_t\sigma_a = (0.0318\ in^2)(60200\ LB/in^2) = \underline{1914\ LB}$ SIMILAR TO SAE GR.5

$F = 1914\ LB \times 4.448\ N/LB = 8513\ N = 8.51\ kN$

g) ALUM, 2024-T4: $\sigma_a = 0.5(68) = 34\ ksi$ ; $\underline{F = 1081\ LB}$

h) S43 000, ANN.; $\sigma_a = 0.5(75) = 37.5\ ksi$; $\underline{F = 1193\ LB}$

i) Ti-6Al-4V, ANN.; $\sigma_a = .5(130) = 65\ ksi$ ; $\underline{F = 2067\ LB}$

j) NYLON 66 DRY: $\sigma_a = 0.5(21) = 10.5\ ksi$ ; $\underline{F = 334\ LB}$

k) POLYCARBONATE : $\sigma_a = .5(9) = 4.5\ ksi$ ; $\underline{F = 143\ LB}$

l) ABS (HIGH IMPACT); $\sigma_a = .5(5) = 2.5\ ksi$ ; $\underline{F = 80\ LB}$

<u>ALTERNATE</u> FOR h) : SCREW MAY BE FULL HARD - <u>APP. 6</u>

<u>S43 000</u> $S_u = 90\ ksi$ ; $\sigma_a = 0.5(90) = 45\ ksi$ $\underline{F = 1431\ LB}$

# CHAPTER 20
# MACHINE FRAMES, BOLTED CONNECTIONS, AND WELDED JOINTS

**1.** DIRECT SHEAR, 5 BOLTS, A307

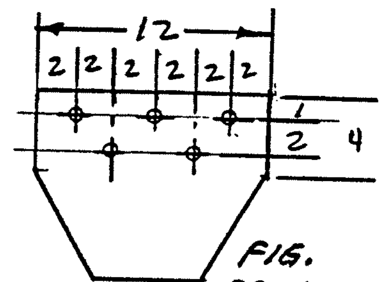


FIG. P20-1

$$F = \frac{12000\ LB}{5} = 2400\ LB/BOLT$$

SINGLE SHEAR

$$\tau = F/A_s$$

$$REQ'D\ A_s = \frac{F}{\tau_d} = \frac{2400\ LB}{10\,000\ LB/IN^2} = 0.240\ IN^2$$

$$A_s = \pi D^2/4 \ : \ D = \sqrt{4A_s/\pi} = \sqrt{4(.240)/\pi} = 0.55\ IN$$

USE 9/16-12 UNC BOLTS, $1\frac{3}{4}$ IN LONG

**2.** DIRECT SHEAR, EACH BAR SUPPORTS 4 HANGERS

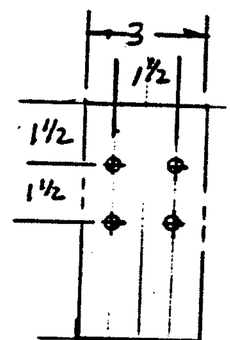


FIG. P20-2

$$F = 4(750) = 3000\ LB = TOTAL\ LOAD\ ON\ JOINT$$

4 BOLTS IN DOUBLE SHEAR, A 307

$$TOTAL\ A_s\ REQ'D = \frac{F}{\tau_d} = \frac{3000\ LB}{10\,000\ LB/IN^2} = 0.30\ IN^2$$

$$A_s = (2)(4)\frac{\pi D^2}{4} = 2\pi D^2$$

$$D = \sqrt{A_s/(2\pi)} = \sqrt{0.30/(2\pi)} = 0.219\ IN$$

USE 1/4-20 UNC BOLTS, 2.00 IN LONG

$\boxed{3.}$

4 BOLTS

HORIZ. DIRECT SHEAR: $F_1 = 5196/4 = 1299 \text{ LB/BOLT} \leftarrow$

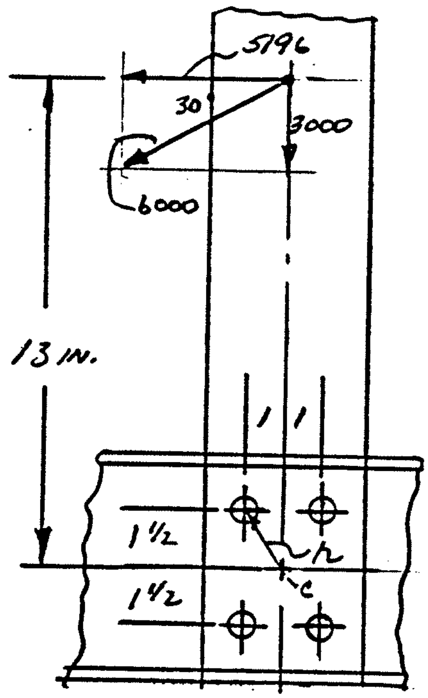
VERTICAL DIR. SHEAR: $F_2 = 3000/4 = 750 \text{ LB} \downarrow$

$M = 5196(13) = 67548 \text{ LB·IN.}$

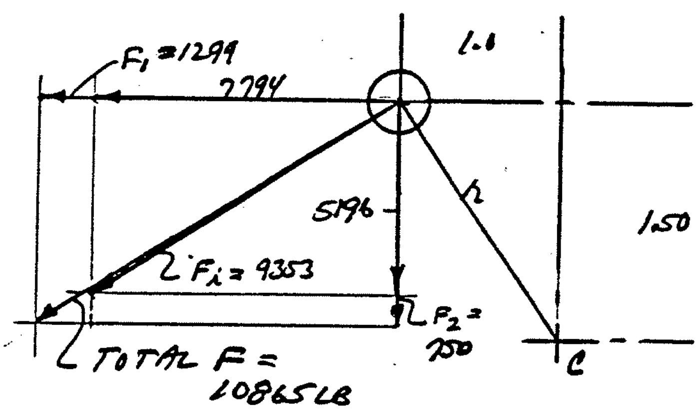
$r = \sqrt{1.50^2 + 1^2} = 1.80 \text{ IN}$ — ALL BOLTS

$\Sigma r^2 = 4r^2 = 4(1.80)^2 = 13.0 \text{ IN}^2$

$$F_i' = \frac{M r_i}{\Sigma r^2} = \frac{(67548)(1.80)}{13.0} = 9353 \text{ LB/BOLT}$$




FORCES ON BOLT AT UPPER LEFT:




TOTAL $F_x = 7794 + 1299 = 9093$

TOTAL $F_y = 5196 + 750 = 5946$

TOTAL $F = \sqrt{9093^2 + 5946^2}$

$F = 10\,865 \text{ LB}$

A 490 BOLTS, DOUBLE SHEAR

$$A_s = \frac{F}{\tau_d} = \frac{10865 \text{ LB}}{22000 \text{ LB/IN}^2} = 0.494 \text{ IN}^2 = 2(\pi D^2/4) = \pi D^2/2$$

$$\text{REQ'D } D = \sqrt{2A_s/\pi} = \sqrt{2(.494)/\pi} = 0.561 \text{ IN.}$$

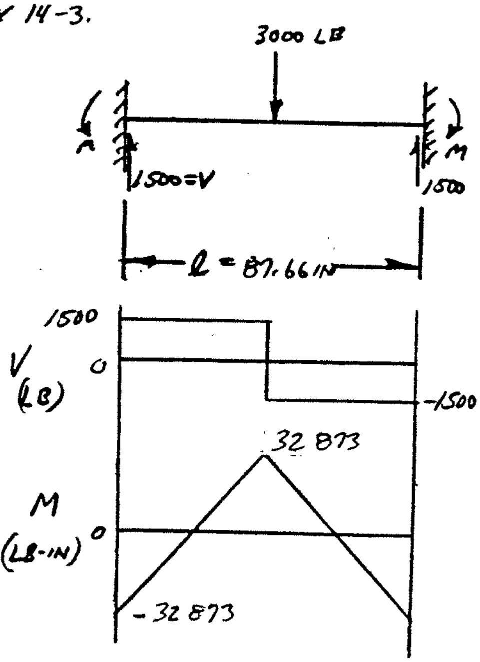
USE 9/16-12 UNC, 3 1/2 IN LONG

**4.** FIXED-END BEAM - CASE e APPENDIX 14-3.

$$M = \frac{wl}{8} = \frac{(3000)(87.66)}{8} = 32873 \text{ LB·IN}$$

EACH SIDE: $M = 32873$ LB·IN.

DIRECT SHEAR $= \frac{1500 \text{ LB}}{8 \text{ BOLTS}} = 188 \text{ LB/BOLT} \downarrow$



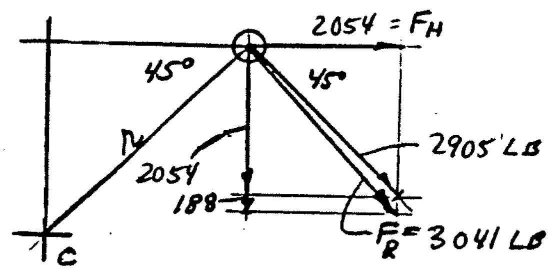

USE 4 BOLTS ON EACH FACE OF COLUMN THROUGH FLANGE AND PLATE.

FORCE ON EACH BOLT DUE TO MOMENT:

$r = 1.414$ IN; $\Sigma r^2 = 8(1.414)^2 = 16.00 \text{ IN}^2$

$$F = \frac{Mr}{\Sigma r^2} = \frac{(32873)(1.414)}{16.00} = 2905 \text{ LB/BOLT}$$

FORCES ON BOLT AT UPPER RIGHT:




TOTAL $F_V = 2054 + 188 = 2242$ LB

$F_R = \sqrt{2242^2 + 2054^2} = 3041$ LB

SPECIFY ASTM A 325 HIGH STRENGTH BOLTS

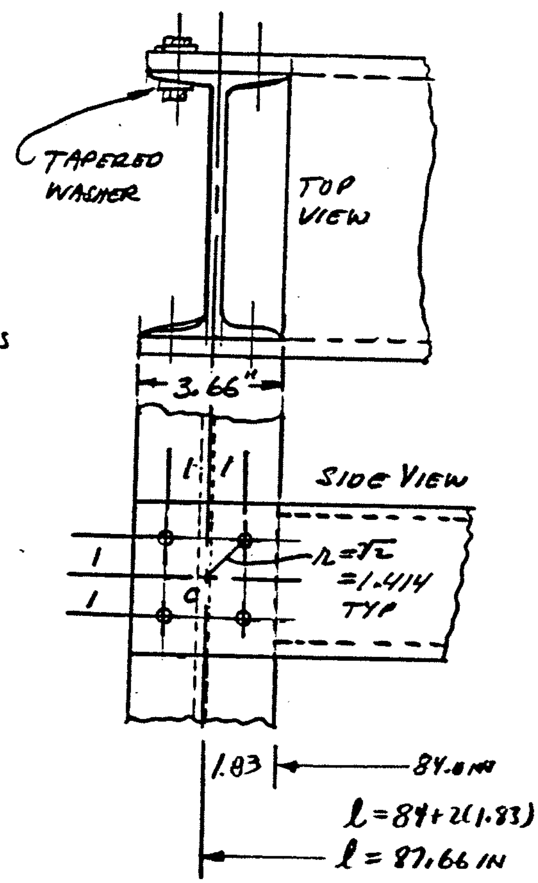
$\tau_d = 17500$ PSI $= F_R/A_s$

REQ'D. $A_s = F/\tau_d = \frac{3041 \text{ LB}}{17500 \text{ LB/IN}^2} = 0.174 \text{ IN}^2$

$A_s = \pi D^2/4$; $D = \sqrt{4A/\pi} = \sqrt{\frac{4(0.174)}{\pi}}$

$D = 0.470$ IN

USE ½-13 UNC BOLTS. NO THREADS IN THE SHEAR PLANE.



$\boxed{5.}$

6 BOLTS, A307, 1400 LB/SIDE

SHEAR: $\frac{1400\text{ LB}}{6} = 233\text{ LB/BOLT} \downarrow$

$M = (1400\text{ LB})(13\text{ IN}) = 18200\text{ LB·IN}$

$r_1 = \sqrt{2^2+3^2} = 3.61\text{ IN}$

$r_2 = 2.00\text{ IN}$

$\Sigma r^2 = 4(3.61)^2 + 2(2.00)^2 = 60\text{ IN}^2$

BOLT ①

$F = \frac{M r_1}{\Sigma r^2} = \frac{(18200)(3.61)}{60} = 1095\text{ LB}$

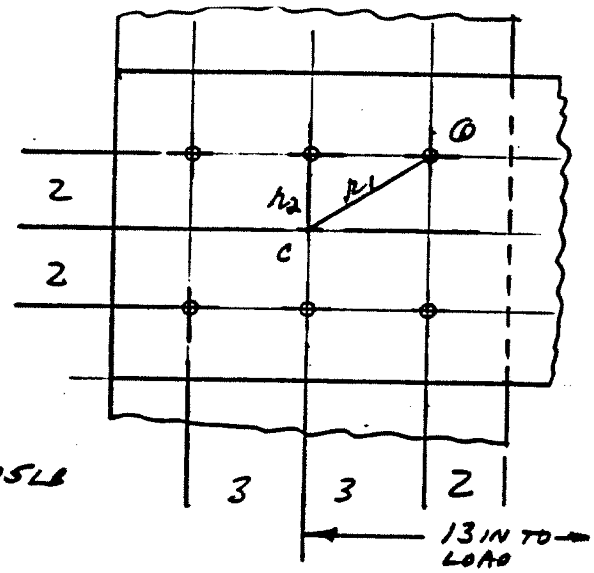
TOTAL $F = 1296$ LB

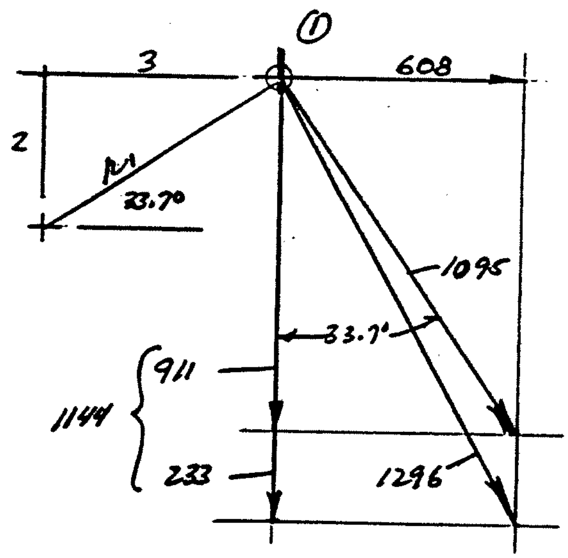
$A_s = \frac{F}{\tau_d} = \frac{1296\text{ LB}}{10000\text{ LB/IN}^2}$

$A_s = 0.130\text{ IN}^2 = \pi D^2/4$

$D = \sqrt{\frac{4A_s}{\pi}} = \sqrt{\frac{4(.130)}{\pi}} = 0.406\text{ IN.}$

USE 7/16-14 BOLTS



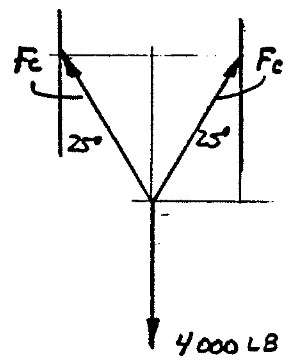



6. FIG. P20-6

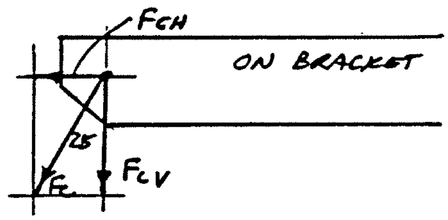
VERTICAL COMPONENT OF FORCE IN EACH CABLE MUST BE 2000 LB

$F_C \cos 25^\circ = 2000 \text{ LB} = F_{CV}$

$F_C = 2000/\cos 25^\circ = 2207 \text{ LB}$




$F_{CH} = F_C \sin 25^\circ$
$= 2207 \sin 25^\circ$
$F_{CH} = 933 \text{ LB}$

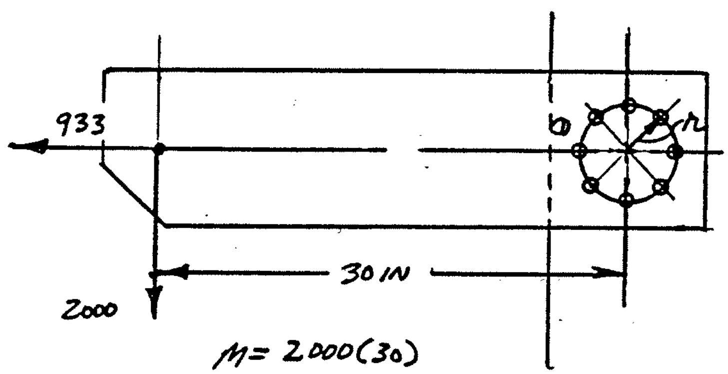



8 BOLTS, $r = 3.00$ IN
A325

$\Sigma r^2 = 8(3.0)^2 = 72 \text{ IN}^2$

$F_i = \dfrac{M r}{\Sigma r^2} = \dfrac{60000(3)}{72}$

$F_i = 2500 \text{ LB}$




$M = 2000(30)$
$= 60\,000 \text{ LB}\cdot\text{IN}$

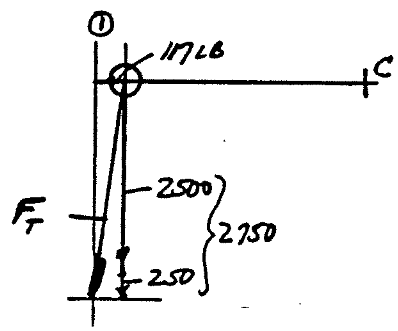
SHEAR-HORIZ. $F_1 = 933/8 = 117 \text{ LB} \leftarrow$

SHEAR-VERT. $F_2 = 2000/8 = 250 \text{ LB} \downarrow$

$F_T = \sqrt{2750^2 + 117^2} = 2753 \text{ LB}$

$A_s = \dfrac{F_T}{\tau_d} = \dfrac{2753}{17500} = 0.157 \text{ IN}^2 = \pi D^2/4$

$D = \sqrt{4A_s/\pi} = \sqrt{4(.157)/\pi} = 0.448 \text{ IN}$




USE 1/2-13 BOLTS

## WELDED JOINTS

**7.** FIGURE P20-4 DIRECT SHEAR; $f = V/A_w$

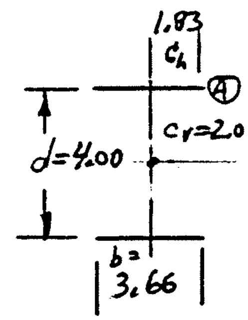
WELD BOTH VERTICAL SIDES, 4.0 IN. LONG.: $A_w = 2(4) = 8.0$ IN

$$f = \frac{V}{A_w} = \frac{12000\ LB}{8.0\ IN} = 1500\ LB/IN$$

FOR A36 STEEL AND E60 ELECTRODE; $f_a = 9600$ LB/IN/IN

$$w = \frac{f}{f_a} = \frac{1500\ LB/IN}{9600\ LB/IN/IN} = 0.156\ IN;\ \text{USE } w = 3/16\ \text{IN LEG.}$$

**8.** FROM PROBLEM 4 - AT EACH END OF THE BEAM, V=1500 LB
T = 32 873 LB·IN - SHARED EQUALLY ON FRONT AND BACK OF PLATES THAT ARE WELDED TO S7X15.3 COLUMN.
WELD ALONG TOP AND BOTTOM OF PLATE.




CASE 3 - FIG. 20-8: $A_w = 2b = 2(3.66) = 7.32$ IN.

$$J_w = (b^3 + 3bd^2)/6 = (3.66^2 + 3(3.66)(4.00)^2)/6$$

$J_w = 37.45\ IN^3$ TORSION

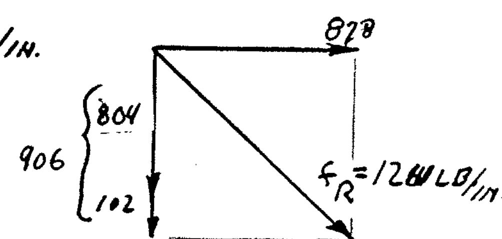
ON EACH WELD PATTERN: V = 750 LB, T = 16 436 LB·IN.

AT (A): $f_1 = \frac{V}{A_w} = \frac{750\ LB}{7.32\ IN} = 102\ LB/IN \downarrow$

$$f_2 = \frac{T c_v}{J_w} = \frac{(16436\ LB\cdot IN.)(2.0\ IN)}{37.45\ IN^3} = 878\ LB/IN \rightarrow$$

$$f_3 = \frac{T c_h}{J_w} = \frac{(16\,436)(1.83)}{37.45} = 804\ LB/IN \downarrow$$

VECTOR SUM OF FORCES: $F_R = 1211$ LB/IN.



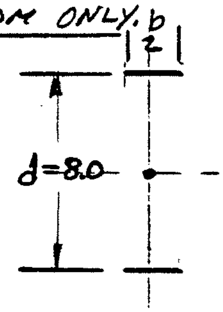

USE E60 ELECTRODE:

$$w = \frac{f_R}{f_a} = \frac{1211}{9600} = 0.131\ IN$$

USE $w = 3/16$ IN = 0.188 IN - MINIMUM FOR 1/2 IN PLATE.

9.

FIGURE P20-5 : TRIM SIDES SO THAT 2.00 IN EXTEND ONTO RIGID STRUCTURE. WELD TOP AND BOTTOM ONLY.




EACH SIDE CARRIES 1400 LB

TORQUE ON WELD : $T = 1400(8 + b/2)$

$$T = 1400\,(8+1) = 12600\ \text{LB}\cdot\text{IN}$$

$$A_w = 2b = 2(2) = 4.00\ \text{IN}$$

$$J_w = \frac{b^3 + 3bd^2}{6} = \frac{(2)^3 + 3(2)(8)^2}{6} = 65.3\ \text{IN}^3$$

CASE 3 - FIGURE 20-8

$$f_1 = \frac{V}{A_w} = \frac{1400\ \text{LB}}{4.00\ \text{IN}} = 350\ \text{LB/IN} \downarrow$$

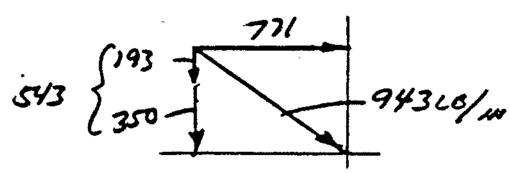
$$f_2 = \frac{T c_v}{J_w} = \frac{(12600\ \text{LB}\cdot\text{IN})(4.00)\ \text{IN}}{65.3\ \text{IN}^3} = 771\ \text{LB/IN} \rightarrow$$

$$f_3 = \frac{T c_h}{J_w} = \frac{(12600)(1.00)}{65.3} = 193\ \text{LB/IN} \downarrow$$

E60 ELECTRODE

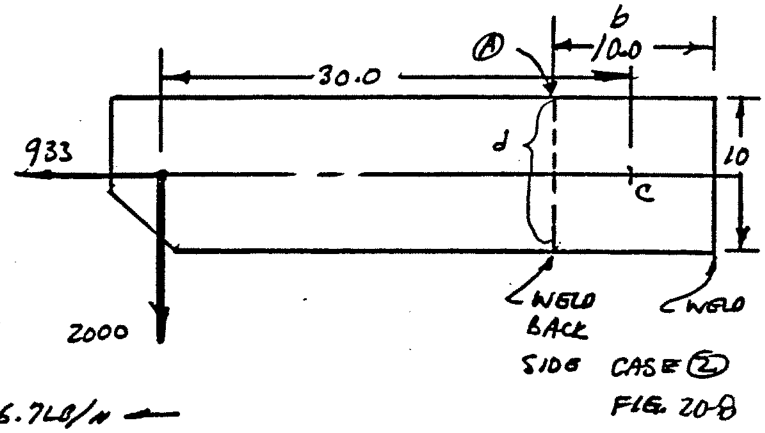
$$w = \frac{943}{9600} = 0.098\ \text{IN}$$

USE $w = 3/16'' = 0.188\ \text{IN}$ (MIN)



$\boxed{10.}$

FIG. P20-6




$A_w = 2(10) = 20\ IN$

$J_w = \dfrac{d(3b^2+d^2)}{6}$

$J_w = \dfrac{10(3(10)^2+10^2)}{6} = 667\ IN^3$

AT A

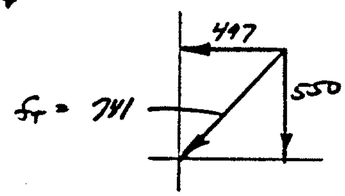
$f_1 = SHEAR = 933/20 = 46.7\ LB/IN \leftarrow$

$f_2 = SHEAR = 2000/20 = 100\ LB/IN \downarrow$

$f_3 = TORSION = \dfrac{T c_v}{J_w} = \dfrac{2000(30)(5)}{667} = 450\ LB/IN \leftarrow$

$f_4 = TORSION = \dfrac{T c_h}{J_w} = f_3 = 450\ LB/IN \downarrow$

$\left.\begin{array}{l} f_1 + f_3 = 497\ LB/IN \\ f_2 + f_4 = 550\ LB/IN \end{array}\right\} f_r = 741\ LB/IN$

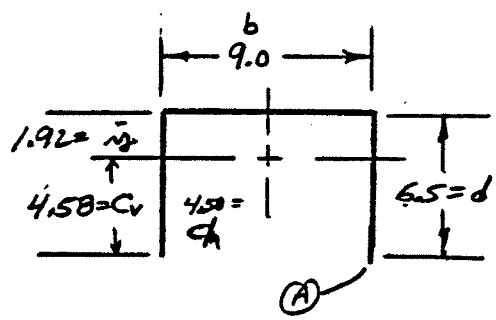



$w = \dfrac{f_r}{f_w} = \dfrac{741}{9600} = 0.077\ IN$ — USE $w = 3/16''$ (MIN)

$\boxed{11.}$

FIGURE P20-11

CASE 6, FIG. 20-8




$A_w = b + 2d = 9 + 2(6.5) = 22.0\ IN.$

$\bar{y} = \dfrac{d^2}{b+2d} = \dfrac{(6.5)^2}{9+2(6.5)} = 1.92\ IN.$

$S_w = \dfrac{d^2(2b+d)}{3(b+d)} = \dfrac{(6.5)^2(2(9)+6.5)}{3(9+6.5)} = 22.3\ IN^2$ (BOTTOM)

$J_w = \dfrac{(b+2d)^3}{12} - \dfrac{d^2(b+d)^2}{b+2d} = 887. - 461 = 426\ IN^3$

**11.** (CONTINUED) AT (A):

$f_1 = \text{SHEAR} = \frac{V}{A_W} = \frac{4200\text{ LB}}{22.0\text{ IN}} = 191\text{ LB/IN} \downarrow$

$f_2 = \text{BENDING} = \frac{M}{S_W} = \frac{4200(21.5)}{22.3} = 4056\text{ LB/IN}$ (INTO WALL)

$f_3 = \text{TORSION (HORIZ.)} = \frac{T c_v}{J_W} = \frac{1200(16.5)(4.58)}{426} = 213\text{ LB/IN} \leftarrow$

$f_4 = \text{TORSION (VERT)} = \frac{T c_h}{J_W} = \frac{1200(16.5)(4.50)}{426} = 209 \downarrow$

$f_1 + f_4 = 191 + 209 = 400\text{ LB/IN} \downarrow$

$\text{RESULTANT} = f_T = \sqrt{f_2^2 + f_3^2 + (f_1+f_4)^2} = \sqrt{4056^2 + 213^2 + 400^2}$

$f_T = 4081\text{ LB/IN}$ USE E70 ELECTRODE

$\text{REQ'D } w = \frac{4081}{11200} = 0.364\text{ IN}$ ; USE $3/8'' = w$

**12.** FIG P20-11 WITH $P_2 = 0$ ; NO TORSION

$f_1 = \text{SHEAR} = \frac{V}{A_W} = \frac{3000}{22.0} = 136\text{ LB/IN} \downarrow$

$f_2 = \text{BENDING} = \frac{M}{S_W} = \frac{3000(21.5)}{22.3} = 2892\text{ LB/IN}$ (INTO WALL)

$f_T = \sqrt{f_1^2 + f_2^2} = \sqrt{136^2 + 2892^2} = 2896\text{ LB/IN}$

USE E60 ELECTRODE:

$w = \frac{2896}{9600} = 0.302\text{ IN}$ — USE 5/16 IN, 0.312 IN.

$\boxed{13.}$ FIGURE P20-1 $\quad P = 4000$ LB ; 6061 ALUMINUM

DIRECT SHEAR – DATA FROM TABLE 19-2

$$\tau = \frac{P}{A_s} = \frac{P}{0.707\,w\,L} \qquad \longrightarrow = t \text{ THROAT WIDTH}$$

$A_s$ = SHEAR AREA = $(0.707\,w) \times$ LENGTH OF WELD

FOR 4043 FILLER ALLOY, $\tau_{ALLOW} = 5000$ PSI

LET $L = 2(4.00\text{ IN}) = 8.00$ IN (VERTICAL SIDES OF BRACKET)

$$\text{REQ'D } w = \frac{P}{0.707\,L\,\tau_{ALLOW}} = \frac{4000 \text{ LB}}{0.707(8.0)\text{IN}(5000\text{ LB/IN}^2)} = 0.141 \text{ IN.}$$

USE 3/16 IN. WELD (MIN).

$\boxed{14.}$ FIGURE P20-14 $\quad$ DIRECT SHEAR, 6061 ALUMINUM
4043 FILLER ALLOY

LENGTH OF WELD $= L = \pi D = \pi(4.50) = 14.1$ IN

$$\text{AS IN 13: } w = \frac{P}{.707\,L\,\tau_a} = \frac{1500}{(.707)(14.1)(5000)} = 0.030 \text{ IN.}$$

USE 3/16"

COULD ALSO CONSIDER PARTIAL WELD.

$\boxed{15}$ FIGURE P20-15 $\quad$ DIRECT SHEAR, 6063 ALUM.
4043 FILLER

LET $w = 3/16 = 0.188$ IN

SOLVE FOR $L$

$$L = \frac{P}{0.707\,w\,\tau_{ALLOW}} = \frac{800 \text{ LB}}{(.707)(.188)(5000)\text{ LB/IN}} = 1.20 \text{ IN.}$$

DISTRIBUTE PARTIAL WELDS TOTALING 1.20 IN EQUALLY ON BOTH SIDES OF TAB.

$\boxed{16.}$ FIGURE P20-16 DIRECT SHEAR, 3003 ALUMINUM
4043 FILLER ALLOY

AS IN PROBLEM 14:

$$\text{REQ'D. } w = \frac{P}{0.707\, L\, \tau_a}$$

$$\tau_a = 5000 \text{ PSI}$$

$$L = 2(\pi D) = 2\pi(2.0\text{ IN}) = 12.57 \text{ IN.} \quad \underline{\text{WELD ALL AROUND}}$$

$$w = \frac{10\,000 \text{ LB}}{(.707)(12.57)\text{IN}(5000 \text{ LB/IN}^2)} = 0.225 \text{ IN}$$

USE $\underline{w = 1/4 \text{ IN} = 0.250 \text{ IN}}$

$\boxed{17.}$ MATERIAL COMPARISON

$F = 4800 \text{ LB}$ ; $N = 2$ ; $\sigma_d = s_y/2$

REQ'D $A = F/\sigma_d$ ; $D = \sqrt{4A/\pi}$

COMPUTE WEIGHT PER INCH OF LENGTH

$V = A \times 1.0 \text{ IN}$

$W = \rho_w V$

| MATERIAL | $s_y$ (KSI) | $\sigma_d$ (KSI) | $A$ (IN²) | $D$ (IN) | $V$ (IN³) | $\rho_w$ (LB/IN³) | $W$ (LB) |
|---|---|---|---|---|---|---|---|
| a) 1020 HR | 30 | 15 | .32 | .638 | .32 | .283 | .0906 |
| b) 5160 OQT 1300 | 100 | 50 | .096 | .350 | .096 | .283 | .0272 |
| c) ALUM. 2014-T6 | 60 | 30 | .16 | .451 | .16 | .100 | .0160 |
| d) ALUM 7075-T6 | 73 | 36.5 | .132 | .409 | .132 | .100 | .0132 |
| e) Ti-6Al-4V (ANNEALED) | 120 | 60 | .080 | .319 | .080 | .160 | .0128 |
| f) Ti-3Al-13V-11Cr | 175 | 87.5 | .055 | .264 | .055 | .160 | .0088 |

# CHAPTER 21
# ELECTRIC MOTORS AND CONTROLS

Questions 1 - 8: See Sections 21-2 and 21-3.

9. Standard frequency for AC power in the U.S. is 60 hertz.

10. Standard frequency for AC power in Europe is 50 hertz.

11. Single phase AC power at 115 and 230 volts.

12. Two conductors plus a ground wire.

13. 480V, three phase is preferred because the current would be lower and the size of the motor would be smaller.

14. Synchronous speed is the speed at which an AC motor tends to run at zero load. $n_s = 120(f)/p$, where $f$ is the frequency of the power and $p$ is the number of poles in the motor.

15. Full-load speed is the speed of the motor when it is delivering its rated torque.

16. In U.S.: $n_s = 120(f)/p = 120(60)/4 = 1800$ rpm
In France: $n_s = 120(f)/p = 120(50)/4 = 1500$ rpm

17. 2-pole motor. Zero-load speed approximately 3600 rpm.

18. $n_s = 120(f)/p = 120(400)/4 = 12000$ rpm

19. Two speed motor; 1725 rpm and 1140 rpm

20. Variable frequency control

21,22,24 - See Section 21-8.

23. National Electrical Manufacturers Association

25. TEFC - Totally enclosed - fan-cooled. See Section 21-8.

26. TENV - Totally enclosed - non-ventilated. Section 21-8.

27. NEMA Design 9 - Hazardous locations. Flour can explode.

28. TENV because motor may get bathed in water during cleaning and to protect food from contaminants from the motor.

30. Locked rotor torque is the torque that a motor can exert when the rotor is at rest. Also called starting torque.

31. A poorer speed regulation means that the motor would slow down more when subjected to an increase in torque.

32. Breakdown torque is the maximum torque a motor can develop during the increase in speed after start or the torque at which a motor would be stalled if the torque is increased after it is running.

33. Split-phase; capacitor-start; permanent-split capacitor; shaded pole.

34. a) Single phase, split-phase AC motor because of the moderate starting torque and the change of torque when the switch cuts out the starting winding.

b) From Table 21-2, full-load speed = 1140 rpm
$T = 63000(P)/n = 63000(.75)/1140 = 41.4$ lb-in
c) Starting torque = 150%(F.L. torque)
$T_s = 1.5(41.4) = 62.2$ lb-in (approximate)
d) Breakdown = 350%(F.L. torque)
$T_b = 3.5(41.4) = 145$ lb-in (approximate)

35. 2-pole; 1.50 kW rated power.

b) From Table 21-2, full-load speed = 3450 rpm
$n = (3450\ \text{rev/min})(2\pi\ \text{rad/rev})(1\ \text{min}/60\ \text{s}) = 361\ \text{rad/s}$
$T = P/n = (1.5\text{x}10^3\ \text{N-m/s})/(361\ \text{rad/s}) = 4.15\ \text{N-m}$
c) Starting torque = 1.5(4.15 N-m) = 6.23 N-m
d) Breakdown torque = 3.5(4.15 N-m) = 14.5 N-m

36. Fan requires about 18 lb-in of torque at 1725 rpm; low starting torque; assume fan cools motor. Recommend 4-pole, single phase permanent split capacitor AC motor.
Power = Tn/63000 = (18)(1725)/63000 = 0.49 hp (use 1/2 hp)

37. Full load torque about 0.5 N-m at 3450 rpm; high starting torque (about 2.8xF.L.T.) due to starting compressor against high pressure in the system. Recommend capacitor start, single phase, 2-pole, AC motor.
$n = (3450\ \text{rev/min})(2\pi\ \text{rad/rev})(1\ \text{min}/60\ \text{s}) = 361\ \text{rad/s}$
$P = Tn = (0.5\ \text{N-m})(361\ \text{rad/s}) = 181\ \text{N-m/s} = 181$ watts

38. Speed is adjusted by varying the resistance in the rotor circuit through an external resistance control.

39. F.L. speed = synchronous speed = 720 rpm. (Table 21-2)

40. Pull-out torque is the torque that would disengage the motor from its synchronous speed and cause it to stop.

41. Universal motors are very small and light weight for a given power rating. Some vacuums, appliances, and hand tools utilize the high speed of rotation effectively.

42. A universal motor can operate on DC or almost any frequency of AC voltage when operating near its full-load point.

43. Batteries, generators, rectified AC, HYDROGEN FUEL CELLS.

44. See Table 21-7.

45. SCR - Silicon controlled rectifier. Used to produce DC power from AC.

46. SCR controls do not produce pure DC power; it has some variation, called ripple, due to the AC input. A low-ripple control would produce a nearly true DC power.

47. Could use a 90V DC motor powered from a NEMA Type K SCR power supply to convert 115 V AC to 90V DC power.

48-50. See Section 21-11.

51. The motor would speed up without limit and may fail catastrophically.

52. Speed is proportional to torque. $T_2 = T_1\ (n_1/n_2)$
$T_2 = (15.0\text{ N-m})(3000/2200) = 20.5\text{ N-m}$

53, 56-61. See Sections 21-9, 21-11, and 21-12.

54. NEMA Size 2 motor starter for 10 hp, 220V AC, 3-phase.

55. NEMA Size 1 motor starter for 1.0 kW, 110V AC, single phase.

# CHAPTER 22
## MOTION CONTROL: CLUTCHES AND BRAKES

**1.** FROM EQ. 22-1: $T = CPK/n = (63025)(5.0)(2.75)/1750 = \underline{495 \text{ LB}\cdot\text{IN}}$

C AND K FROM SECTION 22-4.

SUMMARY OF RESULTS FOR PROBLEMS 2-7.

| PROB. | C | P | K | n | TORQUE |
|---|---|---|---|---|---|
| 2 | 5252 | 75 hp | 5.0 | 2500 | 788 LB·FT |
| 3 | 63025 | 0.50 hp | 1.5 | 1150 | 41 LB·IN |
| 4 | 63025 | 5.0 hp | 2.75 | 180 | 4814 LB·IN |
| 5-1 | 63025 | 5.0 hp | 1.0 | 1750 | 180 LB·IN. |
| 5-2 | 5252 | 75 hp | 1.0 | 2500 | 158 LB·FT |
| 5-3 | 63025 | 0.50 hp | 1.0 | 1150 | 27.4 LB·IN |
| 5-4 | 63025 | 5.0 hp | 1.0 | 180 | 1751 LB·IN |
| 6 | 9549 | 20 kW | 2.75 | 3450 | 152 N·m |
| 7(a) | 9549 | 50 kW | 4.0 | 900 | 2122 N·m CLUTCH |
| (b) | 9549 | 50 kW | 1.0 | 900 | 531 N·m BRAKE |

**8.** DISK: D = 24.0 IN; $R_1$ = 12.0 IN; L = 2.50 IN; $R_2$ = 0; STEEL

$$Wk^2 = \frac{L(R_1^4 - R_2^4)}{323.9} = \frac{2.50(12.0^4 - 0)}{323.9} \text{ LB-FT}^2 = 160.0 \text{ LB-FT}^2$$

$$T = \frac{Wk^2(\Delta n)}{308t} = \frac{(160)(550)}{308(2.0)} = 142.9 \text{ LB-FT}$$

**9.**

| | $R_1$ | $R_2$ | L | $Wk^2$ |
|---|---|---|---|---|
| SHAFT | 0.625 | 0 | 16.0 | 0.00754 |
| COUPLING | 1.50 | .625 | 2.25 | 0.0341 |
| BRG. 1 | 1.00 | .625 | 1.80 | 0.0047 |
| HUB | 2.00 | .625 | 1.00 | 0.0489 |
| GEAR | 6.00 | .625 | 3.00 | 12.0023 |
| BRG 2 | 1.00 | .625 | 1.80 | 0.0047 |
| | | | TOTAL | 12.102 LB-FT² |

$$T = \frac{Wk^2(\Delta n)}{308t} = \frac{(12.102)(775)}{308(1.50)} = \underline{60.9 \text{ LB-FT}}$$

$\boxed{10.}$

NEGLECT CLUTCH AND SHORT SHAFT BETWEEN CLUTCH AND GEAR A.

SPEED OF SHAFT 2: $n_2 = 1750\,(4.00/15.00) = 466.7$ RPM

IN EQ. 22-4 $(n/n_e)^2 = (n_2/n_1)^2 = (466.7/1750)^2 = 0.0711$

| | $R_1$ | $R_2$ | $L$ | $Wk^2$ | $Wk^2_e$ |
|---|---|---|---|---|---|
| GEAR A | 2.00 | 0 | 3.00 | 0.1482 | 0.1482 |
| GEAR B | 7.50 | 1.25 | 3.00 | 29.2833 | 2.0824 |
| SHAFT 2 | 1.25 | 0 | 54.0 | 0.4070 | 0.0289 |
| HUB 1 | 2.50 | 1.25 | 5.0 | 0.5653 | 0.0402 |
| HUB 2 | 2.50 | 1.25 | 5.0 | 0.5653 | 0.0402 |
| END PLATES | 9.00 | 1.25 | 2.00 | 40.4974 | 2.8798 |
| HOLLOW CYL | 9.00 | 7.50 | 30.0 | 314.6284 | 22.3736 |
| | | | TOTAL | | 27.5933 LB-FT² |

$$T = \frac{Wk^2_e(\Delta n)}{308\,t} = \frac{(27.5933)(1750)}{308(1.50)} = 104.5 \text{ LB-FT}$$

$\boxed{11.}$

LOAD SPEED = 50 FT/MIN = V

$$\text{DRUM SPEED} = \frac{V}{R} = \frac{50\text{ FT}}{\text{MIN}} \times \frac{1}{4.00\text{ IN}} \times \frac{12\text{ IN}}{\text{FT}} = 150 \text{ RAD/MIN} = \omega$$

$$\text{RPM OF DRUM} = n = \frac{150\text{ RAD}}{\text{MIN}} \times \frac{\text{REV}}{2\pi\text{ RAD}} = 23.87 \text{ RPM}$$

| | $R_1$ | $R_2$ | $L$ | $Wk^2$ |
|---|---|---|---|---|
| SHAFT | 0.75 | 0 | 24.0 | 0.0234 |
| END PLATES | 4.00 | .75 | 3.0 | 2.3682 |
| HOLLOW CYL. | 4.00 | 3.00 | 13.0 | 7.0238 |
| | | | TOTAL | 9.4154 LB-FT² |

$$\text{LOAD: } Wk^2_e = W\left(\frac{V}{\omega}\right)^2 = 600\text{ LB}\left(\frac{50\text{ FT/MIN}}{150\text{ RAD/MIN}}\right)^2 = 66.6667 \text{ LB-FT}^2$$

TOTAL $Wk^2 = 9.4154 + 66.6667 = 76.08$ LB-FT²

$$\text{BRAKING TORQUE} = \frac{Wk^2(\Delta n)}{308\,t} = \frac{(76.08)(23.87)}{308(0.25)} = 23.6 \text{ LB-FT}$$

ADDITIONAL TORQUE TO HOLD LOAD = 600 LB(4 IN)(FT/12 IN) = 200 LB-FT

TOTAL BRAKE TORQUE = 223.6 LB-FT

12. (a) CLUTCH ON MOTOR SHAFT:

MOTOR SPEED = CLUTCH SPEED = 1150 RPM = $n_c$

BARREL SPEED = 38 RPM = $n$

$(n/n_c)^2 = (38/1150)^2 = 0.001092$

| | $R_1$ | $R_2$ | $L$ | $Wk^2$ | $Wk_e^2$ |
|---|---|---|---|---|---|
| WORM | 1.75 | 0 | 8.00 | 0.2316 | 0.2316 |
| WORMGEAR | 8.00 | 0 | 2.50 | 31.6147 | 0.0344 |
| 2 HUBS | 4.00 | 2.00 | 4.00 | 2.9639 | 0.0032 |
| END PLATES | 14.00 | 2.00 | 4.00 | 474.2204 | 0.5178 |
| HOLLOW CYL. | 14.00 | 12.00 | 18.00 | 982.5255 | 1.0728 |
| SHAFT | 2.00 | 0 | 36.0 | 1.7783 | 0.0019 |
| | | | | | 1.8617 LB-FT$^2$ |

$$T = \frac{Wk_e^2(\Delta n)}{308t} = \frac{(1.8617)(1150)}{308(2.0)} = \underline{\underline{3.48 \text{ LB-FT}}}$$

(b) CLUTCH ON WORMGEAR SHAFT. ONLY BARREL AND HUBS ARE ACCELERATED TO 38 RPM = CLUTCH SPEED.

| | $Wk^2$ |
|---|---|
| 2 HUBS | 2.9639 |
| END PLATES | 474.2204 |
| HOLLOW CYL. | 982.5255 |
| SHAFT | 1.7783 |
| TOTAL | 1461.5 LB-FT$^2$ |

$$T = \frac{Wk^2(\Delta n)}{308\,t}$$

$$T = \frac{(1461.5)(38)}{308(2.0)} = \underline{90.2 \text{ LB-FT.}}$$

13. $T_f = fNR_m$; $\underline{R_m} = T_f/fN = (75 \text{ LB}\cdot\text{IN})/(.25)(150)\text{ LB} = \underline{2.00 \text{ IN}}$

$\underline{P_f} = \dfrac{T_f n}{63000}$ hp $= \dfrac{(75)(1150)}{63000} = \underline{1.37 \text{ hp}}$

LET $WR = 0.10 \text{ hp/in}^2 = P_f/A$

$\underline{A} = P_f/WR = 1.37 \text{ hp}/0.10 \text{ hp/in}^2 = \underline{13.7 \text{ IN}^2}$

$A = \pi(R_o^2 - R_i^2)$, $R_m = (R_o + R_i)/2$ OR $2R_m = R_o + R_i$

TRY $R_o = 1.5R_i$ : $2R_m = 1.5R_i + R_i = 2.5R_i$; $R_i = \dfrac{2R_m}{2.5} = \dfrac{2.00}{1.25} = 1.60 \text{ in}$

$R_o = 1.5(1.60) = 2.40 \text{ in}$; $A = \pi(2.40^2 - 1.60^2) = \underset{\text{LOW}}{10.05}$

SIMILARLY, FOR $\underline{R_o = 1.75R_i}$

$\underline{R_i = 1.45 \text{ in}}$; $\underline{R_o = 2.55 \text{ in}}$; $\underline{A = 13.75 \text{ in}^2 \text{ OK}}$

**14.** FROM PROB. 9: $T_f = 64$ LB·IN; TRY $R_m = 1.50$ IN; $f = 0.25$

$$N = \frac{T_f}{f R_m} = \frac{64.0}{(.25)(1.50)} = 170 \text{ LB}$$

$$P_f = \frac{T_f\, n}{63000} = \frac{(64)(775)}{63000} = 0.79 \text{ hp}$$

FOR $WR = 0.10$ hp/in²; $A = P_f/WR = 7.9$ in²

TRY $R_o = 1.50 R_i$; $R_m = (R_o + R_i)/2$

$2R_m = 1.5R_i + R_i = 2.5R_i$

$R_i = R_m/1.25 = 1.50$ in$/1.25 = 1.20$ IN; $R_o = 1.5(1.2) = 1.80$ in

$A = \pi(R_o^2 - R_i^2) = 5.65$ IN² (LOW)

TRY $R_o = 2.0 R_i$; $2R_m = 2.0R_i + R_i = 3.0R_i$; $R_i = R_m/1.5 = 1.0$ in

$R_o = 2.0(1.0) = 2.0$ in; $A = \pi(2.0^2 - 1.0^2) = 9.42$ in² OK

**15.** FROM EQ. 22-13:

$$F_a = \frac{T_f(\sin\alpha + f\cos\alpha)}{f R_m} = \frac{15 \text{ lb·ft}(\sin 12° + .25\cos 12°)}{(0.25)(3.0 \text{ in})} \times \frac{12 \text{ in}}{\text{ft}}$$

$$F_a = 109 \text{ lb}$$

**16.** $T_f = 64$ lb·in; Let $f = 0.25$; $R_m = 2.0$ in; $\alpha = 12°$

$$F_a = \frac{(64)(\sin 12° + .25\cos 12°)}{(.25)(1.0)} = 116 \text{ lb}$$

**17.** $T_f = 150$ lb·ft $\times$ 12 in/ft $= 1800$ lb·in $= F_f D_o/2$

$F_f = 2T_f/D_o = 2(1800)/12.0 = 300$ lb

$$W = \frac{F_f\left(\frac{a}{f} - b\right)}{L} = \frac{(300 \text{ lb})\left(\frac{4.0}{.25} - 5.0\right)\text{in}}{24.0 \text{ in}} = 138 \text{ lb}$$

**18.** FOR SELF ACTUATION; $W < 0$.

FOR $W = 0$, $a/f - b = 0$ OR $b = a/f = 4.0 \text{ in}/0.25 = 16.0$ IN

$b > 16.0$ IN FOR SELF ACTUATION

19. $T_f = 100\ LB\cdot FT \times 12\ IN/FT = 1200\ LB\cdot IN$

TRY $\underline{D_0 = 10.0\ IN}$; $\underline{f = 0.25}$

$$\underline{F_f} = \frac{2T_f}{D_0} = \frac{2(1200)}{10} = \underline{240\ LB}$$

IN FIG 22-17 (C); LET $\underline{a = 2.0\ IN}$; $\underline{b = 6.00\ IN}$; $\underline{f = 0.25}$; $\underline{L = 18\ IN}$

$$\underline{W} = \frac{F_f\left(\frac{a}{f} - b\right)}{L} = \frac{240\left(\frac{2.0}{.25} - 6.0\right)}{18} = \underline{26.7\ LB}$$

20. $T_f = 100\ LB\cdot FT \times 12\ IN/FT = 1200\ LB\cdot IN$

SELECT WOVEN ASBESTOS; $f = .25$; $p \approx 30\ psi$

IN FIG 22-19: $r = 6.0\ IN$; $C = 9.00\ IN$; $L = 20.0\ IN$

$\theta_1 = 45°$; $\theta_2 = 135°$

FROM EQ. 22-18:

$$w = \frac{T_f}{r^2 f\, p(\cos\theta_1 - \cos\theta_2)} = \frac{1200\ LB\cdot IN}{(6.0)^2(0.25)(30)(\cos 45° - \cos 135°)} = 3.14\ IN$$

USE $w = 3.25\ IN.$; $P = 30\ psi\ (3.14/3.25) = 29.0\ psi$

$$\theta_2 - \theta_1 = 135° - 45° = 90° \times \pi/180 = \pi/2 = 1.57\ rad.$$

(EQ. 22-20)

$$M_N = 0.25(29.0)(3.25)(6.0)(9.0)[2(1.57) - \sin 270° + \sin 90°]$$

$$M_N = 6540\ lb\text{-}in$$

(EQ. 22-21)

$$M_f = -(.25)(29.0)(3.25)(6.0)[6.0(\cos 45 - \cos 135) + (.25)(9.0)(\cos 270° - \cos 90°)]$$

$$M_f = -1200\ lb\cdot IN$$

(EQ. 22-19)

$$W = (M_N - M_f)/L = (6540 - 1200)/20 = 267\ LB = \text{ACTIVATION FORCE}$$

CHECK WEAR RATIO

$$P_f = T_f n/63000 = 1200(480)/63000 = 9.14\ hp$$

(EQ. 22-23)

$$A = 2wr \sin\left(\frac{\theta_2 - \theta_1}{2}\right) = 2(3.25)(6.0)\sin 45° = 27.6\ in^2$$

$$WR = \frac{P_f}{A} = \frac{9.14\ hp}{27.6\ in^2} = 0.33\ hp/in^2$$ SOMEWHAT HIGH — INTERMITTENT SERVICE ONLY.

$\boxed{21.}$ BAND BRAKE: $T_f = 75\,LB \cdot FT\,(12\,in/FT) = 900\,LB \cdot IN$; $n = 350\,RPM$
USE WOVEN ASBESTOS, $p_{MAX} = 25.0\,psi$; $f = 0.25$
TRY $r = 6.0\,IN$; $\theta = 210°\ (3.67\,RAD)$; $w = 2.5\,IN$

$$P_1 = p_{MAX}\, r\, w = (25.0)(6.0)(2.5) = 375\,LB$$

$$P_2 = \frac{P_1}{e^{f\theta}} = \frac{375\,lb}{e^{(.25)(3.67)}} = 150\,LB$$

$$T_f = (P_1 - P_2)r = (375 - 150)(6.0) = 1350\,LB \cdot IN\ (HIGH)$$

TRY $r = 5.50\,IN$; $\theta = 210°$; $w = 2.0\,IN$

$$P_1 = (25.0)(5.50)(2.0) = 275\,LB$$

$$P_2 = \frac{275}{e^{(.25)(3.67)}} = 110\,LB$$

$$T_f = (275 - 110)(5.5) = 908\,LB \cdot IN\ \underline{OK}$$

FOR SIMPLE BAND BRAKE: LET $a = 5.50\,IN$; $L = 12.0\,IN$

$$W = P_2\,(a/L) = 110(5.5/12) = 50.4\,lb$$

WEAR RATIO:

$$A = 2\pi r w \frac{\theta}{360} = 2\pi(5.5)(2.0)\frac{210}{360} = 40.3\,in^2$$

$$P_f = \frac{T_f n}{63000} = \frac{(908)(350)}{63000} = 5.04\,hp$$

$$WR = P_f/A = 5.04/40.3 = 0.125\,hp/in^2\ \underline{OK}$$